Generation Life Tables for Czech Pension Funds

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Abstract

The system of pension funds with the governmental support exists in the Czech Republic since 1994 and nowadays it has nearly 4.5 mio. participants. The funds can face a serious problem of underestimation of the reserves since the calculations based on the Life Tables (LT) published for the Czech republic do not correspond to the actual development in the portfolios of participants. Namely, the LT do not take into account (1) the substantial improvement of mortality anticipated for the Czech Republic in comparison with its previous level and (2) the selection effect consisting in the increased interest of "healthy" persons in this voluntary insurance. The paper takes the generation LT, which have been constructed for the Czech Pension Funds (see Cipra (1998)) according to the approach used in Germany respecting these facts. One shows some consequences for annuity prices and compares the results if either the generation LT or the (current) LT are applied.

1. Introduction

The system of pension funds with the governmental support in the Czech Republic presents supplementary voluntary (contribution defined) pension scheme supported by the state. The governmental support consists in the state contributions added quarterly to the participant contributions and in a tax relief for participants and contributing employers.

Nowadays, the Czech pension funds are successful since they offer very advantageous and profitable form of individual savings just due to the governmental support and tax relief.

One of the potential problems of the Czech pension funds consists in demography, and this paper concentrates explicitly on it. The pension funds must create technical reserves for future payments and they can underestimate these reserves substantially because the considered time horizons are long and the calculations based on the current LT do not correspond to the actual development in the portfolios of participants, i.e. to (1) the improvement of mortality in time and to (2) the selection effect:

(1) The well-known fact in developed countries is that the male and female mortality improves in time (usually in the whole age range). If the symbol $q_x(t)$ ($q_y(t)$, respectively) denotes the probability of male death in age x and time t (the probability of female death in age y and time t, respectively) then one can see that these changes in the Czech Republic (especially after 1990) are very radical, e.g.

male: $q_{50}(2008) = 0.006\,122 = 6.1$ ‰
$q_{50}(1990) = 10.3$ ‰
$q_{50}(1930) = 12.9$ ‰

$q_{70}(2008) = 33.8$ ‰
$q_{70}(1990) = 58.8$ ‰
$q_{70}(1930) = 59.5$ ‰

female: $q_{50}(2008) = 0.002\,916 = 2.9$ ‰
$q_{50}(1990) = 3.7$ ‰
$q_{50}(1930) = 9.1$ ‰

$q_{70}(2008) = 15.9$ ‰
$q_{70}(1990) = 34.6$ ‰
$q_{70}(1930) = 52.1$ ‰

Therefore the corresponding risk of underestimation of mortality for insurers is substantial.

(2) Another factor is the selection effect in portfolios of participants. The voluntary pension system is attractive for persons without subjective health problems so that the average length of life in such portfolios is higher than the one in the global population.

In order to reduce such problems the pension funds can modify their LT using

(1) projections either in the classical LT (this approach is simple but inaccurate) or in the generation LT (in the generation LT the probabilities of death depend not only on age but also on time, see the symbols $q_x(t)$ and $q_y(t)$);

(2) selection coefficients for pension portfolios (these coefficients describe the ratio of the mortality in pension portfolios to the mortality in global population).

Moreover, two different conceptions are possible:

(a) the pension funds construct their LT using own data;

(b) or they modify the global LT (i.e. the LT for global population) respecting (1) and (2).

In this paper the generation LT for Czech pension funds constructed using the conception (b) by means of suitable modifications of the global LT by the Czech Statistical Office (CZSO) are presented (see also Cipra (1998)).. The approach is similar to the one used in the German life tables DAV-Sterbetafel 1994 R suggested just for the pension insurance (see e.g. Lühr (1986) and Schmithals and Schütz (1995)). Some consequences for annuity prices are shown, e.g. one compares the results if either the generation LT or the (current) LT 2008 (CZSO) are applied. The fact that the generation LT have been constructed so early as in 1996 enables to compare their projections with the reality. Other works can be mentioned in this context (e.g. Khalaf-Allah, Haberman and Verrall (1996), Koissi (2006)).

2. Projection and selection effect in generation LT

The construction of the generation LT performed in the year 1996 is based on the classical LT for the Czech region for periods 1899/1902, 1909/1912, 1920/1922, 1929/1932, 1949/1951, 1960/1961, 1971, 1981, 1990, 1991, 1992, 1993, 1994, 1995, 1996 with the age range 0, 1, ..., 103 (the data for the highest ages of this range have been achieved in some cases by means of extrapolations).

2.1. Regression lines for logarithmic probabilities of death. In time graphs of $\ln q_x(t)$ and $\ln q_y(t)$ for particular ages one can use linear regression since the logarithmic transformation discovers decreasing linear trends (see Fig. 1a and 1b for ages 30, 40, 50, 60, 70, 80). The corresponding regression lines can be written for a fixed male age x as

$$\ln q_x(t) \sim B(x) - F(x) \cdot t, \quad t = 1900, \dots, 1996 \tag{2.1}$$

(similarly for a fixed female age y) and can be estimated using the data. E.g. for the male age 60 (the female age 60, respectively) one obtains

$$\ln q_{60}(t) \sim 4.131\,07 - 0.003\,98\, t \quad (\ln q_{60}(t) \sim 20.193\,85 - 0.012\,49\, t, \text{ resp.}), \qquad t = 1900, \dots, 1996.$$

If we confine ourselves only to seven years 1990, 1991, ..., 1996 then the estimates are different, e.g. again for the male age 60 (the female age 60, respectively)

$$\ln q_{60}(t) \sim 72.989\,94 - 0.038\,53\,t \quad (\ln q_{60}(t) \sim 36.872\,09 - 0.020\,85\,t, \text{resp.}), \quad t = 1990, \ldots, 1996.$$

The estimated coefficients $F(x)$ obtained for the long period 1900-96 and for the short period 1990-96 are shown graphically in Fig. 2a (similarly in Fig. 2b for females). The short estimates show high volatility but on the other hand they have higher impact in comparison with the ones based on older data before 1990. Therefore we have used the regression coefficients $F(x)$ estimated for the long period 1900-96 and multiplied by suitable coefficients r_x so that the values $r_x F(x)$ lie approximately between the long and short estimates $F(x)$ (similarly for females). The coefficients r_x have been chosen for males as

$$r_x = \begin{cases} 1.1 & , \ 0 \le x \le 30, \\ 1.1 + 0.1\,(x - 30) & , 31 \le x \le 59, \\ 4 & , 60 \le x \le 95, \\ r_x F(x) = r_{95} F(95) & , 96 \le x \le 103 \end{cases} \tag{2.2}$$

and for females as

$$r_y = \begin{cases} 1.1 & , \ 0 \le y \le 50, \\ 1.1 + 0.05\,(y - 50) & , 51 \le y \le 60, \\ 1.6 + 0.1\,(y - 60) & , 61 \le y \le 84, \\ 4 & , 85 \le y \le 90, \\ r_y F(y) = r_x F(x) & , 91 \le y \le 103. \end{cases} \tag{2.3}$$

The final modification of $r_x F(x)$ and $r_y F(y)$ to the final form $G(x)$ and $G(y)$ respects the following objectives:

– the sequences $G(x)$ and $G(y)$ should be non-increasing for increasing age so that the probabilities of death corresponding to the model (2.1) do not decrease for increasing age;
– the sequences $G(x)$ and $G(y)$ should satisfy $G(x) \le G(y)$ so that the female mortality stays lower than the male mortality.

The values $r_x F(x)$ and $G(x)$ are shown graphically in Fig. 2a (similarly in Fig. 2b for females).

2.2. Choice of basic LT and modification due to selection effect. The basic LT with probabilities of death q_x^B and q_y^B will be the basis for construction of projections that are necessary in the framework of the final generation LT. For this purpose, the classical LT for the period 1996 in the Czech Republic with probabilities of death $q_x(1996)$ and $q_y(1996)$ have been used.

As the selection effect is concerned the corresponding reduction coefficients f_x and f_y respecting this effect should be estimated using data of particular pension funds by means of the formula

$$f_x = \frac{\sum_t T_x(t)}{\sum_t q_x(t)\, L_x(t)}, \tag{2.4}$$

where $T_x(t)$ is the observed number of male deaths in age x and calendar year t, $L_x(t)$ is the observed number of male lives in age x and calendar year t and $q_x(t)$ is the probability of death in age x and calendar year t from the official LT (similarly for the female coefficients f_y). In the formulas of the type of (2.4) we obviously form ratios of the observed mortality in the given pension fund and the mortality for the global population.

Due to lack of data we have used for our purpose the reduction coefficients estimated by means of the data from the reinsurance company Munich Re (on the whole 367 000 “male-years“ and 442 000 “female-years“ in the period 1967-1992):

$$f_x = f_y = \begin{cases} 0.9 & , \quad 0 \le x \le 20, \\ 0.9 - 0.01\,(x-20) & , \; 21 \le x \le 29, \\ 0.8 & , \; 30 \le x \le 50, \\ 0.8 - 0.02\,(x-50) & , \; 51 \le x \le 59, \\ 0.6 & , \; 60 \le x \le 65, \\ 0.6 + 0.015\,(x-65) & , \; 66 \le x \le 74, \\ 0.75 & , \; 75 \le x \le 103. \end{cases} \tag{2.5}$$

The reduced probabilities of death $f_x \cdot q_x(1996)$ which are auxiliary values for the construction of the basic LT are shown graphically in Fig. 3a (similarly in Fig. 3b for females).

2.3. Safety charge. The last step in the construction of the basic LT is a safety charge respecting the risk of statistical estimation. One of possible approaches is the one suggested by Canadian actuaries (see Canadian Institute of Actuaries (1990)). According to this approach the basic LT should be

$$q_x^B = f_x q_x(1996) - \frac{c}{e_x(1996)}, \tag{2.6}$$

where $e_x(1996)$ is the average length of life in age x and calendar year 1996 and c is a suitable constant (one recommends the value 0.015 for a very strict safety charge while the most benevolent value in practice can be 0.003 75). The safety charge (2.6) has several useful properties, e.g. it increases absolutely but decreases relatively with respect to $f_x\, q_x(1996)$ for increasing age.

We have used another approach: the safety charge s_x^{α} in the relation

$$q_x^B = f_x q_x(1996) - s_x^{\alpha} \tag{2.7}$$

is constructed in such a way that the model with a portfolio of a medium size fulfils

$$\mathrm{P}\left(T \geq \sum_x (f_x q_x(1996) - s_x^{\alpha}) L_x \right) = 1 - \alpha\,. \tag{2.8}$$

The value $1 - \alpha$ is the prescribed confidence level for the safety charge, L_x is the observed number of male lives in age x, T_x is the observed number of male deaths in age x and

$$T = \sum_x T_x \tag{2.9}$$

is the total number of deaths in the model (if one uses such a safety charge with confidence level $1 - \alpha$ then the number of deaths in the model presents the lower bound for the observed number of deaths).

The safety charge s_x^{α} with the property (2.8) can be derived in the following way: T_x are random variables with binomial distribution

$$T_x \sim \mathrm{Bi}\,(L_x, q_x)\,.$$

Therefore the probability distribution of the random variable T in (2.9) can be approximated by the normal one

$$T = \sum T_x \sim \mathrm{N}\,(\mathrm{E}(T), \mathrm{var}(T)),$$

where

$$\mathrm{E}(T) = \sum L_x q_x \ , \quad \mathrm{var}(T) = \sum L_x q_x (1 - q_x).$$

Therefore it holds

$$\mathrm{P}\left(\frac{T - \mathrm{E}(T)}{\mathrm{var}(T)} \geq -u_{1-\alpha} \right) = 1 - \alpha$$

where $u_{1-\alpha}$ is the corresponding normal quantile. After some arrangements one obtains

$$\mathrm{P}\left\{ T \geq \sum_x L_x \left(f_x q_x(1996) - u_{1-\alpha} \frac{\sqrt{\mathrm{var}(T)}}{\sum \sqrt{\mathrm{var}(T_x)}} \sqrt{\frac{q_x(1-q_x)}{L_x}} \right) \right\} = 1 - \alpha\,.$$

Hence the safety charge can be taken as

$$s_x^{\alpha} = u_{1-\alpha} \frac{\sqrt{\mathrm{var}(T)}}{\sum \sqrt{\mathrm{var}(T_x)}} \sqrt{\frac{q_x(1-q_x)}{L_x}}\,. \tag{2.10}$$

In our case we have used the value $\alpha = 0.01$ (i.e. the safety charge is constructed with the confidence level 99 per cent so that one uses $u_{0.99} = 2.326$) and the model with the same age structure as in the classical LT for the period 1996 in the Czech Republic. Moreover the model size has been chosen so that e.g. the number of males in age above 59 is approximately 11 000 and the number of females in age above 59 is approximately 16 000).

The resulting basic probabilities of death q_x^B are given in the Tab. 1a and Fig. 3a (for females in the Tab. 1b and Fig. 3b).

2.4. Construction of generation LT. Now the construction of the generation LT is obvious: the probabilities of death $q_x(t)$ in age x and calendar year t can be calculated as

$$q_x(t) = \mathrm{e}^{-G(x)(t-1996)} q_x^B \tag{2.11}$$

since (2.11) is equivalent to the relation $\ln q_x(t) = \ln q_x{}^B - G(x)(t - 1996)$ with the basic probabilities of death $q_x{}^B$ corresponding to calendar year 1996 (one has to shift only the time origin and to use the coefficients $G(x)$ instead of $F(x)$ in the model (2.1)). The probabilities of death $q_y(t)$ in age y and calendar year t for females are calculated similarly.

It is more comfortable to rewrite the formulas of the type (2.11) to the explicit generation form

$$q_x^\tau = \mathrm{e}^{-G(x)(x+\tau-1996)} q_x^B \tag{2.12}$$

where $q_x{}^\tau$ is the probability of death of a male from generation τ in age x. The values $G(x)$ and $q_x{}^B$, which are necessary for application of this formula, are given in Tab. 1a (similarly in Tab. 1b for females).

2.5. An exampe. The application of the generation LT can be demonstrated by means of the example with a male from generation 1936 and his life annuity starting in age 60 (i.e. in year 1996).

In this case we use the formula (2.12) for τ = 1936 and for particular ages x = 60, 61, ... , i.e.

$$q_x^{1936} = \mathrm{e}^{-G(x)(x+1936-1996)} q_x^B = \mathrm{e}^{-G(x)(x-60)} q_x^B \ , \quad x = 60, 61, \dots .$$

We'll obtain

$$q_{60}^{1936} = \mathrm{e}^{-G(60)(60-60)} q_{60}^B = q_{60}^B = 0.011\,316 \, ,$$
$$q_{61}^{1936} = \mathrm{e}^{-G(61)(61-60)} q_{61}^B = \mathrm{e}^{-G(61)} q_{61}^B = \mathrm{e}^{-0.015\,231} 0.012\,278 = 0.012\,092 \, ,$$
etc.

Fig. 4a and Tab. 1a present male probabilities of death for generations 1935, 1955 and 1975 (similarly Fig. 4b and Tab. 1b for females) as other examples of the described methodology.

Tab. 3 compares the life expectancy e_{60} for males and females in age 60 for various starting years (1996, 2000, 2005, 2010, ..., 2030). E.g. for the starting year 2005 (i.e. for the generation 1945) the male values are $e_{60}{}^{1945}$ = 22.98. Using the classical LT for the period 1996 in the Czech Republic (see the last row of Tab. 3) this value is substantially lower e_{60} = 16.25.

3. Approximation of generation LT by means of age shifts

The practical application of the formulas of the type (2.12) can be cumbersome. Therefore one can accept an approximate approach where fixed LT are shifted depending on the given generation.

We'll choose the LT for generation 1955 as the ones to be shifted (this generation lies in the centre of the most interesting generations for the Czech pension funds nowadays). The probabilities q_x^{1955} and q_y^{1955} (see Tab. 1a for males and Tab. 1b for females) are adjusted to form increasing sequences (with exception of the lowest and highest ages) and are given in Tab. 2a (q_x for males and q_y for females).

One wants to shift the probabilities of death q_x using age shifts $h(\tau,x)$ so that the following approximation for q_x^{τ} from the generation LT is achieved

$$q_{x+h(\tau,x)} = q_x^{\tau} \; . \tag{3.1}$$

Moreover one can use the following linear interpolation for the shifts $h(\tau,x)$

$$h(\tau,x) = h_0(\tau,x) + \frac{q_x^{\tau} - q_{x+h_0(\tau,x)}}{q_{x+h_0(\tau,x)+1} - q_{x+h_0(\tau,x)}} \tag{3.2}$$

where $h_0(\tau,x)$ is an integer chosen unambiguously (due to increasing form of q_x) in such a way that

$$q_{x+h_0(\tau,x)} < q_x^{\tau} \le q_{x+h_0(\tau,x)+1} \; .$$

In order to exclude the dependence of the shifts $h(\tau,x)$ on age x and to obtain the shifts $h(\tau)$ depending only on generation τ one can use an approximation by means of suitable averages

$$h(\tau) = \frac{1}{k-j+1} \sum_{x=j}^{k} h(\tau,x) \tag{3.3}$$

(in our case we have used the lower bound $j = \max(60, 1997 - \tau)$ and the upper bound $k = 85$).

The corresponding age shifts $h(\tau)$ calculated according to (3.3) for generations $\tau = 1917$, 1918, ... , 1990 and rounded to integers are given in Tab. 2b. Since e.g. for males it is $h(1975) = -4$, one shifts q_x from Tab. 2a so that these LT become by 4 years "younger". Especially one has e.g. $q_{60}^{1975} = q_{60-4} = q_{56} = 0.006\,498$ which can be compared with the "exact value" $q_{60}^{1975} = 0.006\,079$ from Tab. 1a.

4. Consequences for pensions

The generation LT described in this contribution present a useful instrument to solve some problems in the context of pension insurance. The fact that the differences due to application of the generation LT are substantial can be demonstrated by means of Tab. 4, where monthly life annuities starting at the age 65 in the year 2008 and corresponding to the capital 1 mio. CZK are given. One presents two alternatives corresponding to the formulas $1\,000\,000 / 12e_{60}$, $1\,000\,000 / 12\ddot{a}_{60}(2.5\%)$. E.g. the second alternative (the technical interest rate 2.5 per cent) gives for a male in age 65 with starting year 2008 the monthly annuity 5,307 CZK. This can be compared with the classical calculation (i.e. the classical LT for the year 2008 in the Czech Republic) providing overestimated result 6,620 CZK.

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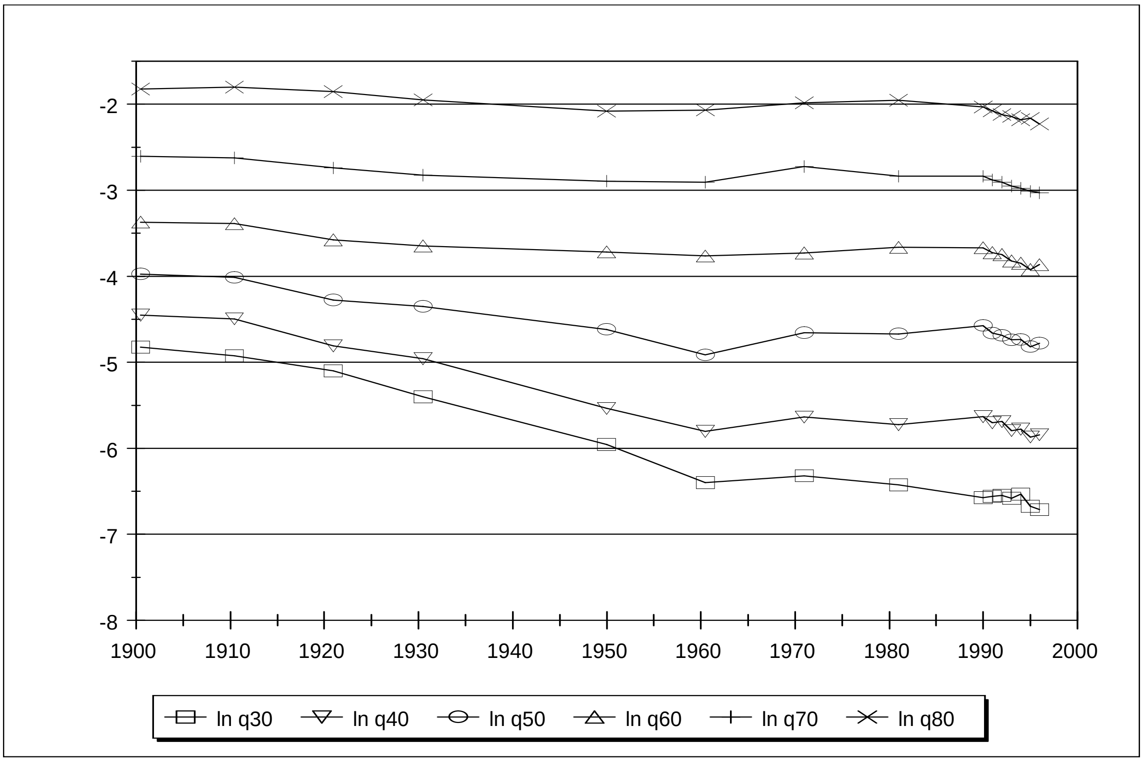

Fig. 1a. Development of mortality for some ages (Czech Republic: male)

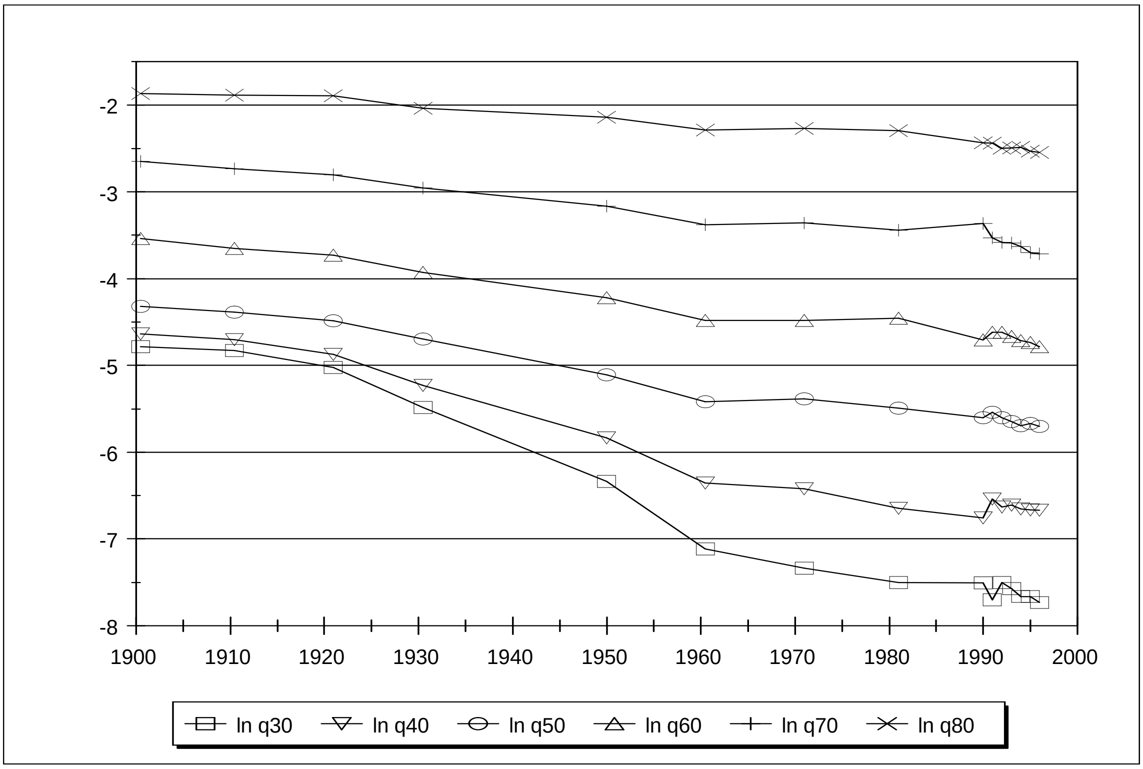

Fig. 1b. Development of mortality for some ages (Czech Republic: female)

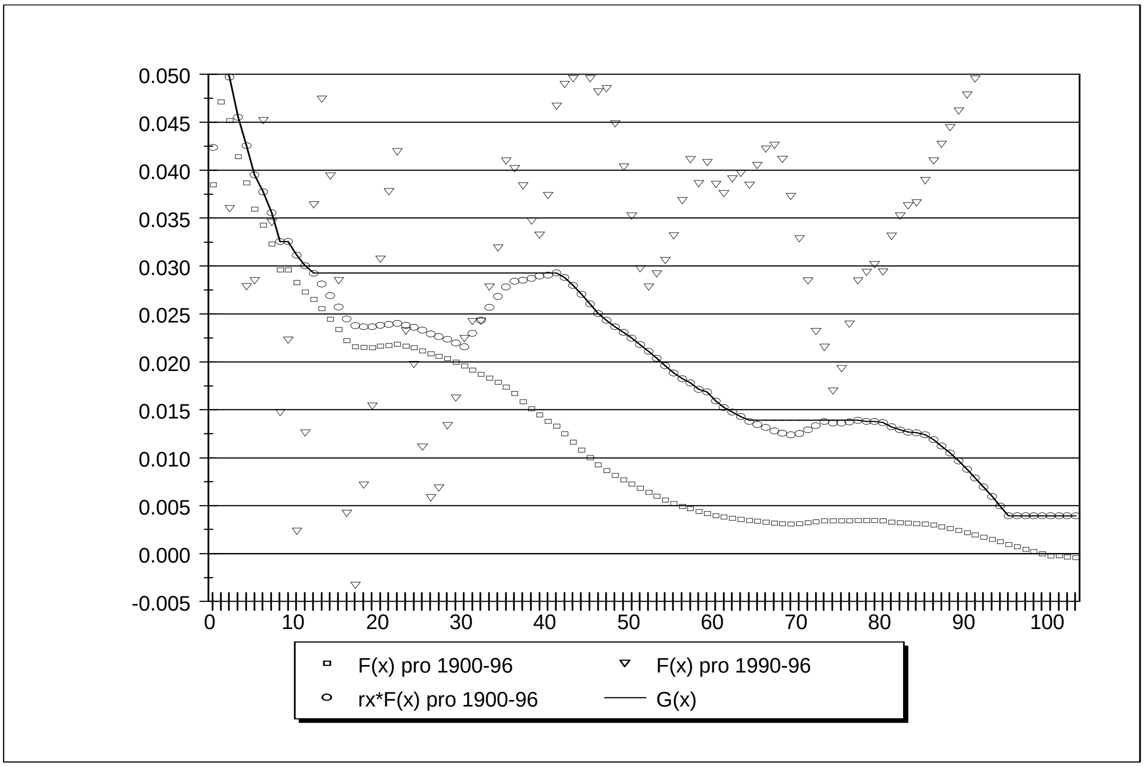

Fig. 2a. Coefficients $F(x)$ (male)

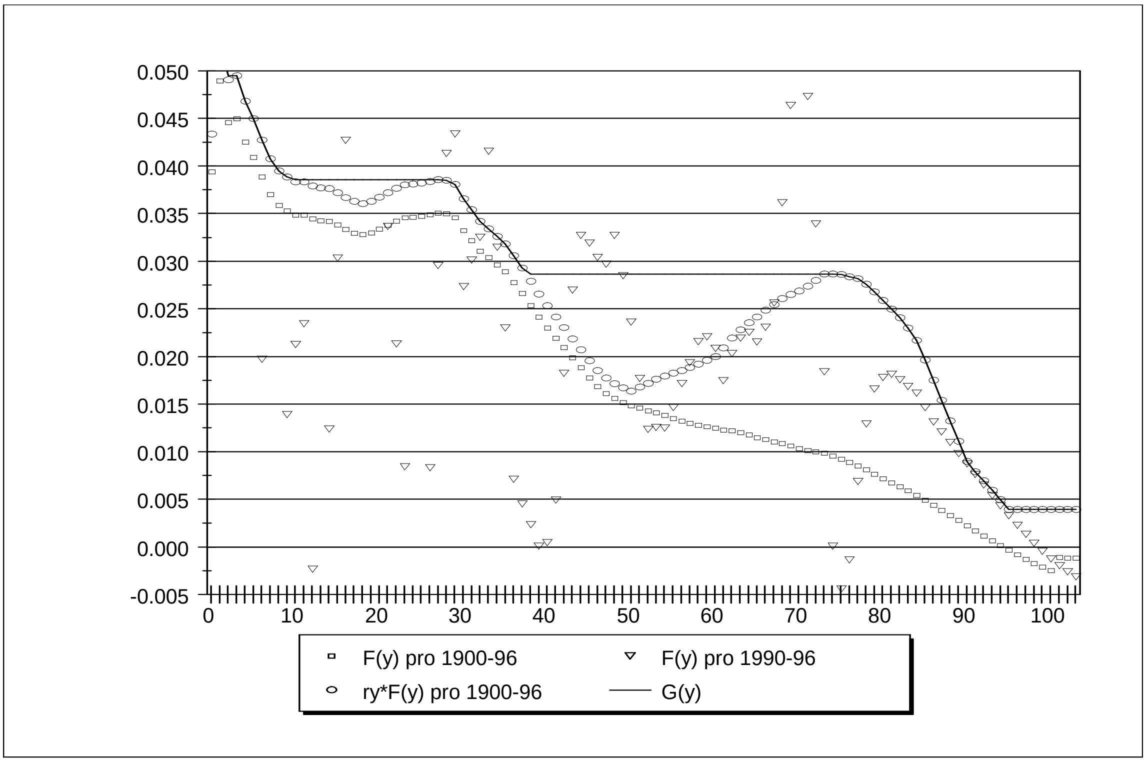

Fig. 2b. Coefficients $F(y)$ (female)

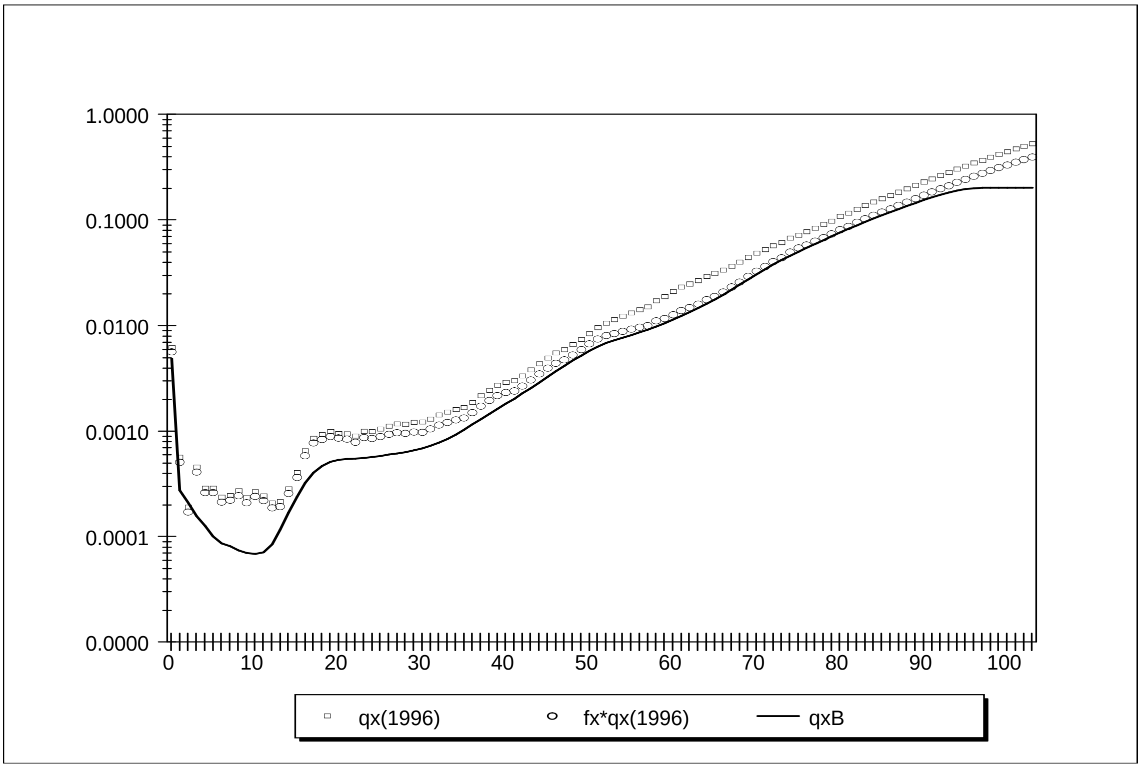

Fig. 3a. Basic probabilities of death (male)

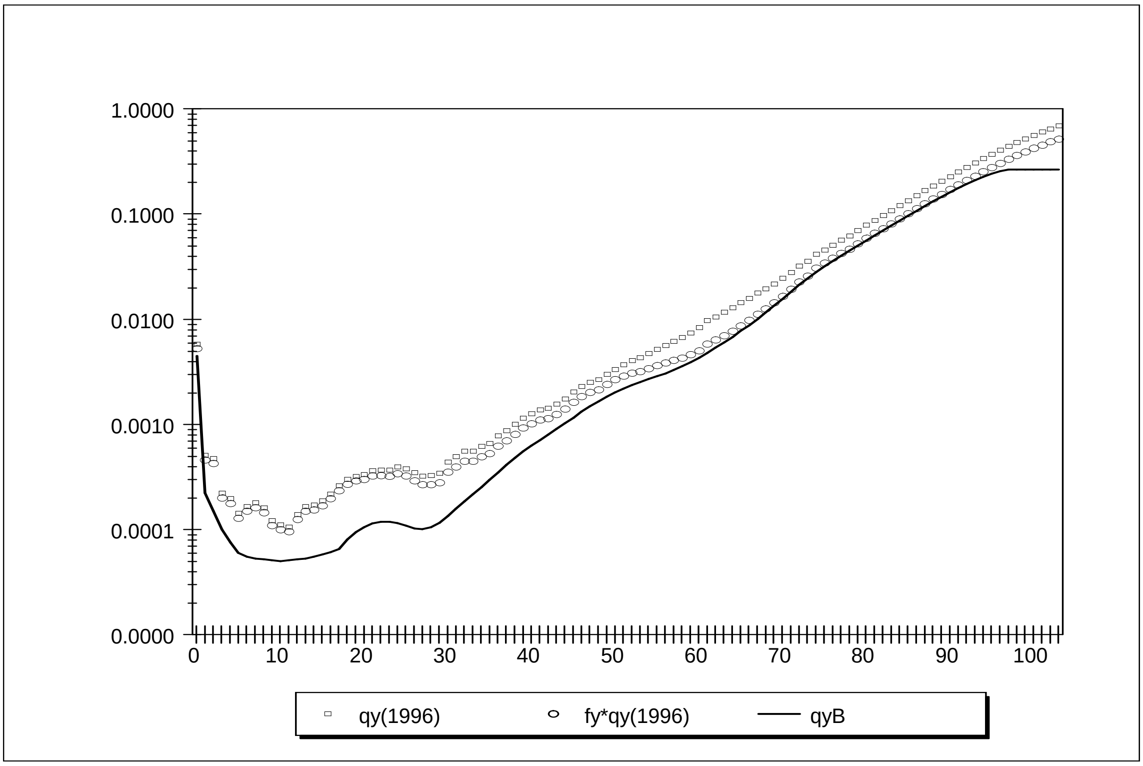

Fig. 3b. Basic probabilities of death (female)

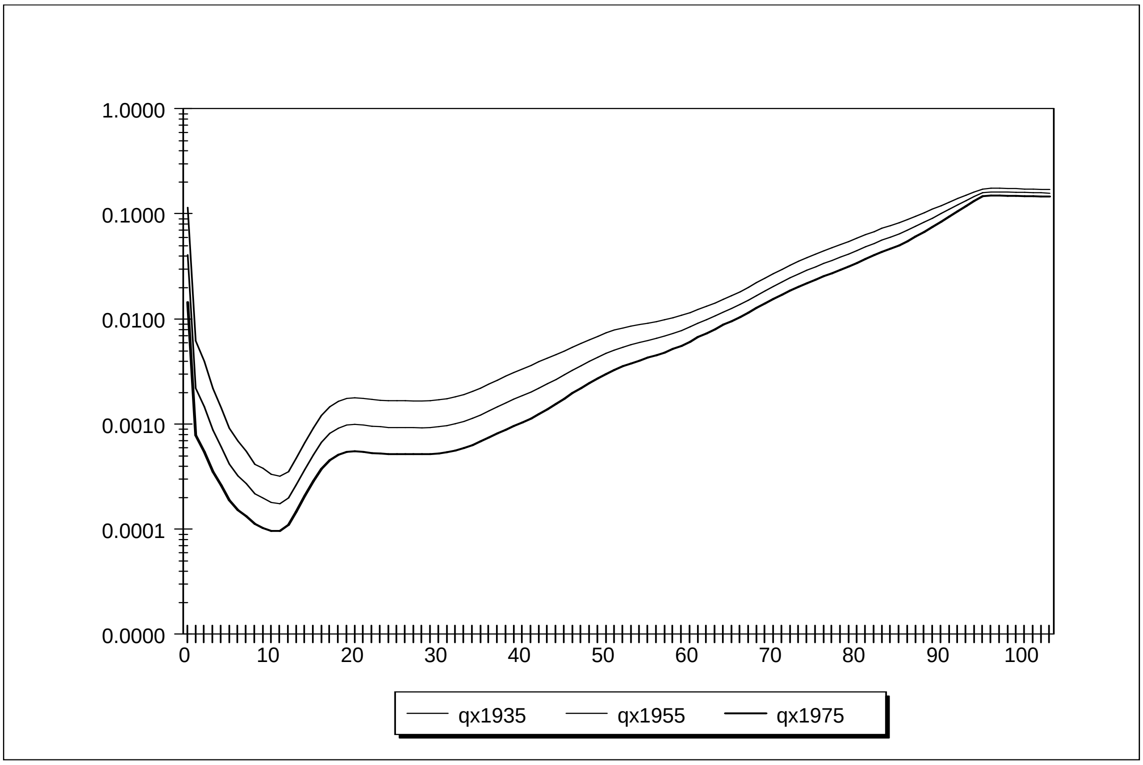

Fig. 4a. Generation probabilities of death (male)

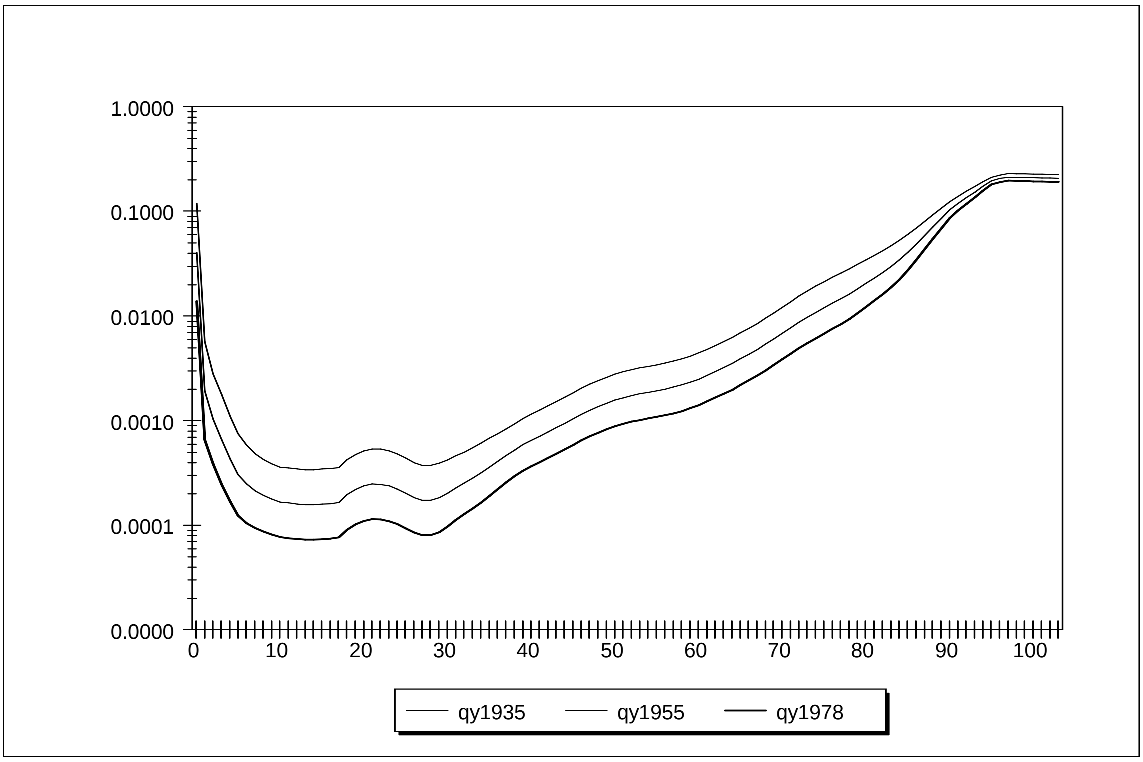

Fig. 4b. Generation probabilities of death (female)

Table 1a. Generation LT and application for some generations (Czech Republic: male)

x	$G(x)$	q_x^B	q_x^{1935}	q_x^{1955}	q_x^{1975}
0	0.051872	0.004842	0.114614	0.040615	0.014392
1	0.051872	0.000275	0.006181	0.002190	0.000776
2	0.049725	0.000210	0.003948	0.001460	0.000540
3	0.045547	0.000155	0.002176	0.000875	0.000352
4	0.042553	0.000125	0.001414	0.000604	0.000258
5	0.039540	0.000100	0.000913	0.000414	0.000188
6	0.037734	0.000086	0.000685	0.000322	0.000151
7	0.035557	0.000081	0.000551	0.000270	0.000133
8	0.032560	0.000073	0.000412	0.000215	0.000112
9	0.032560	0.000069	0.000377	0.000197	0.000103
10	0.031125	0.000068	0.000333	0.000179	0.000096
11	0.030047	0.000071	0.000318	0.000174	0.000096
12	0.029285	0.000084	0.000353	0.000196	0.000109
13	0.029285	0.000116	0.000471	0.000262	0.000146
14	0.029285	0.000168	0.000665	0.000370	0.000206
15	0.029285	0.000239	0.000918	0.000511	0.000285
16	0.029285	0.000322	0.001201	0.000669	0.000372
17	0.029285	0.000402	0.001458	0.000811	0.000452
18	0.029285	0.000465	0.001638	0.000912	0.000508
19	0.029285	0.000510	0.001743	0.000971	0.000540
20	0.029285	0.000533	0.001771	0.000986	0.000549
21	0.029285	0.000541	0.001747	0.000973	0.000541
22	0.029285	0.000545	0.001707	0.000950	0.000529
23	0.029285	0.000553	0.001683	0.000937	0.000522
24	0.029285	0.000563	0.001664	0.000926	0.000516
25	0.029285	0.000578	0.001659	0.000924	0.000514
26	0.029285	0.000595	0.001659	0.000923	0.000514
27	0.029285	0.000612	0.001657	0.000923	0.000514
28	0.029285	0.000629	0.001654	0.000921	0.000513
29	0.029285	0.000652	0.001665	0.000927	0.000516
30	0.029285	0.000681	0.001689	0.000940	0.000523
31	0.029285	0.000720	0.001733	0.000965	0.000537
32	0.029285	0.000771	0.001802	0.001003	0.000558
33	0.029285	0.000836	0.001898	0.001057	0.000588
34	0.029285	0.000919	0.002026	0.001128	0.000628
35	0.029285	0.001023	0.002190	0.001219	0.000679
36	0.029285	0.001149	0.002390	0.001331	0.000741
37	0.029285	0.001292	0.002609	0.001453	0.000809
38	0.029285	0.001448	0.002840	0.001581	0.000880
39	0.029285	0.001617	0.003080	0.001715	0.000955
40	0.029285	0.001802	0.003333	0.001855	0.001033
41	0.029285	0.002009	0.003609	0.002009	0.001119
42	0.028824	0.002257	0.003903	0.002193	0.001232
43	0.027982	0.002545	0.004212	0.002407	0.001375
44	0.027073	0.002875	0.004556	0.002651	0.001543
45	0.026021	0.003249	0.004926	0.002927	0.001740
46	0.025058	0.003666	0.005339	0.003235	0.001960
47	0.024321	0.004126	0.005799	0.003566	0.002192
48	0.023661	0.004632	0.006300	0.003925	0.002445
49	0.023068	0.005171	0.006821	0.004300	0.002711
50	0.022495	0.005726	0.007333	0.004676	0.002982

x	$G(x)$	q_x^B	q_x^{1935}	q_x^{1955}	q_x^{1975}
51	0.021824	0.006267	0.007796	0.005039	0.003256
52	0.021087	0.006772	0.008187	0.005370	0.003522
53	0.020384	0.007223	0.008503	0.005656	0.003762
54	0.019579	0.007656	0.008781	0.005936	0.004013
55	0.018838	0.008084	0.009052	0.006210	0.004261
56	0.018254	0.008544	0.009361	0.006498	0.004510
57	0.017819	0.009082	0.009753	0.006829	0.004782
58	0.017143	0.009729	0.010243	0.007270	0.005159
59	0.016883	0.010463	0.010822	0.007721	0.005508
60	0.015932	0.011316	0.011498	0.008361	0.006079
61	0.015231	0.012278	0.012278	0.009054	0.006676
62	0.014763	0.013340	0.013144	0.009784	0.007283
63	0.014294	0.014534	0.014124	0.010612	0.007973
64	0.013896	0.015918	0.015268	0.011563	0.008757
65	0.013896	0.017522	0.016575	0.012553	0.009507
66	0.013896	0.019426	0.018122	0.013725	0.010395
67	0.013896	0.021664	0.019931	0.015095	0.011432
68	0.013896	0.024248	0.022000	0.016662	0.012619
69	0.013896	0.027140	0.024284	0.018392	0.013929
70	0.013896	0.030366	0.026796	0.020294	0.015370
71	0.013896	0.033880	0.029484	0.022330	0.016912
72	0.013896	0.037625	0.032292	0.024456	0.018522
73	0.013896	0.041563	0.035179	0.026643	0.020178
74	0.013896	0.045730	0.038172	0.028910	0.021895
75	0.013896	0.050011	0.041169	0.031180	0.023614
76	0.013896	0.054464	0.044217	0.033488	0.025362
77	0.013896	0.059187	0.047388	0.035890	0.027181
78	0.013798	0.064252	0.050817	0.038562	0.029262
79	0.013794	0.069693	0.054370	0.041262	0.031314
80	0.013648	0.075615	0.058343	0.044406	0.033798
81	0.013236	0.081931	0.062876	0.048253	0.037030
82	0.012929	0.088604	0.067537	0.052148	0.040266
83	0.012668	0.095597	0.072345	0.056153	0.043585
84	0.012586	0.102883	0.077023	0.059882	0.046556
85	0.012401	0.110455	0.082023	0.064006	0.049947
86	0.011876	0.118401	0.087986	0.069385	0.054715
87	0.011223	0.126733	0.094660	0.075629	0.060424
88	0.010512	0.135436	0.101969	0.082635	0.066966
89	0.009669	0.144466	0.110200	0.090823	0.074853
90	0.008801	0.153735	0.119105	0.099882	0.083762
91	0.007887	0.163083	0.128722	0.109939	0.093896
92	0.006937	0.172303	0.138964	0.120963	0.105294
93	0.005958	0.181110	0.149674	0.132861	0.117936
94	0.004958	0.189093	0.160553	0.145396	0.131670
95	0.003943	0.195645	0.171096	0.158120	0.146129
96	0.003943	0.199868	0.174102	0.160898	0.148696
97	0.003943	0.200416	0.173891	0.160703	0.148516
98	0.003943	0.200416	0.173207	0.160071	0.147931
99	0.003943	0.200416	0.172525	0.159441	0.147349
100	0.003943	0.200416	0.171846	0.158813	0.146769
101	0.003943	0.200416	0.171170	0.158188	0.146192
102	0.003943	0.200416	0.170496	0.157566	0.145616
103	0.003943	0.200416	0.169825	0.156946	0.145043

Table 1b. Generation LT and application for some generations (Czech Republic: female)

y	$G(y)$	q_y^B	q_y^{1935}	q_y^{1955}	q_y^{1975}
0	0.053841	0.004448	0.118704	0.040440	0.013777
1	0.053841	0.000223	0.005645	0.001923	0.000655
2	0.049493	0.000150	0.002781	0.001034	0.000384
3	0.049493	0.000100	0.001765	0.000656	0.000244
4	0.046795	0.000075	0.001080	0.000424	0.000166
5	0.044985	0.000060	0.000745	0.000303	0.000123
6	0.042730	0.000055	0.000577	0.000245	0.000104
7	0.040752	0.000053	0.000479	0.000212	0.000094
8	0.039464	0.000052	0.000421	0.000191	0.000087
9	0.038841	0.000051	0.000384	0.000177	0.000081
10	0.038557	0.000050	0.000357	0.000165	0.000076
11	0.038557	0.000051	0.000351	0.000162	0.000075
12	0.038557	0.000052	0.000344	0.000159	0.000074
13	0.038557	0.000053	0.000337	0.000156	0.000072
14	0.038557	0.000055	0.000337	0.000156	0.000072
15	0.038557	0.000058	0.000342	0.000158	0.000073
16	0.038557	0.000061	0.000346	0.000160	0.000074
17	0.038557	0.000065	0.000354	0.000164	0.000076
18	0.038557	0.000080	0.000421	0.000195	0.000090
19	0.038557	0.000094	0.000474	0.000219	0.000101
20	0.038557	0.000105	0.000511	0.000237	0.000109
21	0.038557	0.000114	0.000532	0.000246	0.000114
22	0.038557	0.000118	0.000530	0.000245	0.000113
23	0.038557	0.000118	0.000509	0.000235	0.000109
24	0.038557	0.000115	0.000477	0.000221	0.000102
25	0.038557	0.000108	0.000435	0.000201	0.000093
26	0.038557	0.000103	0.000395	0.000183	0.000085
27	0.038557	0.000100	0.000372	0.000172	0.000080
28	0.038495	0.000105	0.000373	0.000173	0.000080
29	0.038043	0.000116	0.000391	0.000183	0.000085
30	0.036553	0.000135	0.000418	0.000201	0.000097
31	0.035404	0.000158	0.000457	0.000225	0.000111
32	0.034172	0.000185	0.000499	0.000252	0.000127
33	0.033406	0.000215	0.000549	0.000281	0.000144
34	0.032591	0.000251	0.000606	0.000316	0.000165
35	0.031802	0.000295	0.000674	0.000357	0.000189
36	0.030570	0.000348	0.000748	0.000406	0.000220
37	0.029290	0.000411	0.000830	0.000462	0.000257
38	0.028661	0.000480	0.000927	0.000523	0.000295
39	0.028661	0.000552	0.001037	0.000585	0.000330
40	0.028661	0.000627	0.001144	0.000645	0.000364
41	0.028661	0.000706	0.001253	0.000706	0.000398
42	0.028661	0.000795	0.001371	0.000773	0.000436
43	0.028661	0.000900	0.001507	0.000850	0.000479
44	0.028661	0.001021	0.001661	0.000936	0.000528
45	0.028661	0.001160	0.001835	0.001035	0.000583
46	0.028661	0.001316	0.002023	0.001140	0.000643
47	0.028661	0.001483	0.002216	0.001249	0.000704
48	0.028661	0.001657	0.002405	0.001356	0.000764
49	0.028661	0.001836	0.002589	0.001459	0.000823
50	0.028661	0.002014	0.002760	0.001556	0.000877

y	$G(y)$	q_y^B	q_y^{1935}	q_y^{1955}	q_y^{1975}
51	0.028661	0.002187	0.002913	0.001642	0.000926
52	0.028661	0.002357	0.003050	0.001720	0.000969
53	0.028661	0.002524	0.003174	0.001789	0.001009
54	0.028661	0.002692	0.003290	0.001855	0.001045
55	0.028661	0.002866	0.003403	0.001918	0.001081
56	0.028661	0.003055	0.003525	0.001987	0.001120
57	0.028661	0.003280	0.003678	0.002073	0.001169
58	0.028661	0.003551	0.003870	0.002182	0.001230
59	0.028661	0.003881	0.004110	0.002317	0.001306
60	0.028661	0.004282	0.004407	0.002484	0.001400
61	0.028661	0.004768	0.004768	0.002688	0.001515
62	0.028661	0.005330	0.005179	0.002920	0.001646
63	0.028661	0.005992	0.005658	0.003190	0.001798
64	0.028661	0.006765	0.006208	0.003499	0.001973
65	0.028661	0.007676	0.006845	0.003858	0.002175
66	0.028661	0.008750	0.007582	0.004274	0.002409
67	0.028661	0.010037	0.008451	0.004764	0.002685
68	0.028661	0.011572	0.009469	0.005338	0.003009
69	0.028661	0.013400	0.010654	0.006006	0.003385
70	0.028661	0.015595	0.012049	0.006792	0.003829
71	0.028661	0.018173	0.013644	0.007691	0.004336
72	0.028661	0.021103	0.015397	0.008679	0.004893
73	0.028661	0.024362	0.017272	0.009736	0.005488
74	0.028661	0.027911	0.019229	0.010840	0.006110
75	0.028625	0.031663	0.021208	0.011964	0.006749
76	0.028367	0.035676	0.023312	0.013219	0.007495
77	0.028172	0.040022	0.025500	0.014516	0.008263
78	0.027575	0.044763	0.028011	0.016137	0.009296
79	0.026792	0.050016	0.030880	0.018070	0.010574
80	0.025879	0.055880	0.034175	0.020367	0.012138
81	0.024991	0.062369	0.037836	0.022953	0.013924
82	0.024052	0.069578	0.041987	0.025953	0.016043
83	0.022999	0.077591	0.046781	0.029532	0.018643
84	0.021678	0.086454	0.052510	0.034037	0.022063
85	0.019626	0.096254	0.060097	0.040587	0.027411
86	0.017509	0.107048	0.069100	0.048685	0.034301
87	0.015384	0.118833	0.079657	0.058560	0.043050
88	0.013239	0.131624	0.092064	0.070647	0.054212
89	0.011088	0.145415	0.106606	0.085404	0.068418
90	0.008941	0.160161	0.123581	0.103347	0.086425
91	0.007887	0.175783	0.138746	0.118500	0.101208
92	0.006937	0.192124	0.154950	0.134878	0.117406
93	0.005958	0.208876	0.172620	0.153229	0.136017
94	0.004958	0.225529	0.191489	0.173412	0.157041
95	0.003943	0.241237	0.210967	0.194968	0.180182
96	0.003943	0.254596	0.221773	0.204954	0.189411
97	0.003943	0.263228	0.228391	0.211070	0.195063
98	0.003943	0.263228	0.227492	0.210239	0.194295
99	0.003943	0.263228	0.226597	0.209412	0.193530
100	0.003943	0.263228	0.225705	0.208588	0.192769
101	0.003943	0.263228	0.224817	0.207767	0.192010
102	0.003943	0.263228	0.223932	0.206949	0.191254
103	0.003943	0.263228	0.223051	0.206135	0.190502

Table 2a. Generation LT by means of shifts

x,y	q_x	q_y	x,y	q_x	q_y
0	0.000174	0.000003	52	0.005370	0.001720
1	0.000174	0.000003	53	0.005656	0.001789
2	0.000174	0.000003	54	0.005936	0.001855
3	0.000174	0.000003	55	0.006210	0.001918
4	0.000174	0.000003	56	0.006498	0.001987
5	0.000174	0.000003	57	0.006829	0.002073
6	0.000174	0.000003	58	0.007270	0.002182
7	0.000174	0.000003	59	0.007721	0.002317
8	0.000174	0.000003	60	0.008361	0.002484
9	0.000174	0.000003	61	0.009054	0.002688
10	0.000174	0.000003	62	0.009748	0.002920
11	0.000174	0.000005	63	0.010612	0.003190
12	0.000196	0.000016	64	0.011563	0.003499
13	0.000262	0.000037	65	0.012553	0.003858
14	0.000370	0.000064	66	0.013725	0.004274
15	0.000511	0.000095	67	0.015095	0.004764
16	0.000669	0.000129	68	0.016662	0.005338
17	0.000811	0.000164	69	0.018392	0.006006
18	0.000912	0.000195	70	0.020294	0.006792
19	0.000971	0.000219	71	0.022330	0.007691
20	0.000986	0.000237	72	0.024456	0.008679
21	0.000987	0.000246	73	0.026643	0.009736
22	0.000989	0.000246	74	0.028910	0.010840
23	0.000990	0.000247	75	0.031180	0.011964
24	0.000992	0.000247	76	0.033488	0.013219
25	0.000993	0.000248	77	0.035890	0.014516
26	0.000995	0.000248	78	0.038562	0.016137
27	0.000996	0.000249	79	0.041262	0.018070
28	0.000998	0.000249	80	0.044406	0.020367
29	0.000999	0.000250	81	0.048253	0.022953
30	0.001001	0.000250	82	0.052148	0.025953
31	0.001002	0.000251	83	0.056153	0.029532
32	0.001003	0.000252	84	0.059882	0.034037
33	0.001057	0.000281	85	0.064006	0.040587
34	0.001128	0.000316	86	0.069385	0.048685
35	0.001219	0.000357	87	0.075629	0.058560
36	0.001331	0.000406	88	0.082635	0.070647
37	0.001453	0.000462	89	0.090823	0.085404
38	0.001581	0.000523	90	0.099882	0.103347
39	0.001715	0.000585	91	0.109939	0.118500
40	0.001855	0.000645	92	0.120963	0.134878
41	0.002009	0.000706	93	0.132861	0.153229
42	0.002193	0.000773	94	0.145396	0.173412
43	0.002407	0.000850	95	0.158120	0.194968
44	0.002651	0.000936	96	0.160898	0.204954
45	0.002927	0.001035	97	0.160898	0.211070
46	0.003235	0.001140	98	0.160898	0.211070
47	0.003566	0.001249	99	0.160898	0.211070
48	0.003925	0.001356	100	0.160898	0.211070
49	0.004300	0.001459	101	0.160898	0.211070
50	0.004676	0.001556	102	0.160898	0.211070
51	0.005039	0.001642	103	0.160898	0.211070

Table 2b. Generation LT by means of shifts

	Age shifts			*Age shifts*	
Year	*Male*	*Female*	*Year*	*Male*	*Female*
1917	6	6	1954	0	0
1918	6	6	1955	0	0
1919	6	6	1956	0	0
1920	6	6	1957	0	–1
1921	6	6	1958	–1	–1
1922	6	6	1959	–1	–1
1923	6	6	1960	–1	–1
1924	6	6	1961	–1	–2
1925	5	6	1962	–1	–2
1926	5	6	1963	–1	–2
1927	5	6	1964	–2	–2
1928	5	6	1965	–2	–3
1929	5	6	1966	–2	–3
1930	4	5	1967	–2	–3
1931	4	5	1968	–2	–4
1932	4	5	1969	–2	–4
1933	4	5	1970	–3	–4
1934	4	5	1971	–3	–5
1935	3	5	1972	–3	–5
1936	3	4	1973	–3	–5
1937	3	4	1974	–3	–5
1938	3	4	1975	–4	–5
1939	3	4	1976	–4	–5
1940	3	4	1977	–4	–5
1941	2	3	1978	–4	–5
1942	2	3	1979	–4	–5
1943	2	3	1980	–4	–5
1944	2	3	1981	–4	–6
1945	2	2	1982	–5	–6
1946	2	2	1983	–5	–6
1947	1	2	1984	–5	–6
1948	1	2	1985	–5	–6
1949	1	1	1986	–6	–6
1950	1	1	1987	–6	–6
1951	1	1	1988	–6	–6
1952	0	1	1989	–6	–6
1953	0	0	1990	–6	–6

Table 3. Life expectancy

Start of pension	*Males in age of 60*	*Females in age of 60*
	e_{60}	e_{60}
1996	21.87	26.43
2000	22.36	27.09
2005	22.98	27.87
2010	23.58	28.61
2015	24.18	29.30
2020	24.77	29.95
2025	25.35	30.55
2030	25.92	31.11
LT 1996	16.25	20.39

Table 4. Monthly life annuity (in CZK) based on the capital of 1 mio. CZK (the starting age 65 in the year 2008)

Starting year 2008	*Males in age of 65 - calculation using:*		*Females in age of 65 - calculation using:*	
	e_{60}	$\ddot{a}_{60}(2.5\%)$	e_{60}	$\ddot{a}_{60}(2.5\%)$
Classical LT 2008	5,331	6,620	4,413	5,626
Generation LT	4,011	5,307	3,307	4,500