The Lebanese pension system facing the demographic change: An OLG model with labor market frictions.

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Very preliminary version

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Abstract

In this article, we evaluate the macroeconomic effects of population ageing in Lebanon using an innovative OLG model which takes into account the presence of search and matching frictions in the labor market.

Keywords: Pensions; Population ageing; OLG models; Labor market frictions

JEL codes : J11, E62, H55

1. Introduction

Social security policies will face the same challenges: to satisfy the desire of national solidarity expressed by the populations and to guarantee the sustainability of the security systems. These challenges are difficult to meet given the population ageing problem that developed countries and also developing countries will face in the next decades. In fact, even developing countries will live a fall in fertility and mortality rates. With respect to developed countries, developing countries are characterized by a small size of the social security system, a large informal sector and a low women's labor participation. Although there is an extensive literature analyzing the effects of population ageing in developed countries, only few works are focused on developing countries (see, for instance, Loumrhari, 2014, for Morocco; Abdessalem and Chekki, 2016, for Tunisia; Georges and Seckin, 2016, for Turkey;).

Lebanon is a very interesting case study for several reasons. First of all, the Lebanese social security system is widely considered as inadequate and unfair. While public employees enjoy a generous system and are covered by health insurance, in the private sector the coverage is quite limited and individuals obtain a lump-sum payment upon retirement instead of being covered by a pension plan. In addition, they and their families lose the health insurance, at the time when they are much needed. According to a report of ILO (2008), more than half of the elderly can be considered as poor, as they have difficulties to ensure their basic needs (e.g. health services, water, electricity and housing). The funded nature of the private sector pension scheme implies that there is no redistribution across generations in order to protect the most vulnerable people. The size of the Lebanese social security system is quite small, even compared with that of other emerging countries. In 2010, the public pension expenditure represents 3% of GDP while in other emerging economies it represents 6% of GDP. The inadequacy of the social security system is partially compensated by the existence of informal transfers from family members and from charitable institutions. In addition, the demographic changes that are expected to occur in the next decades will clearly put pressure on the financing of the social security system. The second reason is that the Lebanese labor market, as in other developing economies, is characterized by a large informal sector, a high unemployment rate and a low labor participation of women. With respect to other developing countries, the Lebanese labor market is also characterized by high migration flows of highly educated young people. In fact, from one hand, the investment in human capital in Lebanon is higher with respect to other MENA countries (IMF, 2016) and, from the other hand, the lack of appropriate employment opportunities induces many highly educated people to migrate. For people who decide to stay in Lebanon, the problem of skill mismatch explains the high level of unemployment of high educated people. The third reason is that Lebanon is facing a demographic transition which will strongly affect the structure of the population.

The objective of this article is to analyze the effects of the demographic transition in Lebanon on the financial situation of the existent pension schemes and on the macroeconomic dynamic of the

country. Our analysis is focused on the functioning of the labor market and the problem of the sustainability of the pension system.

The analysis is carried out using a simulation OLG model (Auerbach and Kotlikoff, 1987) which is calibrated in order to reproduce the main features of the Lebanese economy observed in the recent years. The main innovation of our model, with respect to standard OLG models existing in the literature, is that we introduce search and matching frictions in the labor market as in de la Croix et al. (2012). However, with respect to de la Croix et al. (2012), the matching function is defined for each education level (see Charlot, 2005). This permits to take into account the difficulty, especially for highly educated people, to find a job which is an important characteristic of the labor market in Lebanon.

Our results show that the public pension scheme is clearly unsustainable since the system would produce extremely high deficits. In contrast, the private scheme can be considered as sustainable, even though this result is explained by the fact that the generosity of the system is extremely low. In addition, population ageing produces negative effects at the macroeconomic level by reducing the ratio between investments and GDP, by reducing the economic growth rate. The only positive macroeconomic effect is related to the fact that the unemployment rate is expected to decrease over time thanks to the increase in the marginal productivity of labor and the reduction in the interest rate.

The article is organized as follows: section 2 describes the demographic evolution, the pension schemes and the characteristics of the labor market. In section 3, we describe the OLG model used in our analysis. In section 4 we describe the calibration procedure and in section 5 we present our results. Section 6 concludes.

2. Demographic evolution, pension system and the labor market in Lebanon

2.1 The demographic evolution in Lebanon

First of all, it is important to highlight that the analysis of the Lebanese demographic evolution is made it difficult because of the lack of official statistics. In order to avoid tensions between the 18 religious communities that coexist in Lebanon, the last census of the population has been realized in 1932. However, demographic data exist. Recent estimations take into account the consequences of the war with Israel in 2006 and the war in Syria. According to PopulationData.net (2017), the population growth rate has fallen from 1.7% in 2006 to 0.7% in 2007. Then, it started to increase from 2009 with a peak in 2012 and 2013 (6.9% and 7%, respectively).

Our study is based on the demographic estimates of the Union Nations (World Population Prospects: The 2017 Revision) that provides a complete dataset of the Lebanese population, distinguished by age and gender, from 1950 to 2100. The main features of the Lebanese population transition are summarized in Table 1.

First of all, the Lebanese population strongly increased between 1950 and 2000 (from 1.3 million to 3.2 million) and should increase until 2020 and reach 6 million. Then, according to the Union Nations projections, total population is expected to decrease and reach 4.3 million in 2100.

Table 1 also shows the strong decline in the fertility rates which is mainly explained by the improvement in the education level of women which has important implications on the choices concerning the family. The fertility rate was 5.7 children per woman in 1950 and was only 1.7 in 2010. According to the projections of the Union Nations, the fertility rate should raise in the next years at the level that guarantee the generational renewal.

Life expectancy at birth has strongly increased from 62.2 for women and 58.9 for men in 1950 to 81.7 for women and 78.3 for men in 2015. According to the United Nations projections, they should progressively increase and reach 92 for women and 90 for men in 2100.

Clearly, the demographic transition described above will induce a significant change in the structure of the population age structure. Young individuals aged 14 and less, which represented 34% of the total population in 1950, would represent less than 14% of the total population starting from 2050. The working-age population, with respect to the total population, should increase from 58.5% in 1950 to 69% in 2020; then, it should decrease and reach 51% in 2100. Individuals aged 65 and more, with respect to the total population, should increase from 7.3% in 1950 to 23.3% in 2050 and to 35.2% in 2100.

The population ageing problem that will characterize the Lebanese economy during the next decades can be summarized by analyzing the evolution of the old-age dependency ratio. As shown in figure 1, the old-age dependency ratio, computed as the ratio between the number of individuals aged 65 and more to the number of individuals aged 15-64, should increase in a spectacular way starting from 2020: it was equal to 12% in 2010, it should be equal to 37% in 2050 and stabilize at a level slightly lower than 70% starting from 2070.

2.2 The Lebanese pension system

The Lebanese social security system, established in 1963, includes four different branches: health and maternity insurance, family and educational allowances, end of service indemnity system, and security of accidents of labor and sickness due to work injury (though this branch has never been implemented).

The pension system is characterized by the coexistence of several schemes that are differentiated according to the method of financing and the computation rules of benefits. The two main schemes are the public sector pension system, which covers civil servants and the military personnel, and the private sector pension system which covers private sector employees, contractual government employees, and other categories (taxi drivers, newspaper vendors, and voluntary self-employed subscribers, etc.)

The private sector pension system is a special case where employees receive upon retirement an end-of-service indemnity instead of monthly pensions. After the payment of the lump-sum indemnity, which is considered as insufficient to provide a decent retirement (see for instance Rached, 2012), the retiree loses his health insurance.

Even though the end-of-service indemnity was introduced in 1963 as a temporary scheme, no reform has been introduced to protect the elderly against poverty which is explained by the low coverage and the low amount of the end-of-service indemnity. In 2004 and 2011, two (unsuccessful) attempts aimed at reforming the existing social security system have been proposed. In particular, the 2004 draft bill proposed a flat indemnity payment to individuals who have not made enough contributions to the scheme and the transformation of the lump-sum payment into an annuity. However, no extension of the scheme's coverage was proposed, implying that the employees in the informal sector would still be not covered by the social security system. The 2011 draft bill proposed the introduction of a Notional Defined Contribution system (as in Sweden, Italy, Latvia and Poland).

In what follows, we describe the most important features of the two schemes that are considered in our empirical analysis. For a detailed description of the Lebanese pension system, see Robalino (2005), Rached (2012) and Jarmuzek and Nakhle (2018).

2.1 The public sector pension system

The public sector pension scheme is a PAYG system with defined-benefit pensions which covers civil and military servants. The system does not impose a minimum retirement age even though individuals must have made at least 20 years of contribution to the scheme. In addition, the maximum retirement age is 64 for civil servants and between 58 and 64, depending on the rank, for military. Public employees pay contributions of 6% of their wages.

Individuals with more than 20 years of service can choose between lifetime pensions or an end-of-service lump-sum. The lifetime pension is computed as 2/800 times the last monthly basic salary times the number of years of service. The maximum replacement rate is 85% of the final salary. The end-of-service compensation is computed as 85% of last monthly salary times one month for each year of service for the first 10 years, two months for each year of service in excess of 10 years but less than 20 years, and three months for each year of service in excess of 20 years. Individuals with more than 40 years of service, obtain a combination of lifetime pension and end-of-service compensation: 85% of the last monthly basic salary as a life-pension and an end-of-service compensation at 85% of last monthly basic salary times 3 months for each year of service that exceeds 40 years.

Concerning the pension indexation, there are not well-defined rules. The last pension increase was granted in 2012.

As noted by Marwan (2016), the system is characterized by different distorting mechanisms: an extra lump-sum payment is allowed for retirees with more than 40 years of service; additional years of service are accounted for the military; reversibility pensions are exceptionally generous since unmarried, divorced and widowed daughters of retirees keep the full pension benefits for life.

2.2 The private sector pension system

The private sector pension scheme is a funded system which covers private sector employees, contractual government employees, taxi drivers, newspaper and magazine vendors, local councilors and voluntary self-employed subscribers. It is a defined-contribution scheme administered by the National Social Security Fund (NSSF) which is an independent institution established in 1963 and controlled by the Council of Ministers and the Ministry of Labor. Contributions are paid only by employers and the contribution rate is 8.5% (of which 0.5% to the NSSF administration). They are collected in individual accounts and accrue an interest rate determined by NSSF investments, usually the rate on government bonds as most of NSSF investments are in Treasury bonds.

The maximum retirement age is 64. Individuals can retire if they have reached age 60, with at least 20 years of service. They receive an end-of-service indemnity which is a lump-sum transfer equal to the accumulated contributions associated with past employers, and one month of earnings for each year of service with final employer (up to 20 years, with half a month for each year in excess of 20 years, for those aged more than 60).

After retirement, pensioners lose all benefits when they need them the most. They only receive one lump-sum payment at retirement and have no health coverage benefits thereafter.

2.3 The labor market in Lebanon

First of all it is important to highlight that data on the Lebanese labor market are in some cases insufficient and, in other cases, contradictory. Concerning the labor market, our study is based on data provided by the Central Administration for Statistics (CAS), which is the main source of data concerning employment, unemployment and wages in Lebanon thanks to the living conditions household surveys conducted the in 2004, 2007 and 2009, and by the ILO.

As shown in tables 2 and 3, the Lebanese labor market is characterized by a low level of labor participation, especially for women. Focusing on people aged 15-64, the labor participation rate in 2007 was 72.9% for men and 23.9% for women and, on average, 47.6%. Interestingly, the women's labor participation is higher for highly educated women (45%) and extremely low for primary and secondary education levels (between 13% and 19%). In contrast, for men, the participation rate is higher for men with primary and lower secondary education (78.7% and

76.9%, respectively) then for men with upper secondary education and university levels (59% and 62.5, respectively).

As shown in table 4 and 5, the unemployment rate is 7.8% in 2004, 9.1% in 2007 and 6.4% in 2009 and is higher for women (9.5% in 2004, 10.1% in 2007 and 10.4% in 2009 vs 7.3% in 2004, 8.6% in 2007 and 5% in 2009). In addition, the unemployment rate is higher for highly educated people (11.2% in 2007 and 8.8% in 2009), which is related to the difficulty for those people to find a job given the problem of mismatch in the labor market.

Consequently, the employment rate (see table 4) is very low (39.5% in 2007 and 44.6% in 2009). Although the employment rate is much higher for men (61.1% vs. 19% in 2007 and 69.2% vs. 20.4% in 2009), high educated men display a quite low employment rate.

The public sector employs 144 605 in 2007 and 162 659 in 2012, which constitute respectively 24% and 23% of formal works. More than 80% are men (which is explained by the fact that the public sector includes the public army) and between 80% and 85% of the public workers have a secondary school level.

Concerning the informal sector, as noted by Gohlke-Rouhayem et al. (2016), the informal sector in Lebanon includes not only illegal activities but also, in most cases, legal activities which are not registered and therefore are not accounted in national statistics. According to the World Bank, the informal sector represents 50% of GDP and 71% of all employees do not contribute to any social security system. In contrast, according to ILO (2015), the informal sector represents 30% of GDP and includes 44% of workers which are deprived of social protection.

The inadequacy of skills of job seekers to the needs of hiring firms constitutes a big challenge for the Lebanese economy. In particular, the high level of specialization of recent graduates does not match the needs of the economy. In addition, 55% of firms consider that the lack of soft skills (including leadership, communication and writing skills) of potential workers is a constraint to their activity (Angel-Urdinola et al., 2013). The high cost of education in Lebanon, in particular for private high schools and private universities, induces the job seekers to demand wages in excess of those set by the market (CAS, 2012). In many cases, they choose to emigrate in order to take advantage of better paid job opportunities. Workers who choose to remain in the local labor market have in general a medium or elementary level of education, which widens the gap between job seekers and the needs of firms.

Additional information necessary to calibrate our OLG model has been obtained using a household survey which includes a sample of 10827 individuals. After reducing our sample to people aged 15-64 and after controlling for missing values, we retain a total of 6164 observations. The main characteristics of these data are reported in tables from A1 to A6 in Appendix 1. In particular, these data allow (i) to identify the composition of the population by age, gender and education level; (ii) to determine the repartition of workers (classified by age, gender and education level) by the economic sector of activity (public sector, private sector, informal sector); (iii) to estimate the wage equation in the three economic sectors.

3 The OLG model with labor market frictions

3.1 General description

The model used in our analysis is an OLG model of the type of Auerbach-Kotlikoff (1987), similar to that built by De la Croix et al. (2012) who introduced labor market frictions *à la* Pissariedes (2000) in simulation OLG models.

In our model, 17 generations coexist at each period. Individuals are classified according to their age g, gender s, education level e, and occupational category c. In particular, we consider 17 age groups (1 for individuals aged 15-19, 2 for individuals aged 20-24,…, 17 for individuals aged 95-99), men ($s1$) and women ($s2$), 5 education levels ($e1$ for the preschool level, $e2$ for the primary school level, $e3$ for the intermediate school level, $e4$ for the secondary school level, and $e5$ for the university level) and 3 occupational categories ($c1$ for the public sector, $c2$ for the private sector and $c3$ for informal workers). The first two categories correspond to the Lebanese pension schemes analyzed in our article, while the third category corresponds to individuals that are not covered by the social security system.

We assume that a representative firm produces one good using labor and capital in a perfectly competitive market. In contrast, the labor market is assumed to be characterized by the presence of search and matching frictions. This modelling allows us to endogenize the unemployment rate and to take into account for the difficulty, especially for highly educated young people, to find a job, which is an important feature of the Lebanese economy.

Each period is made up of 5 years and the time horizon of the model is 1960-2200 even though the simulation results, for convenience, are reported only for the period 2010-2060.

In what follows, we describe the characteristics of our OLG model.

3.2 Individuals

3.2.1 Demographics

In our model, individuals can live up to 17 periods. The demographic evolution is defined by the following equations:

$$Pop_{g,s,t} = \left(1 + x_{s,t}\right) \cdot Pop_{g,s,t-1} \quad \text{with } g = 1$$

$$Pop_{g+1,s,t+1} = \beta_{g+1,s,t+1} \cdot Pop_{g,s,t} \quad \text{with } g = 2, \dots, 16$$

where $Pop_{g,s,t}$ is the number of people with age g and gender s; $x_{s,t}$ is a measure of the fertility rate; $\beta_{g,s,t}$ is the unconditional probability (differentiated by gender) to be alive at age g at time t (with $\beta_{17,s,t+16} < \beta_{16,s,t+15} < \cdots < \beta_{2,s,t+1}$).

Individuals by age and gender are (exogenously) grouped by education level. The number of people with age g, gender s and education level e is noted by $N_{g,s,e,t}$ and given by:

$$N_{g,s,e,t} = Pop_{g,s,e,c,t} \cdot \varphi^{e}_{g,s,e,t}$$

where $\varphi^{e}_{g,s,e,t}$ is supposed to be exogenous.

3.2.2 Activity, inactivity, unemployment

We assume that people aged 65 and more (i.e. $g \geq 11$) are retired. Until age 64 (i.e. $g \leq 10$), people may be inactive or active, and active people may work or be unemployed.

We note by $z_{g,s,e,t}$ the (exogenous) labor participation rate differentiated by age, gender and education level. Thus, the number of people participating in the labor market (by age g, gender s and education level e) is exogenous and given by:

$$z_{g,s,e,t} \cdot N_{g,s,e,t}$$

The number of workers (by age g, gender s, education level e and occupational category c) depends on the endogenous unemployment rate (which depends on the frictions in the labor market) and is given by:

$$L_{g,s,e,c,t} = \left(1 - u_{g,s,e,t}\right) \cdot z_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} = n_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

where $u_{g,s,e,t}$ is the unemployment rate and $n_{g,s,e,t}$ is the employment rate (both differentiated by age, gender and education level), which are related by the following relation: $n_{g,s,e,t} = \left(1 - u_{g,s,e,t}\right) \cdot z_{g,s,e,t}$. Note that the previous expression implies that $u_{g,s,e,t} \cdot z_{g,s,e,t} = z_{g,s,e,t} - n_{g,s,e,t}$. Clearly, individuals may be inactive, workers or unemployed, implying that $\left(1 - z_{g,s,t}\right) + n_{g,s,e,t} + u_{g,s,e,t} \cdot z_{g,s,t} = 1$.

The number of workers (by age, gender and education level) are exogenously distributed among the three occupational categories. In particular, the number of workers (by age g, gender s, education level e and occupational category c) is given by:

$$n_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

where $\varphi^{c}_{g,s,e,c,t}$ is supposed to be exogenous.

Concerning people aged 65 and more, the number of individuals aged 65 and more (by age g, gender s and education level e) is given by:

$$z_{g,s,e,t}^{pens} \cdot N_{g,s,e,t}$$

where $z_{g,s,e,t}^{pens}$ is computed as the average of the participation rate during the last three periods before retiring.

The number of individuals aged 65 and more (by age g, gender s, education level e and category c) is given by:

$$N_{g,s,e,t} \cdot z_{g,s,e,t}^{pens} \cdot \varphi_{g,s,e,c,t}^{c}$$

where the variables $\varphi_{g,s,e,c,t}^{c}$ are again supposed to be exogenous and reflect the occupational status during the working life.

Clearly, only individuals aged 65 and more that belong to the first two categories (i.e. public sector and private sector) earn pensions. Thus, people that are not covered by the pension system are people that were previously inactive and people that previously worked in the informal sector.

3.2.3 The individual budget constraint and choices

At each period, individuals have to choose the optimal level of consumption in order to maximize their expected intertemporal utility given an intertemporal budget constraint. Assuming the absence of random shocks and taking into account rational expectations, individuals have perfect foresight. Thus, in the absence of unexpected shocks, they perfectly predict the evolution of any economic variables of the model. We also assume the absence of liquidity constraints.

The expected intertemporal utility function, for an individual belonging to the first age class in t, of gender s and education level e, is given by:

$$U_{s,e,t} = E_t \left[\sum_{g=1}^{17} \beta_{g,s,t+g-1} \cdot \left(\frac{1}{1+\rho} \right)^{g-1} \cdot \ln c_{g,s,e,t+g-1} \right]$$

where $c_{g,s,e,t}$ is the level of consumption, ρ is the intertemporal preference rate, $\beta_{g,s,t}$ (as previously mentioned) is the unconditional probability (differentiated by gender) to be alive at age g at time t.

The instantaneous budget constraint for all individuals (except those belonging to the last age class), is given by:

$$k_{g+1,s,e,t+1} = [1 + r_t \cdot (1 - \tau_t)] \cdot k_{g,s,e,t} + (1 - \tau_t) \cdot \alpha^{\pi}_{g,s,e,t} \cdot \pi_t + \sum_c \left(1 - \tau_t - \tau^{contr}_{c,t}\right) \cdot n_{g,s,e,t} \cdot w_{e,c,t} \cdot A_{g,s,e,c,t} \cdot \varphi^{c}_{g,s,e,c,t} + \sum_c (1 - \tau_t) \cdot z^{pens}_{g,s,t} \cdot pens_{g,s,e,c,t} \cdot \varphi^{c}_{g,s,e,c,t} + transf_{g,s,e,t} - c_{g,s,e,t}$$

The instantaneous budget constraint for individuals belonging to the last age class, is given by:

$$c_{g,s,e,t} = [1 + r_t \cdot (1 - \tau_t)] \cdot k_{g,s,e,t} + (1 - \tau_t) \cdot \alpha^{\pi}_{g,s,e,t} \cdot \pi_t + \sum_c \varphi_{g,s,e,c,t} \cdot z^{pens}_{g,s,t} \cdot pens_{g,s,e,c,t} + transf_{g,s,e,t}$$

$k_{g,s,e,t}$ is the wealth owned at the beginning of t by individuals aged g, with gender s and education level e; r_t is the interest rate (equal to the marginal productivity of capital, net of depreciation), $\alpha^{\pi}_{g,s,e,t} \cdot \pi_t$ is the dividends (where $\alpha^{\pi}_{g,s,e,t}$ is the fraction of total capital owned by individuals aged g, with gender s and education level e and π_t is the profit of the representative firm); τ_t is the tax rate (applied on labor incomes, capital incomes, and dividends); $\tau^{contr}_{c,t}$ is the contribution rate applied by each pension scheme; $n_{g,s,e,c,t}$ is the (endogenous) employment rate; $w_{e,t} \cdot A_{g,s,e,c,t}$ is the wage earned by an individual aged g, with gender s, education level e and occupational category c), where $w_{e,t}$ is the (negotiated) wage per unit of effective labor which is differentiated by education, and $A_{g,s,e,c,t}$ is the (exogenous) productivity level (by age, gender, education level and category) that is supposed to grow over time according to an exogenous rate reflecting the technological progress; $pens_{g,s,e,c,t}$ is the pension benefit (in the public sector) or the end-of-service indemnity in the private sector; $transf_{g,s,e,t}$ represents intergenerational transfers, which considers: (i) involuntary bequests (if people die before the last age class) that are redistributed to their children (supposed to be aged 20 years less than their parents); (ii) transfers perceived by individuals aged 65 and more and paid by their children (supposed to be aged 20 years less than their parents) if the pension is lower than a given threshold; in particular, the amount of the transfer is computed, for each education level, such that the pension plus the transfer is equal to 50% of the average labor income.

Pensions are determined according to the following rules. In the public sector ($c = c1$), the first pension is computed as 85% of the last wage:

$$pens_{11,s,e,c1,t} = 0.85 \cdot w_{e,t-1} \cdot A_{10,s,e,c1,t-1}$$

Then, we assume that pensions are indexed on prices and, thus, remain constant over time:

$$pens_{g+1,s,e,c1,t+1} = pens_{g,s,e,c1,t} \quad \text{for } g > 11$$

In the private sector ($c = c2$), the end-of-service indemnity is computed by capitalizing past contributions:

$$pens_{11,s,e,c2,t} = \sum_{g=1}^{10} \tau_{c,t+g-11}^{contr} \cdot w_{e,t+g-11} \cdot A_{g,s,e,c2,t+g-11} \cdot \prod_{g'=g}^{10} (1 + r_{t+g'-11})$$

Then, we have:

$$pens_{g+1,s,e,c2,t+1} = 0 \quad \text{for } g > 11$$

The optimal consumption path is described by the following Euler equation:

$$\frac{c_{g+1,s,e,t+1}}{c_{g,s,e,t}} = \frac{1 + r_{t+1} \cdot (1 - \tau_{t+1})}{1 + \rho} \cdot \frac{\beta_{g,s,t}}{\beta_{g+1,s,t+1}}$$

Thus, the intertemporal evolution of consumption depends on the (net of taxation) interest rate, the intertemporal preference rate and the conditional probability to be alive in $t + 1$ for an individual aged g in t with gender s.

3.3 The labor market

3.3.1 The matching function

The number of jobs created at the beginning of each period, for each education level e, noted by $M_{e,t}$, is given by the following matching functions:

$$M_{e,t} = f(V_{e,t}, \Omega_{e,t})$$

where $V_{e,t}$ is the number of vacancies posted by the representative firm (by education level e) and $\Omega_{e,t}$ is the number of people searching for a job (by education level e).

The probability to find a job for an individual with education e who searches for a job is:

$$p_{e,t} = \frac{M_{e,t}}{\Omega_{e,t}}$$

Note that this probability is assumed to be the same independently of the age and the gender of the individual.

The probability that a vacancy is filled is:

$$q_{e,t} = \frac{M_{e,t}}{V_{e,t}}$$

3.3.2 Number of individuals searching for a job

With respect to a standard model with search and matching frictions in the labor market, in an OLG model it is necessary to distinguish individuals according their age.

For people belonging to the first age class, i.e. individuals that enter the labor market for the first time, the number of individual searching for a job (by education level) is equal to the number of people participating in the labor market:

$$\Omega_{g,s,e,t} = z_{g,s,e,t} \cdot N_{g,s,e,t} \quad \text{with } g = 1$$

Concerning the other age classes ($g > 1$), we have to distinguish three cases.

i) The participation rate remains constant ($z_{g,s,t} = z_{g-1,s,t-1}$). The number of people looking for a job in t depends on the number of individuals that were unemployed in $t-1$ (and are still alive in t) and on the number of individuals that worked in $t-1$ and lose their job at the beginning of the current period. We note by χ the separation rate, which is assumed to be exogenous, constant and independent of the age, gender and education level. Then, the number of people searching for a job (by education level) is:

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,t} + \chi \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t} \quad \text{with } g > 1 \text{ and } z_{g,s,e,t} = z_{g-1,s,e,t-1}$$

ii) $z_{g,s,t} > z_{g-1,s,t-1}$. In this case, some individuals enter the labor market in t and thus search for a job. Thus, the number of people searching for a job (by education level) is:

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,t} + \chi_c \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,c,t} + \left(z_{g,s,e,t} - z_{g-1,s,e,t-1}\right) \cdot N_{g,s,e,t} \quad \text{with } g > 1 \text{ and } z_{g,s,e,t} > z_{g-1,s,e,t-1}$$

In the previous expression, the first two components in the RHS are as in the first case, while the last component represents the number of individuals that enter the labor market.

iii) $z_{g,s,t} < z_{g-1,s,t-1}$. In this case, some individuals exit the labor market in t. Thus, the number of people searching for a job (by education level) is:

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,t} + \chi \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t} - u_{g-1,s,e,t-1} \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t} - \chi \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t} \quad \text{with } g > 1 \text{ and } z_{g,s,e,t} < z_{g-1,s,e,t-1}$$

In the previous expression, the first two components in the RHS are as in the first case, while the third component represents the number of individuals that were unemployed in $t-1$ and that are still alive in t and become inactive; the fourth component represents the number of individuals that worked in $t-1$, lose their job at the beginning of the current period and don't look for a job since they become inactive.

In a more compact way, for individuals aged $g > 1$, the number of individuals searching for a job can be written as:[1]

$$\Omega_{g,s,e,t} = \left[z_{g,s,e,t} - (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \mu_{g,s,e,t}\right] \cdot N_{g,s,e,t} \quad \text{with } g > 1$$

with:

$$\mu_{g,s,e,t} = \begin{cases} 1 & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} \geq 0 \\ \dfrac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} < 0 \end{cases}$$

Note that the previous expression is different with respect of that in the article of de la Croix et al. (2012 ; equation 7) since in the article of de la Croix et al. (2012), the labor market participation rates are equal to one independently of the age.

Finally, the total number of people searching for a job (by education level) is:

$$\Omega_{e,t} = \sum_{g,s} \Omega_{g,s,e,t}$$

This variable is used in the matching functions.

3.3.3 Number of workers

The number of workers (by age, gender, education level) of individuals belonging to the first age class is given by the number of individuals that look for a job and find a job (with probability $p_{e,t}$):

[1] For the mathematical details, see Appendix 2a.

$$L_{g,s,e,c,t} = n_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$= p_{e,t} \cdot z_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} \quad \text{if } g = 1$$

Concerning people belonging to the other age classes, the number of workers (by age, gender, education level) is equal to the number of individuals that find a job among those who look for a job plus the number of individuals who worked during the previous period and still work since they are still active and don't lose their job. However, in the case of a fall in the participation rate, individuals that previously worked and become inactive must be removed. In a compact form, the number of workers is given: [2]

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} + \left(1 - p_{e,t}\right) \cdot (1 - \chi) \cdot n_{g-1,s,e,t-1} \cdot \mu_{g,s,e,t}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

if $g > 1$

with:

$$\mu_{g,s,e,t} = \begin{cases} 1 & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} \geq 0 \\ \dfrac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} < 0 \end{cases}$$

Considering that $L_{g,s,e,c,t} = n_{g,s,e,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$, the employment rate $n_{g,s,e,t}$ can be defined in the following way:

$$n_{g,s,e,t} = p_{e,t} \cdot z_{g,s,e,t} \quad \text{if } g = 1$$
$$n_{g,s,e,t} = p_{e,t} \cdot z_{g,s,e,t} + \left(1 - p_{e,t}\right) \cdot (1 - \chi) \cdot n_{g-1,s,e,t-1} \cdot \mu_{g,s,e,t} \quad \text{if } g > 1$$

The effect of a change on the employment rate of an individual aged l at time $t + l$ (with $l \geq 1$) on the employment rate at time of the same individual when aged k at time $t + k$ (with $k > l$) can be defined as follows:

$$\frac{\partial n_{k,s,e,c+k}}{\partial n_{l,s,e,t+l}} = \frac{\partial n_{k,s,e,t+k}}{\partial n_{k-1,s,e,t+k-1}} \cdot \ldots \cdot \frac{\partial n_{l+1,s,e,t+l+1}}{\partial n_{l,s,e,t+l}}$$
$$= \left(1 - p_{e,t+k}\right) \cdot (1 - \chi) \cdot \mu_{k-1,s,e,t+k-1} \cdot \ldots \cdot \left(1 - p_{e,t+l+1}\right) \cdot (1 - \chi) \cdot \mu_{l+1,s,e,t+l+1}$$

Following de la Croix et al. (2012), the value in $t + k$ of an additional job in $t + l$ (differentiated by age, gender, education level and occupational category) is given by:

[2] For the mathematical details, see Appendix 2b.

$$\frac{\partial U_{t+k}}{\partial L_{l,s,e,c,t+l}} \cdot \frac{1}{u'\left(c_{l,s,e,t+l}\right)}$$

$$= \sum_{l \le k} \frac{\beta_{k,s,t+k}}{\beta_{l,s,t+l}} \cdot \frac{u'\left(c_{k,s,e,t+k}\right)}{u'\left(c_{l,s,e,t+l}\right)} \cdot \frac{\left(1-\tau_{t+k}-\tau_{c,t+k}^{contr}\right) \cdot w_{e,t+k} \cdot A_{k,s,e,c,t+k}}{(1+\rho)^{k-l}} \cdot \frac{\partial n_{k,s,e,t+k}}{\partial n_{l,s,e,t+l}}$$

Considering the Euler equation $u'\left(c_{k,s,e,t}\right) = u'\left(c_{k+1,s,e,t+1}\right) \cdot \frac{\beta_{k,s,t}}{\beta_{k+1,s,t+1}} \cdot \frac{1+(1-\tau_{t+1}) \cdot r_{t+1}}{1+\rho}$ which implies that $\frac{1}{1+(1-\tau_{t+1}) \cdot r_{t+1}} = \frac{u'\left(c_{k,s,e,t}\right)}{u'\left(c_{k+1,s,e,t}\right)} \cdot \frac{\beta_{k+1,s,t+1}}{\beta_{k,s,t}} \cdot (1+\rho)$, we find a more clear economic interpretation:

$$\frac{\partial U_{t+k}}{\partial L_{l,s,e,c,t+l}} \cdot \frac{1}{u'\left(c_{l,s,e,t+l}\right)} = \sum_{l \le k} \frac{\left(1-\tau_{t+k}-\tau_{c,t+k}^{contr}\right) \cdot w_{e,t+k} \cdot A_{k,s,e,c,t+k}}{\prod_{l'>l}^{k} \left[1+(1-\tau_{t+l'}) \cdot r_{t+l'}\right]} \cdot \frac{\partial n_{k,s,e,t+k}}{\partial n_{l,s,e,t+l}}$$

Thus, the value in $t+k$ of an additional job obtained in $t+l$ is equal to the present value of the net wages (until the retirement age) weighted by the effect of this additional job on the employment rate.

3.4 Number of vacancies (by education level and occupational category)

The technology of the representative firm is described by a standard Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \cdot Z_t^{1-\alpha}$$

where K_t is the capital employed and Z_t is the number of units of effective labor supplied by all workers, defined as follows:

$$Z_t = \sum_{g,s,e,c} A_{g,s,e,c,t} \cdot N_{g,s,e,t} \cdot n_{g,s,e,t} \cdot \varphi_{g,s,e,c,t}^{c}$$

At each period, the representative firm chooses the optimal demand of capital and the optimal number of vacancies (by education level). This choice is made in order to maximize the value of the firm given the technological constraint and the probability that a vacancy is filled.

A vacancy implies a cost (assumed to be exogenous and growing at a rate equal to the productivity growth rate), noted by $c\_search_{e,c,t}$. The profit function is:

$$\pi_t = Y_t - (r_t + \delta) \cdot K_t - \sum_{g,s,e,c} w_{e,t} \cdot A_{g,s,e,c,t} \cdot N_{g,s,e,t} \cdot n_{g,s,e,t} \cdot \varphi^c_{g,s,e,c,t} - \sum_e c\_search_{e,t} \cdot V_{e,t}$$

The value of the firm, noted by W_t, is defined as the present value of all expected profits. Thus:

$$W_t = \sum_{t'} \frac{\pi_{t'}}{\prod_{t''<t'} (1 + r_{t''})}$$

The optimal demand of capital is determined such that the marginal productivity of capital is equal to the real cost.

$$\frac{\partial Y_t}{\partial K_t} = r_t + \delta$$

The optimal number of vacancies posted, by education level and occupational category, is determined such that the increase in the value of the firm related to an additional vacancy (marginal revenue) is equal to the search cost (marginal cost):

$$c\_search_{e,t} = q_{e,t} \cdot \sum_{g,s,c} \frac{\Omega_{g,s,e,t}}{\Omega_{e,t}} \cdot \frac{\partial W_t}{\partial L_{g,s,e,c,t}} \cdot \varphi^c_{g,s,e,c,t}$$

where, the marginal revenue is equal to the average increase in the value of the firm with an additional job and weighted by the probability that the vacancy is filled.

In particular, the increase in the value of the firm with an additional job is equal to the present value (until the retirement age) of all the differences between the marginal productivity of labor and the negotiated wage, weighted by the survival probability and the probability that the job is not destroyed:

$$\frac{\partial W_t}{\partial L_{g,s,e,c,t}} = \sum_{t'} \beta_{g+k,s,t+k} \cdot \frac{mpl_{t+k} \cdot A_{g+k,s,e,c,t+k} - w_{e,t+k} \cdot A_{g+k,s,e,c,t+k}}{\prod_{t''<t'} (1 + r_{t''})} \cdot (1 - \chi)^k$$

where mpl_t is the marginal productivity of labor:

$$\frac{\partial Y_t}{\partial Z_t} = mpl_t$$

3.5 Wage bargaining

The wage per unit of effective labor is assumed to be negotiated at each period and for each level of education. In particular, it is determined in order to maximize the following expression:

$$\left[\sum_{g,s,c} \frac{\partial W_t}{\partial L_{g,s,e,c,t}} \cdot \frac{\Omega_{g,s,e,t}}{\Omega_{e,t}} \cdot \varphi^{c}_{g,s,e,c,t}\right]^{1-\eta} \cdot \left[\sum_{g,s,c} \frac{\partial U_t}{\partial L_{g,s,e,c,t}} \cdot \frac{1}{u'\left(c_{g,s,e,t}\right)} \cdot \frac{\Omega_{g,s,e,t}}{\Omega_{e,t}} \cdot \varphi^{c}_{g,s,e,c,t}\right]^{\eta}$$

The optimality condition is the following:

$$\eta \cdot \sum_{g,s,c} \frac{\Omega_{g,s,e,t}}{\Omega_{e,t}} \cdot \frac{\partial W_t}{\partial L_{g,s,e,c,t}} \cdot \varphi^{c}_{g,s,e,c,t} = (1-\eta) \cdot \sum_{g,s,c} \frac{\Omega_{g,s,e,t}}{\Omega_{e,t}} \cdot \frac{\partial U_t}{\partial L_{g,s,e,c,t}} \cdot \frac{1}{u'\left(c_{g,s,e,t}\right)} \cdot \varphi^{c}_{g,s,e,c,t}$$

Thus, the wage per unit of effective labor, for each education level, is determined to share the total surplus created by an additional job among the representative agent and workers, according to their bargaining weights.

3.6 The pension system

For both the public sector and the private sector, the budget surplus is computed as follows:

$$S^{pens}_{c,t} = \tau^{contr}_{c,t} \cdot \sum_{g,s,e} n_{g,s,e,t} \cdot w_{e,t} \cdot A_{g,s,e,c,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} - \sum_{g,s,e} z^{pens}_{g,s,e,t} \cdot pens_{g,s,e,c,t} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Pension reserves (if $B_{c,t}$ is positive) change over time as follows:

$$B_{c,t+1} = B_{c,t} \cdot (1 + r_t) + S^{pens}_{c,t}$$

3.7 The government

We assume that the ratio between government purchases and GDP is exogenous and constant:

$$\frac{G_t}{Y_t} = \alpha_g$$

We also assume that the public financial situation is balanced at each period:

$$G_t = \tau_t \cdot \sum_{g,s,e} \left[r_t \cdot k_{g,s,e,t} + \alpha^{\pi}_{g,s,e,t} \cdot \pi_t + \sum_{c} n_{g,s,e,c,t} \cdot w_{e,t} \cdot A_{g,s,e,c,t} \right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

The tax rate is endogenously determined to equal public expenditures and public revenues.

3.8 The market equilibrium

The market clearing condition in the capital and in the goods markets are:

$$K_t = \sum_{g,s,e,c} k_{g,s,e,t} \cdot N_{g,s,e,c,t} + \sum_{c} B_{c,t}$$

$$Y_t = \sum_{g,s,e} c_{g,s,e,t} \cdot N_{g,s,e,t} + I_t + G_t + \sum\nolimits_{e} c\_search_{e,t} \cdot V_{e,t}$$

4. Model calibration

The model is calibrated in two steps. In the first step, the model is calibrated in steady state implying that the economy, in the absence of shocks, is assumed to be on its balanced growth path. In the second step, after the introduction of the demographic shock, some parameters and exogenous variables (in particular the initial stocks) have been calibrated in order to reproduce, for the most recent years, the main macroeconomic variables (GDP and GDP components) and those concerning the labor market (employment rates and unemployment rates by age, gender and education level) and the pension schemes (expenditures and contributions).

To build and calibrate our OLG model we use demographic data used from the United Nations, macroeconomic data from the national accounting provided by CAS, data about the situation of the pension schemes, and microeconomic data about the labor market situation provided by CAS and the micro data set. In particular, the micro data set is used to determine the value of the parameters $\varphi^{e}_{g,s,e,t}$ and $\varphi^{c}_{g,s,e,c,t}$ which permit, respectively to decompose the population (by age and gender) by education level and the number of workers (by age, gender and education level) by occupation category.

In tables 8, we compare the most important available data observed in 2010 with those generated by our OLG model for the same year. We show that our calibration procedure is quite accurate in reproducing observed data. In table 9, we show the results of our model for year 2010 concerning labor market variables (labor participation, unemployment, informality, etc.) at the national level and by age, gender and education level.

5. Simulations

5.1 Simulated shocks

In our base scenario, we analyze the economic consequences of population ageing in the absence of any reform. The base scenario is built by simulating four shocks simultaneously: i) the demographic shock (i.e. the change in fertility rates $x_{s,t}$ and in survival probabilities $\beta_{g,s,t}$) that modifies the age structure of the population; ii) the change in the composition of the population

by education level (i.e. the change in the parameters $\varphi^{e}_{g,s,e,t}$); iii) the change in labor participation rates (i.e. the parameters $z_{g,s,e,t}$); iv) the change in the composition of the workers by occupational category (i.e. the change in the parameters $\varphi^{c}_{g,s,e,c,t}$). These changes are introduced as exogenous shocks in the model.

As previously mentioned, the demographic shock will induce a strong increase in the old-age dependency ratio (see figure 1). In addition, the composition of the population by education level will be strongly affected. As shown in figure 2, the fraction of low educated people (preschool level and primary school level) is expected to significantly reduce in the next decades, while the fraction of high educated people with a secondary school level should strongly increase. Interestingly, the fraction of people with a university level is expected to slightly decrease over time. This situation is explained by the fact that highly educated young people decide to migrate, which compensates the high investment in education observed in Lebanon. Note that the evolution depicted in figure 2 has been determined by projecting the microeconomic data observed in 2010 concerning the composition of the population by education level and by assuming that the survival probabilities are not dependent on the education level.

5.2 Base scenario

In this section, we present the results of the base scenario which evaluates the effects of the demographic transition in Lebanon assuming that no reforms will be introduced.

First of all, the shocks indicated in section 5.1 will affect the level of capital supply and labor supply. The capital accumulation is affected because of different reasons. In particular, population ageing will worsen the financial situation of the two pension schemes and thus reduce the level of savings in the economy; individuals expect to live longer and, thus, decide to save more. Labor supply is directly affected by the shocks indicated in section 5.1: the demographic shock, the change in the labor participation rates, in the composition of the population by education level and in the composition of the number of workers by occupational category.

The main macroeconomic results are summarized in table 10.

As in standard OLG models, we find that population ageing increases the capital per unit of effective labor (i.e. the ratio K_t/Z_t). This is explained by the fact that even if capital supply is negatively affected by the shock, labor supply is even more affected since the population ageing implies that the number of workers with respect to the total population strongly falls.

The evolution of the capital per unit of effective labor affects positively the marginal productivity of labor and negatively the marginal productivity of capital. The increase in the marginal productivity of labor implies that the wage (for each education level) that firms and jobseekers negotiate will also increase. In contrast, the fall in the marginal productivity of capital implies a decline in the interest rate (computed as the difference between the marginal productivity of capital and the depreciation rate).

As previously said, population ageing affects negatively the accumulation of capital since the level of aggregate savings declines. This implies a strong fall in the ratio between investments and GDP (from 22% observed in 2010 to less than 10% starting from 2050).

Clearly, the evolution of capital and labor supply affects the evolution of the economic growth. Table 10 shows that the growth rate of GDP and GDP per person will strongly reduce in the next decades.

The effects concerning the labor market are reported in table 11. From one hand, the demographic shock affects the number of people searching for a job. On the other hand, the effects on the marginal productivity of labor and on the interest rate have important consequences on the number of vacancies posted by the representative firm.

As found by de la Croix et al. (2012), population ageing, by simultaneously increasing the marginal productivity of labor (which increases the gain for the firm to post a vacancy) and reducing the interest rate (which increases the present value of all the gains for the firm to post a vacancy), induces firms to create more vacancies for each education level. This affects positively the number of new jobs created, for each education level, which induces a decrease in the unemployment rate, both at the national level (from 6.4% in 2010 to 3.4% in 2060) and for each education level.

The effects on the pension schemes are reported in table 12. Clearly, population ageing will induce a strong increase in the number of retirees and in the ratio between the number of retirees and the number of workers, both at the national level and for each pension scheme. The evolution of the ratio between the number of retirees and the number of workers reflects the evolution of the old-age dependency ratio. As shown in table 12, the ratio between contributions and GDP remains essentially constant both for the public sector system and the private sector system. In contrast, the ratio between pension expenditures and GDP rises dramatically for the public pension scheme (from 3.1% in 2010 to 14.3% in 2060), while, for the private pension scheme the increase is less important (from 0.5% in 2010 to 3.8% in 2060). Consequently, the public pension scheme produces extremely high deficits (that would be higher than 10% of GDP after 2050) implying that the system is clearly unsustainable. In contrast, without considering the interest on pension reserves, the private scheme produces surpluses until 2040 (implying that pension reserves still increase) and deficits only starting from 2045. The private scheme can be considered as sustainable.

5.3 Alternative scenarios

5.3.1 Increase in women's labor participation

To be done

5.3.2 Reduction in the informal sector

To be done

6 Conclusions

This article evaluates the effects of population ageing in Lebanon, both at the macro level and concerning the sustainability of the pension system. We find that the private scheme, which is funded system that pays a unique lump-sum transfer when people retire, is sustainable. In contrast, we find that the public system, which is a standard PAYG system, is not sustainable. The reason is that this system is very generous given the low level of contribution rate.

However, as many other developing countries, Lebanon is characterized by a low level of labor participation of women and a large size of the informal sector. Thus, contrary to developed countries where the sustainability of their pension systems can be achieved only through standard policies (increase in contribution rates, reduction in pension benefits, increase in the retirement age), different tools are available in the Lebanese case. In particular, Lebanon can face the sustainability problem by introducing incentives for women to participate in the labor market and for informal workers to become formal.

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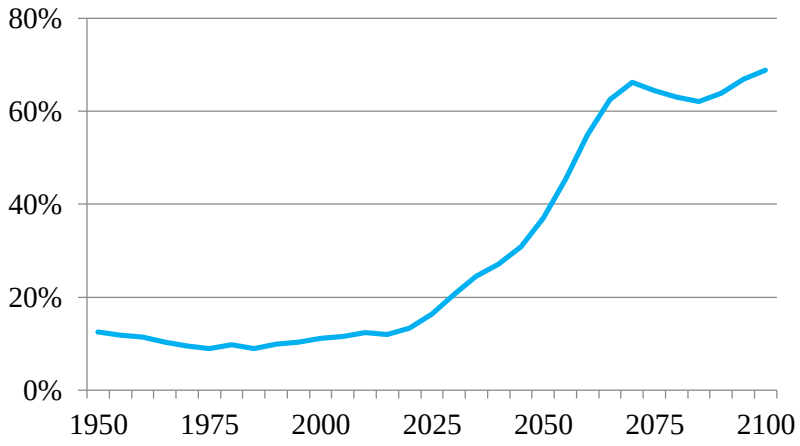
Table 1: Lebanese demographic evolution

	Total population	Children per woman	Life expectancy at birth (women)	Life expectancy at birth (men)	0-14 / Total population	15-64 / Total population	65+ / Total population
1950	1 335	5.74	62.2	58.9	34.2%	58.5%	7.3%
1975	2 576	4.23	69.4	65.9	39.8%	55.3%	4.9%
2000	3 235	2.01	77.4	73.9	28.6%	64.3%	7.1%
2005	3 987	1.58	79.7	76.0	27.9%	64.6%	7.5%
2010	4 337	1.72	80.9	77.3	23.7%	67.9%	8.4%
2015	5 851	1.95	81.7	78.3	24.0%	67.9%	8.1%
2020	6 020	2.09	82.4	79.2	21.6%	69.1%	9.2%
2025	5 606	2.19	83.1	80.2	19.9%	68.9%	11.3%
2050	5 412	2.21	86.5	84.7	13.8%	62.9%	23.3%
2075	5 062	2.26	89.6	87.8	13.2%	52.8%	34.0%
2100	4 350	2.28	92.0	90.1	13.6%	51.2%	35.2%

Source: World Population Prospects: 2017 Revision

Figure 1: Old age dependency ratio (65 + / 15-64)



Source: World Population Prospects: 2017 Revision

Figure 2: Composition of the population by education level

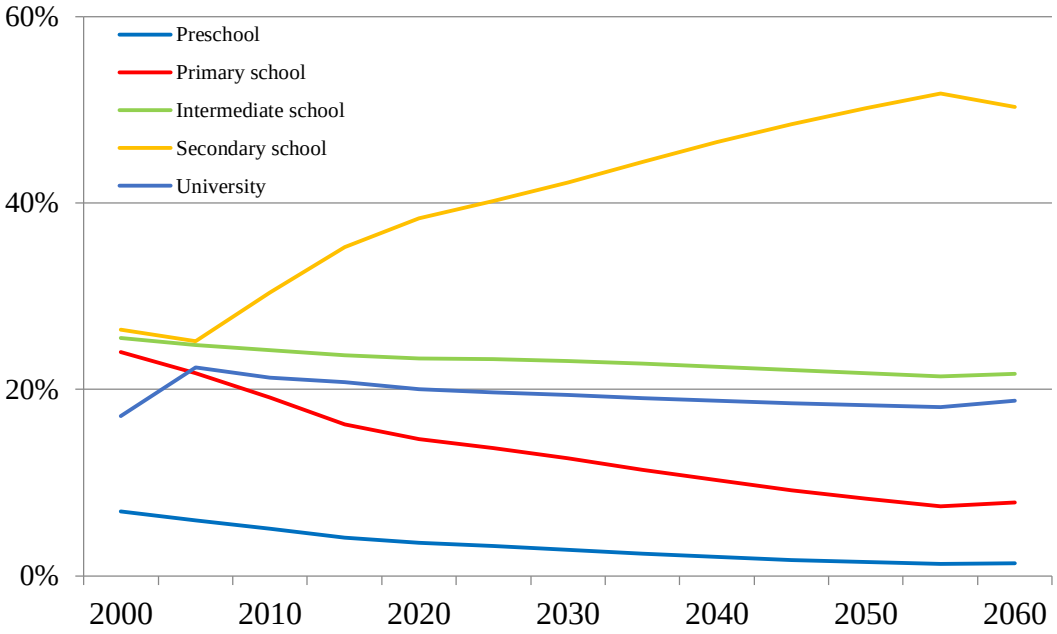


Table 2: Labor participation rate

	Women			Men			Total		
	2004	2007	2009	2004	2007	2009	2004	2007	2009
15-19	7.2%	6.0%	7.4%	25.4%	25.0%	27.9%	16.6%	16.2%	18.4%
20-24	29.1%	30.1%	34.9%	60.5%	59.7%	63.4%	45.0%	45.1%	49.6%
25-29	37.4%	42.4%	47.0%	87.4%	89.2%	94.4%	61.4%	65.0%	70.5%
30-34	28.9%	35.9%	32.8%	93.3%	93.9%	96.7%	60.1%	63.5%	63.2%
35-39	23.7%	24.6%	26.7%	94.1%	94.6%	97.1%	56.0%	56.0%	59.5%
40-44	23.7%	22.1%	23.3%	91.9%	90.2%	95.8%	54.4%	52.4%	57.2%
45-49	22.5%	22.1%	24.9%	91.3%	91.7%	94.4%	54.6%	54.1%	60.4%
50-54	18.1%	16.7%	18.7%	86.6%	83.3%	92.4%	51.2%	48.3%	51.1%
55-59	11.1%	14.5%	15.3%	77.3%	80.6%	90.0%	42.8%	45.5%	52.2%
60-64	7.6%	10.8%	9.2%	65.0%	63.6%	72.0%	34.9%	35.7%	40.7%
> 65	2.9%	1.7%	5.7%	31.5%	28.0%	49.3%	17.0%	14.6%	28.8%
15-64	22.4%	23.9%		73.4%	72.9%		47.1%	47.6%	
Total	20.4%	21.1%	22.8%	68.8%	66.9%	72.8%	44.0%	43.4%	47.6%

Source: CAS and ILO

Table 3: Labor participation rate by education level

	Women		Men		Total	
	2007	2009	2007	2009	2007	2009
Preschool	4.3%	7.6%	43.2%	52.0%	16.5%	22.1%
Primary school	13.2%	13.1%	78.7%	84.9%	49.5%	53.8%
Intermediate school	13.1%	14.2%	76.9%	83.1%	45.5%	50.0%
Secondary school	20.4%	19.3%	59.0%	60.8%	38.9%	39.2%
University	45.4%	47.0%	62.5%	68.9%	54.0%	58.0%
Total	21.1%	22.8%	66.9%	72.8%	43.4%	47.6%

Source: CAS and ILO

Table 4: Unemployment rate

	Women			Men			Total		
	2004	2007	2009	2004	2007	2009	2004	2007	2009
15-19	25.4%	36.0%	30.3%	27.1%	23.9%	17.4%	26.8%	25.9%	19.8%
20-24	17.4%	18.5%	20.5%	17.3%	21.5%	13.1%	17.3%	20.7%	15.6%
25-29	10.7%	12.5%	13.4%	7.8%	11.8%	6.5%	8.7%	12.1%	8.8%
30-34	7.3%	7.7%	7.2%	5.1%	6.2%	3.7%	5.6%	6.6%	4.7%
35-39	5.5%	3.8%	6.1%	3.9%	3.6%	2.1%	4.2%	3.6%	3.1%
40-44	6.7%	5.9%	5.2%	3.5%	2.8%	1.4%	4.3%	3.4%	2.2%
45-49	4.8%	4.4%	4.4%	2.7%	2.4%	1.6%	3.1%	2.8%	2.2%
50-54	1.3%	0.0%	3.0%	3.5%	7.6%	2.0%	3.2%	6.1%	1.6%
55-59	1.3%	10.9%	0.0%	2.5%	4.6%	2.1%	2.4%	5.7%	1.8%
60-64	2.0%	0.0%	1.2%	4.4%	2.4%	3.7%	4.1%	2.0%	3.4%
> 65	0.0%	0.0%		5.1%	5.1%		4.8%	4.9%	
15-64	9.6%	10.2%		7.4%	8.8%		8.0%	9.2%	
Total	9.5%	10.1%	10.4%	7.3%	8.6%	5.0%	7.8%	9.0%	6.4%

Source: CAS and ILO

Table 5: Unemployment rate by education level

	Women		Men		Total	
	2007	2009	2007	2009	2007	2009
Preschool	10.0%	3.7%	6.2%	4.7%	6.7%	4.4%
Primary school	12.1%	9.1%	8.4%	4.1%	8.8%	4.6%
Intermediate school	15.1%	10.5%	8.4%	4.4%	9.3%	5.2%
Secondary school	9.5%	14.3%	9.9%	5.3%	9.9%	7.7%
University	11.5%	11.4%	11.1%	7.0%	11.2%	8.8%
Total	10.2%	10.4%	9.1%	5.0%	9.4%	6.4%

Source: CAS and ILO

Table 6: Employment rate

	Women			Men			Total		
	2004	2007	2009	2004	2007	2009	2004	2007	2009
15-19	5.4%	3.6%	5.2%	18.5%	18.9%	23.0%	12.2%	11.8%	14.8%
20-24	24.1%	24.4%	27.7%	50.1%	46.6%	55.1%	37.2%	35.9%	41.9%
25-29	33.4%	37.3%	40.7%	80.6%	78.4%	88.3%	56.1%	57.2%	64.3%
30-34	26.8%	33.1%	30.4%	88.5%	87.9%	93.1%	56.8%	59.2%	60.2%
35-39	22.4%	23.2%	25.1%	90.4%	91.1%	95.1%	53.7%	53.6%	57.7%
40-44	22.2%	21.4%	22.1%	88.6%	88.2%	94.5%	52.1%	50.6%	55.9%
45-49	21.5%	21.2%	23.8%	88.8%	89.6%	92.9%	52.9%	52.6%	59.1%
50-54	17.8%	16.7%	18.1%	83.5%	77.4%	90.6%	49.6%	45.6%	50.3%
55-59	11.0%	13.2%	15.3%	75.3%	77.6%	88.1%	41.7%	43.4%	51.3%
60-64	7.4%	10.8%	9.1%	62.2%	62.1%	69.3%	33.5%	35.0%	39.3%
> 65	2.9%	1.7%	5.7%	29.9%	26.4%	49.3%	16.2%	14.1%	28.8%
15-64	20.2%	21.4%		68.0%	66.5%		43.3%	43.3%	
Total	18.4%	19.0%	20.4%	63.8%	61.1%	69.2%	40.6%	39.5%	44.6%

Source: CAS and ILO

Table 7: Employment rate by education level

	Women		Men		Total	
	2007	2009	2007	2009	2007	2009
Preschool	3.9%	7.3%	40.5%	49.6%	15.4%	21.1%
Primary school	11.6%	11.9%	72.1%	81.4%	45.1%	51.3%
Intermediate school	11.1%	12.7%	70.5%	79.4%	41.2%	47.4%
Secondary school	18.5%	16.5%	53.1%	57.6%	35.1%	36.2%
University	40.2%	41.6%	55.5%	64.1%	48.0%	52.9%
Total	18.9%	20.4%	60.8%	69.2%	39.3%	44.6%

Source: CAS and ILO

Table 8: Comparison between data and the model results for year 2010

		model	data
GDP		46.795	46.795
Investment / GDP		22.1%	22.1%
Pensions / GDP private sector	Public sector	3.06%	3.06%
	Private sector	0.50%	0.50%
Contributions / GDP private sector	Public sector	0.40%	0.40%
	Private sector	2.17%	2.17%
Wages / GDP in the public sector		6.72%	6.72%
Number of contributors in the public sector	Public sector	160.445	162.659
	Private sector	552.525	546.952
Labor participation rate	Total	49.2%	47.6%
	Women	22.8%	22.8%
	Men	72.8%	72.8%
	Pre-school	29.4%	22.1%
	Primary school	62.8%	53.8%
	Intermediate school	58.0%	50.0%
	Secondary school	45.9%	39.2%
	University	37.7%	58.0%
Unemployment rate	Total	6.4%	6.4%
	Women	9.7%	10.4%
	Men	5.5%	5.0%
	Pre-school	4.4%	4.4%
	Primary school	4.6%	4.6%
	Intermediate school	5.2%	5.2%
	Secondary school	7.7%	7.7%
	University	8.8%	8.8%

Table 9: Calibration results concerning the labor market

	Total					
	Activity rate	Employment rate	Unemployment rate	% of public workers	% of private workers	% of informal workers
Preschool	29.4%	28.1%	4.4%	0.0%	34.1%	65.9%
Primary school	62.8%	59.9%	4.6%	0.0%	28.7%	71.3%
Intermediate school	58.0%	55.0%	5.2%	0.0%	42.8%	57.2%
Secondary school	45.9%	42.4%	7.7%	31.6%	40.5%	27.9%
University	37.7%	34.4%	8.8%	16.5%	59.4%	24.1%
Total	**49.2%**	**46.0%**	**6.4%**	**12.1%**	**41.7%**	**46.2%**

	Women					
	Activity rate	Employment rate	Unemployment rate	% of public workers	% of private workers	% of informal workers
Preschool	8.4%	8.0%	4.1%	0.0%	45.3%	54.7%
Primary school	15.6%	14.9%	4.2%	0.0%	64.5%	35.5%
Intermediate school	18.2%	17.3%	5.0%	0.0%	76.0%	24.0%
Secondary school	29.7%	26.5%	10.7%	15.6%	76.8%	7.6%
University	23.5%	20.4%	13.3%	18.6%	70.0%	11.4%
Total	**22.8%**	**20.6%**	**9.7%**	**11.8%**	**73.2%**	**15.0%**

	Men					
	Activity rate	Employment rate	Unemployment rate	% of public workers	% of private workers	% of informal workers
Preschool	52.6%	50.2%	4.5%	0.0%	32.1%	67.9%
Primary school	91.7%	87.4%	4.6%	0.0%	24.9%	75.1%
Intermediate school	86.3%	81.8%	5.2%	0.0%	37.8%	62.2%
Secondary school	63.0%	59.1%	6.2%	39.2%	23.3%	37.5%
University	53.9%	50.4%	6.6%	15.6%	54.5%	29.9%
Total	**72.8%**	**68.8%**	**5.5%**	**12.2%**	**33.3%**	**54.5%**

Table 10: Base scenario (macro results)

	2010	2020	2030	2040	2050	2060
Capital per unit of effective labor	0.220	0.207	0.248	0.277	0.287	0.276
Wage per unit of effective labor (*e1*)	0.304	0.297	0.317	0.333	0.340	0.342
Wage per unit of effective labor (*e2*)	0.312	0.301	0.323	0.336	0.341	0.344
Wage per unit of effective labor (*e3*)	0.331	0.312	0.336	0.353	0.358	0.356
Wage per unit of effective labor (*e4*)	0.347	0.317	0.347	0.352	0.361	0.369
Wage per unit of effective labor (*e5*)	0.337	0.330	0.344	0.359	0.364	0.364
Interest rate	15.5%	16.2%	14.1%	12.8%	12.5%	12.9%
Investments /GDP	22.1%	9.3%	13.4%	10.4%	6.9%	4.7%
GDP growth rate	8.5%	3.5%	2.1%	2.2%	0.9%	-0.1%
GDP per person growth rate	4.7%	2.6%	2.4%	1.4%	0.5%	0.0%

Table 11: Base scenario (results on the labor market)

		2010	2020	2030	2040	2050	2060
Labor participation rate		49.2%	51.0%	51.3%	50.8%	51.0%	51.4%
Employment rate		46.0%	47.7%	49.4%	48.9%	49.0%	48.9%
Unemployment rate	Pre-school	4.4%	5.1%	4.3%	4.9%	6.4%	8.2%
	Primary school	4.6%	4.9%	4.0%	4.6%	6.0%	7.7%
	Intermediate school	5.2%	5.6%	3.9%	4.0%	4.4%	5.4%
	Secondary school	7.7%	8.3%	4.1%	3.6%	3.4%	4.0%
	University	8.8%	5.6%	2.8%	2.9%	4.1%	7.1%
	Total	6.4%	6.5%	3.8%	3.7%	3.9%	4.9%
Probability to find a job	Pre-school	81.4%	78.1%	81.4%	80.5%	76.7%	71.1%
	Primary school	80.6%	77.8%	81.8%	80.2%	76.2%	70.8%
	Intermediate school	79.3%	72.4%	78.3%	77.8%	76.2%	70.8%
	Secondary school	78.5%	73.4%	81.5%	81.5%	80.0%	74.5%
	University	78.8%	76.5%	77.5%	75.1%	72.1%	67.3%
Probability to fill a vacancy	Pre-school	75.0%	78.2%	75.0%	75.9%	79.7%	85.9%
	Primary school	70.0%	72.5%	69.0%	70.3%	74.0%	79.6%
	Intermediate school	60.0%	65.8%	60.8%	61.1%	62.5%	67.3%
	Secondary school	60.0%	64.1%	57.8%	57.8%	58.9%	63.2%
	University	60.0%	61.9%	61.1%	63.0%	65.6%	70.3%

Table 12: Base scenario (results on the pension schemes)

		2010	2020	2030	2040	2050	2060
Old-age dependency ratio		0.127	0.139	0.216	0.282	0.383	0.556
Retired / Workers	Total	0.133	0.138	0.230	0.315	0.415	0.586
	Public sector	0.153	0.107	0.122	0.193	0.246	0.364
	Private sector	0.066	0.066	0.127	0.200	0.370	0.622
Pensions / GDP	Public sector	3.1%	3.8%	4.3%	7.3%	10.5%	16.9%
	Private sector	0.5%	0.5%	1.2%	1.7%	3.6%	5.6%
Contributions / GDP	Public sector	0.4%	0.5%	0.5%	0.5%	0.6%	0.6%
	Private sector	2.2%	2.3%	2.3%	2.4%	2.2%	2.1%
Surplus / GDP	Public sector	-2.7%	-3.4%	-3.8%	-6.7%	-10.0%	-16.3%
	Private sector	1.7%	1.7%	1.1%	0.6%	-1.4%	-3.6%
Reserves / GDP	Public sector	-15.1%	-18.5%	-31.3%	-46.4%	-75.0%	-134.0%
	Private sector	10.6%	12.5%	19.9%	26.2%	34.5%	45.4%

Appendix 1 - Data coming from the micro data set

Table A1: Number of individuals classified by age. Source: Micro data set

	Women	Men	Total
15-19	344	355	699
20-24	356	373	729
25-29	266	303	569
30-34	240	195	435
35-39	207	183	390
40-44	274	195	469
45-49	324	204	528
50-54	285	203	488
55-59	231	197	428
60-64	209	185	394
65-69	146	151	297
70-74	144	126	270
75-79	116	111	227
80-84	70	63	133
85-89	37	45	82
90-94	8	10	18
95-99	6	2	8
Total	**3263**	**2901**	**6164**

Table A2: Number of individuals classified by education level. Source: Micro data set

	Women	Men	Total
Preschool	437	236	673
Primary school	526	579	1105
Intermediate school	740	761	1501
Secondary school	818	670	1488
University	742	655	1397
Total	**3263**	**2901**	**6164**

Table A3: Number of individuals by economic activity. Source: Micro data set

	Women	Men	Total
Inactive or unemployed	2503	1063	3566
Public sector	155	254	409
Private sector	449	786	1235
Informal sector	156	798	954
Total	**3263**	**2901**	**6164**

Table A4: Repartition of individuals by age, gender and education level. Source: Micro data set

	Women							Men					
	Preschool	Primary	Intermediate	Secondary	University	Total		Preschool	Primary	Intermediate	Secondary	University	Total
15-19	0.6%	2.9%	18.9%	61.0%	16.6%	100.0%		0.8%	8.5%	30.7%	48.2%	11.8%	100.0%
20-24	0.8%	5.3%	14.0%	13.5%	66.3%	100.0%		1.1%	9.7%	17.4%	19.8%	52.0%	100.0%
25-29	1.9%	4.1%	19.5%	21.4%	53.0%	100.0%		1.7%	12.2%	23.8%	18.8%	43.6%	100.0%
30-34	1.3%	12.1%	23.8%	26.7%	36.3%	100.0%		3.6%	14.4%	30.3%	18.5%	33.3%	100.0%
35-39	2.4%	13.5%	28.5%	32.4%	23.2%	100.0%		2.7%	15.8%	30.1%	32.2%	19.1%	100.0%
40-44	7.3%	13.1%	26.6%	34.3%	18.6%	100.0%		5.1%	21.0%	31.8%	25.1%	16.9%	100.0%
45-49	6.2%	19.1%	29.6%	32.7%	12.3%	100.0%		4.9%	27.0%	31.4%	24.0%	12.7%	100.0%
50-54	9.1%	24.2%	31.6%	24.6%	10.5%	100.0%		9.4%	23.6%	34.5%	15.8%	16.7%	100.0%
55-59	19.9%	25.5%	26.4%	16.5%	11.7%	100.0%		8.6%	26.9%	31.0%	18.3%	15.2%	100.0%
60-64	22.5%	31.6%	28.7%	11.5%	5.7%	100.0%		11.4%	33.5%	21.6%	20.0%	13.5%	100.0%
65-69	29.5%	36.3%	19.2%	12.3%	2.7%	100.0%		15.9%	27.2%	27.8%	16.6%	12.6%	100.0%
70-74	45.1%	27.1%	16.7%	9.0%	2.1%	100.0%		20.6%	28.6%	23.0%	19.8%	7.9%	100.0%
75-79	62.9%	22.4%	8.6%	2.6%	3.4%	100.0%		27.0%	35.1%	19.8%	12.6%	5.4%	100.0%
80-84	61.4%	18.6%	12.9%	4.3%	2.9%	100.0%		46.0%	31.7%	11.1%	6.3%	4.8%	100.0%
85-89	73.0%	10.8%	10.8%	5.4%	0.0%	100.0%		44.4%	44.4%	4.4%	4.4%	2.2%	100.0%
90-94	75.0%	25.0%	0.0%	0.0%	0.0%	100.0%		50.0%	40.0%	10.0%	0.0%	0.0%	100.0%
95-99	50.0%	0.0%	33.3%	16.7%	0.0%	100.0%		50.0%	0.0%	50.0%	0.0%	0.0%	100.0%

Table A5: Repartition of workers by age, gender, education level and economic activity. Source: Micro data set

			Public sector	Private sector	Informal sector	Total
15-19	Women	Preschool	-	-	-	-
		Primary school	0.0%	100.0%	0.0%	100.0%
		Intermediate school	0.0%	66.7%	33.3%	100.0%
		Secondary school	33.3%	66.7%	0.0%	100.0%
		University	-	-	-	-
	Men	Preschool	-	-	-	-
		Primary school	0.0%	84.2%	15.8%	100.0%
		Intermediate school	0.0%	91.7%	8.3%	100.0%
		Secondary school	0.0%	88.9%	11.1%	100.0%
		University	16.7%	83.3%	0.0%	100.0%
20-24	Women	Preschool	-	-	-	-
		Primary school	0.0%	100.0%	0.0%	100.0%
		Intermediate school	0.0%	84.6%	15.4%	100.0%
		Secondary school	16.7%	66.7%	16.7%	100.0%
		University	25.6%	69.2%	5.1%	100.0%
	Men	Preschool	0.0%	66.7%	33.3%	100.0%
		Primary school	0.0%	66.7%	33.3%	100.0%
		Intermediate school	0.0%	70.9%	29.1%	100.0%
		Secondary school	50.9%	31.6%	17.5%	100.0%
		University	33.8%	56.9%	9.2%	100.0%
25-29	Women	Preschool	-	-	-	-
		Primary school	0.0%	75.0%	25.0%	100.0%
		Intermediate school	0.0%	70.6%	29.4%	100.0%
		Secondary school	16.0%	76.0%	8.0%	100.0%
		University	25.0%	69.8%	5.2%	100.0%
	Men	Preschool	0.0%	50.0%	50.0%	100.0%
		Primary school	0.0%	72.2%	27.8%	100.0%
		Intermediate school	0.0%	80.3%	19.7%	100.0%
		Secondary school	42.9%	32.7%	24.5%	100.0%
		University	36.7%	54.1%	9.2%	100.0%
30-34	Women	Preschool	-	-	-	-
		Primary school	0.0%	100.0%	0.0%	100.0%
		Intermediate school	0.0%	83.3%	16.7%	100.0%
		Secondary school	17.4%	69.6%	13.0%	100.0%
		University	30.8%	60.0%	9.2%	100.0%
	Men	Preschool	0.0%	50.0%	50.0%	100.0%
		Primary school	0.0%	40.0%	60.0%	100.0%
		Intermediate school	0.0%	57.1%	42.9%	100.0%
		Secondary school	28.1%	37.5%	34.4%	100.0%
		University	26.3%	59.6%	14.0%	100.0%

			Public sector	Private sector	Informal sector	Total
35-39	Women	Preschool	-	-	-	-
		Primary school	0.0%	71.4%	28.6%	100.0%
		Intermediate school	0.0%	50.0%	50.0%	100.0%
		Secondary school	5.3%	73.7%	21.1%	100.0%
		University	27.8%	61.1%	11.1%	100.0%
	Men	Preschool	0.0%	66.7%	33.3%	100.0%
		Primary school	0.0%	39.3%	60.7%	100.0%
		Intermediate school	0.0%	55.8%	44.2%	100.0%
		Secondary school	22.8%	29.8%	47.4%	100.0%
		University	38.2%	50.0%	11.8%	100.0%
40-44	Women	Preschool	0.0%	33.3%	66.7%	100.0%
		Primary school	0.0%	50.0%	50.0%	100.0%
		Intermediate school	0.0%	52.6%	47.4%	100.0%
		Secondary school	21.4%	64.3%	14.3%	100.0%
		University	25.0%	46.9%	28.1%	100.0%
	Men	Preschool	0.0%	42.9%	57.1%	100.0%
		Primary school	0.0%	40.0%	60.0%	100.0%
		Intermediate school	0.0%	40.0%	60.0%	100.0%
		Secondary school	31.1%	22.2%	46.7%	100.0%
		University	18.8%	28.1%	53.1%	100.0%
45-49	Women	Preschool	0.0%	66.7%	33.3%	100.0%
		Primary school	0.0%	60.0%	40.0%	100.0%
		Intermediate school	0.0%	57.9%	42.1%	100.0%
		Secondary school	30.8%	53.8%	15.4%	100.0%
		University	57.1%	38.1%	4.8%	100.0%
	Men	Preschool	0.0%	100.0%	0.0%	100.0%
		Primary school	0.0%	32.6%	67.4%	100.0%
		Intermediate school	0.0%	46.6%	53.4%	100.0%
		Secondary school	26.8%	31.7%	41.5%	100.0%
		University	30.4%	34.8%	34.8%	100.0%
50-54	Women	Preschool	0.0%	66.7%	33.3%	100.0%
		Primary school	0.0%	66.7%	33.3%	100.0%
		Intermediate school	0.0%	46.2%	53.8%	100.0%
		Secondary school	26.1%	34.8%	39.1%	100.0%
		University	40.0%	40.0%	20.0%	100.0%
	Men	Preschool	0.0%	46.2%	53.8%	100.0%
		Primary school	0.0%	30.2%	69.8%	100.0%
		Intermediate school	0.0%	37.3%	62.7%	100.0%
		Secondary school	20.7%	24.1%	55.2%	100.0%
		University	27.3%	30.3%	42.4%	100.0%

			Public sector	Private sector	Informal sector	Total
55-59	Women	Preschool	0.0%	0.0%	100.0%	100.0%
		Primary school	0.0%	33.3%	66.7%	100.0%
		Intermediate school	0.0%	16.7%	83.3%	100.0%
		Secondary school	33.3%	50.0%	16.7%	100.0%
		University	85.7%	14.3%	0.0%	100.0%
	Men	Preschool	0.0%	10.0%	90.0%	100.0%
		Primary school	0.0%	28.9%	71.1%	100.0%
		Intermediate school	0.0%	44.0%	56.0%	100.0%
		Secondary school	31.3%	21.9%	46.9%	100.0%
		University	48.0%	24.0%	28.0%	100.0%
60-64	Women	Preschool	0.0%	66.7%	33.3%	100.0%
		Primary school	0.0%	20.0%	80.0%	100.0%
		Intermediate school	0.0%	66.7%	33.3%	100.0%
		Secondary school	40.0%	40.0%	20.0%	100.0%
		University	83.3%	16.7%	0.0%	100.0%
	Men	Preschool	0.0%	33.3%	66.7%	100.0%
		Primary school	0.0%	29.8%	70.2%	100.0%
		Intermediate school	0.0%	48.1%	51.9%	100.0%
		Secondary school	42.3%	23.1%	34.6%	100.0%
		University	36.8%	21.1%	42.1%	100.0%

Table A6: Wage equation in the public sector, private sector and informal sector. Source: Micro data set

	Public sector	Private sector	Informal sector
Age	0.121***	0.204***	0.225***
	(0.038)	(0.020)	(0.030)
Age square	-0.003	-0.012***	-0.014***
	(0.003)	(0.002)	(0.002)
Men	0.198***	0.257***	0.566***
	(0.041)	(0.030)	(0.055)
Edu_2	-	0.286***	0.228***
	-	(0.082)	(0.083)
Edu_3	-	0.397***	0.357***
	-	(0.079)	(0.084)
Edu_4	-0.236***	0.561***	0.647***
	(0.041)	(0.082)	(0.089)
Edu_5	-	0.941***	0.914***
	-	(0.081)	(0.096)
Constant	9.054***	8.004***	7.864***
	(0.100)	(0.097)	(0.134)
Adjusted R^2	0.28	0.28	0.23
Number of observations	409	1235	954

Appendix 2a

The number of individuals aged $g > 1$ that look for a job in t is given by:

Case 1: $g > 1$ and $z_{g,s,e,t} = z_{g-1,s,e,t-1}$

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,t} + \chi_c \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t}$$

Considering that $u_{g,s,e,t} \cdot z_{g,s,e,t} = z_{g,s,e,t} - n_{g,s,e,c,t}$, we have:

$$\Omega_{g,s,e,t} = \left(z_{g-1,s,e,t-1} - n_{g-1,s,e,t-1}\right) \cdot N_{g,s,e,t} + \chi_c \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t}$$

Thus:

$$\boldsymbol{\Omega_{g,s,e,t}} = \left[\boldsymbol{z_{g-1,s,e,t-1}} - (\mathbf{1} - \boldsymbol{\chi_c}) \cdot \boldsymbol{n_{g-1,s,e,t-1}}\right] \cdot \boldsymbol{N_{g,s,e,t}}$$

Case 2: $g > 1$ and $z_{g,s,e,t} > z_{g-1,s,e,t-1}$

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,c,t} + \chi_c \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,c,t} + \left(z_{g,s,e,t} - z_{g-1,s,e,t-1}\right) \cdot N_{g,s,e,c,t}$$

Considering that $u_{g,s,e,t} \cdot z_{g,s,e,t} = z_{g,s,e,t} - n_{g,s,e,t}$, we have:

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - n_{g-1,s,e,t-1} + \chi \cdot n_{g-1,s,e,t-1} + \left(z_{g,s,e,t} - z_{g-1,s,e,t-1}\right)\right] \cdot N_{g,s,e,t}$$

Thus:

$$\boldsymbol{\Omega_{g,s,e,t}} = \left[\boldsymbol{z_{g,s,e,t}} - (\mathbf{1} - \chi) \cdot \boldsymbol{n_{g-1,s,e,t-1}}\right] \cdot \boldsymbol{N_{g,s,e,t}}$$

Case 3: $g > 1$ and $z_{g,s,e,t} < z_{g-1,s,e,t-1}$

$$\Omega_{g,s,e,t} = u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1} \cdot N_{g,s,e,t} + \chi \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t}$$

$$- \; u_{g-1,s,e,t-1} \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t}$$

$$- \; \chi \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \left(z_{g-1,s,t-1} - z_{g,s,t}\right) \cdot N_{g,s,e,t}$$

Considering that $u_{g,s,e,t} \cdot z_{g,s,e,t} = z_{g,s,e,t} - n_{g,s,e,t}$, we have:

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - n_{g-1,s,e,t-1} + \chi \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[u_{g-1,s,e,t-1} + \chi \cdot \left(1 - u_{g-1,s,e,t-1}\right)\right] \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[\chi + (1-\chi) \cdot u_{g-1,s,e,t-1}\right] \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t}$$

We multiply and divide by $z_{g-1,s,e,t-1}$:

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[\chi + (1-\chi) \cdot u_{g-1,s,e,t-1}\right] \cdot \mathbf{z_{g-1,s,e,t-1}} \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{\mathbf{z_{g-1,s,e,t-1}}} \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[\chi \cdot z_{g-1,s,e,t-1} + (1-\chi) \cdot u_{g-1,s,e,t-1} \cdot z_{g-1,s,e,t-1}\right] \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{z_{g-1,s,e,t-1}} \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[\chi \cdot z_{g-1,s,e,t-1} + (1-\chi) \cdot \left(z_{g-1,s,e,t-1} - n_{g-1,s,e,t-1}\right)\right] \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{z_{g-1,s,e,t-1}} \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[\chi \cdot z_{g-1,s,e,t-1} + (1-\chi) \cdot z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{z_{g-1,s,e,t-1}} \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$$

$$- \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{z_{g-1,s,e,t-1}} \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot \left(1 - \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right) \cdot N_{g,s,e,t}$$

$$\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} \cdot N_{g,s,e,t}$$

Finally:

$$\boldsymbol{\Omega_{g,s,e,t}} = \left[\boldsymbol{z_{g,s,e,t}} - \boldsymbol{(1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}}\right] \cdot \boldsymbol{N_{g,s,e,t}}$$

Considering the three cases together, it is possible to write in a compact form:

$$\Omega_{g,s,e,t} = \left[z_{g,s,e,t} - (1-\chi)\cdot n_{g-1,s,e,t-1}\cdot \mu_{g,s,e,t}\right]\cdot N_{g,s,e,t}$$

with:

$$\mu_{g,s,e,t} = \begin{cases} 1 & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} \geq 0 \\ \dfrac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} < 0 \end{cases}$$

Appendix 2b

The number of workers aged $g > 1$ (differentiated by age, gender, education level and occupational category) is given by:

Case 1: $g > 1$ and $z_{g,s,t} = z_{g-1,s,t-1}$

$$L_{g,s,e,c,t} = p_{e,t} \cdot \Omega_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} + (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Considering that $\Omega_{g,s,e,t} = \left[z_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$, we have:

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g-1,s,e,t-1} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} + (1-\chi) \cdot n_{g-1,s,c,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Thus:

$$\boldsymbol{L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g-1,s,e,t-1} + \left(1 - p_{e,t}\right) \cdot (1-\chi_c) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,c,t} \cdot \varphi^{c}_{g,s,e,c,t}}$$

Case 2: $g > 1$ and $z_{g,s,t} > z_{g-1,s,t-1}$

$$L_{g,s,e,c,t} = p_{e,t} \cdot \Omega_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} + (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Considering that $\Omega_{g,s,e,t} = \left[z_{g,s,e,t} - (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t}$, we have:

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Thus:

$$\boldsymbol{L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} + \left(1 - p_{e,t}\right) \cdot (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}}$$

Case 3: $g > 1$ and $z_{g,s,t} < z_{g-1,s,t-1}$

$$L_{g,s,e,c,t} = p_{e,t} \cdot \Omega_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} + (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t} - (1-\chi) \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Considering that $\Omega_{g,s,e,t} = \left[z_{g,s,e,t} - (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right] \cdot N_{g,s,e,t}$, we have

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,c,t} \cdot \varphi^{c}_{g,s,e,c,t} - (1-\chi) \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \left(z_{g-1,s,e,t-1} - z_{g,s,e,t}\right) \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

We multiply and divide by $z_{g-1,s,e,t-1}$. Then:

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$- (1-\chi) \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \boldsymbol{z_{g-1,s,e,t-1}} \cdot \frac{z_{g-1,s,e,t-1} - z_{g,s,e,t}}{\boldsymbol{z_{g-1,s,e,t-1}}} \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$- (1-\chi) \cdot \left(1 - u_{g-1,s,e,t-1}\right) \cdot \boldsymbol{z_{g-1,s,e,t-1}} \cdot \left(1 - \frac{z_{g,s,e,t}}{\boldsymbol{z_{g-1,s,e,t-1}}}\right) \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

Considering that $u_{g,s,e,t} \cdot z_{g,s,e,t} = z_{g,s,e,t} - n_{g,s,e,t}$, we have:

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,c,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$- \left[(1-\chi) \cdot z_{g-1,s,e,t-1} \cdot \left(1 - \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right) - (1-\chi) \cdot \left(z_{g-1,s,e,t-1} - n_{g-1,s,e,t-1}\right) \cdot \left(1 - \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right)\right]$$
$$\cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$- \left[(1-\chi) \cdot z_{g-1,s,e,t-1} - (1-\chi) \cdot z_{g,s,e,t} - (1-\chi) \cdot z_{g-1,s,e,t-1} \cdot \left(1 - \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right) + (1-\chi)\right.$$
$$\left. \cdot n_{g-1,s,e,t-1} \cdot \left(1 - \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right)\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} - p_{e,t} \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} + (1-\chi) \cdot n_{g-1,s,e,t-1}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$
$$- \left[(1-\chi) \cdot z_{g,s,e,t} - (1-\chi) \cdot z_{g,s,e,t} + (1-\chi) \cdot n_{g-1,s,e,t-1} - (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}\right] \cdot N_{g,s,e,t}$$
$$\cdot \varphi^{c}_{g,s,e,c,t}$$

Finally:

$$\boldsymbol{L_{g,s,e,c,t}} = \left[\boldsymbol{p_{e,t} \cdot z_{g,s,e,t} + (1 - p_{e,t}) \cdot (1-\chi) \cdot n_{g-1,s,e,t-1} \cdot \frac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}}}\right] \cdot \boldsymbol{N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}}$$

Considering the three cases together, it is possible to write in a compact form:

$$L_{g,s,e,c,t} = \left[p_{e,t} \cdot z_{g,s,e,t} + (1 - p_{e,t}) \cdot (1-\chi_c) \cdot n_{g-1,s,e,t-1} \cdot \mu_{g,s,e,t}\right] \cdot N_{g,s,e,t} \cdot \varphi^{c}_{g,s,e,c,t}$$

with:

$$\mu_{g,s,e,t} = \begin{cases} 1 & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} \geq 0 \\ \dfrac{z_{g,s,e,t}}{z_{g-1,s,e,t-1}} & \text{if } z_{g,s,e,t} - z_{g-1,s,e,t-1} < 0 \end{cases}$$
