On the redesign of Pay-As-You-Go Retirement Systems: towards more generational and inter-generational equality

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Abstract: *Retirement PAYG systems work following a combination of two objectives which are not necessarily correlated: poverty alleviation and smoothing benefits to contribution. Between the two, retirement benefits are defined in function of the reference wage in, somehow, an arbitrary way which summarize a set of economic, political, cultural and social environments where this system has evolved. Thus, the retirement formula can result in huge inequalities between the possible contribution-retirement scenarios taken from the profitability point of view. Usually, the years of contributions are associated with a validation factor to calculate pension benefits. Also, an anticipated retirement can imply a reduction factor in proportion with the number of anticipated years. Such a practice, even common in many countries, the used formula is not well adapted to each country-specific circumstances and can lead to huge inter and intra-generational inequalities. In this paper, we consider the Algerian case, and we show that the possible contribution-retirement scenarios allowed in the system lead to Internal Return Rate going from 9% to 15%. We propose and compare a few models allowing to reformulate the first pension benefit in function of final wage, the years of contribution and the age of retirement. Results allowed concentrating the IRRs in into 12% for most of the possible scenarios. Note that, inequalities in salaries were not considered here since wages at age x and year t are supposed to be equals.*

Key-words*: Retirement, PAYG, equality, IRR, scenario, Algeria.*

1. INTRODUCTION

Pay-As-You-Go retirement systems try to combine two principals who are not necessarily correlated: poverty reducing and smoothing retirement benefits to contributions (Bovenberg and Van Ewijk, 2011a). The first principle is based on the intra and inter-generational solidarity and public subsidies in the framework of the social protection function of the state. The second aims to ensure more equity between what contributors pay for retirement and what they receive from it.

One way to evaluate the generosity of a retirement system is to evaluate the Internal Return Rate (IRR) provided for a retirement contract holder (Diseny, 2006; Reznik et al., 2009; Ben Brahem, 2009 Luthen, 2016). In this sense, Inequality within a retirement system can be viewed as a difference between the IRR's provided for different individuals within the same generation or belonging to different generations.

In many countries over the world where retirement works following the PAYG system, retirement benefits are wage linked. The reference wage is either the average wage during the whole working career or the final wage averaged on the 5 or 10 last working years. Then, each year of contribution is associated with an annuitization rate. The replacement rate is estimated by the product of the years of contribution and the annuitization factor. Also, a penalization function is associated with each anticipated year below the legal age of retirement. This formulation, even very common in defining pension benefits under the PAYG system, it remains very discussable since its definition does not obey to well-defined rules. Between two extremes, fully funded and full distribution, a combination is arbitrarily defined in midway between the two according to a set of cultural, historical, political factors (Bovenberg and Van Ewijk, 2011b). Additionally, the formulation of pension benefits in function of the reference wage should be described by a simple formula that can be understood by everybody involved in the retirement contract: the contracted himself and also retirement funds employees. That is why it is important to avoid complicated formulations in defining pension benefits.

If there should be inter and intra-generational transfers within a system, it should be from the high to low earning people.

In this work, assuming that wages are same for all workers aged x in time t, we show that the classical formulation of the retirement benefits leads to huge inequalities not depending on wage. Afterward, we will propose a new formulation of pension benefits in a way to reduce inequalities.

To this end, we evaluate the inequalities within the Algerian retirement system dedicated to salaried employees. We define all the possible scenarios regarding the contribution of career and retirement. Then, we try to reformulate the retirement benefits formula by proposing and evaluating simple models based on the years of contribution, the age of retirement and with secondary importance, the year of retirement.

2. GENERAL CONTEXT

2.1. Design of the Algerian retirement system

The retirement system in Algeria is composed of three schemes, one for militaries, one for the high civil servants and government staff and a civil scheme. This least is arranged into two sub-schemes; the scheme dedicated for salaried employees is managed by "La Caisse Nationale des Retraites CNR" while the one dedicated for self-employed people is managed by "La Caisse d'Assurance Sociale des Non-Salaries CASNOS". The part of retirees covered by CNR is almost 90% of the total covered by both CNR and CASNOS.

In this paper, we focus only on the salaried-employees regime.

The regulatory age of retirement in the Algerian System is 60 years old. The 1st retirement benefit is calculated by the formula:

$$RB_x = 2.5\% * n * \overline{W}_{[x-5,x[}$$

With:

- RB_x is the retirement benefit that a newly retired perceive, age x represents the age of retirement;
- n is the number of years of contribution;
- $\overline{W}_{[x-5,x[}$ is the average wage of the 5 years before retirement (final wage); for simplification issues, we note it W*;

According to this formula, with 32 years of contribution, which represent the max, the first retirement benefit should represent 80% of the final wage. On the other side, the contribution rate for retirement (CRR) has evolved from 5% in 1985 to 18.25% in 2016. Interested readers can see Flici and Planchet (2019) for a more detailed overview of the Algerian retirement system.

2.2. Generational equality

A retirement system tries to combine two principals in defining the relationship contributions to benefits; social protection and life insurance. The social protection component in retirement systems aims to protect low earning people and reduce poverty while the second component tries to keep some linkage between the paid contributions and the expected retirement benefits.

These two objectives are not necessarily well correlated. Hence, when designing a retirement system, policymakers need to define an equilibrium point between the two. Retirement is a tool for social protection addressed to old people. Thus, the state needs to proceed to a transfer of revenues from the reaches to the poorest over the retirement system itself or through taxation. This mechanism allows ensuring an intra and intergenerational sharing between the retirees. On the other side, the insurance mechanism tries to ensure more equality in linking retirement benefits to paid contributions. A retiree needs to perceive the equivalent of what he had paid during the contribution phase. This linkage of contributions – benefits encourage people to improve their contribution effort.

3. THE PROBLEM OF SYSTEMIC INEQUALITY

In order to study the systemic inequalities in the Algerian retirement system, we estimate the "Internal Return Rate" for all the possible scenarios contributions-retirement that can be generated in the Algerian system.

For the needs of our calculations, we set some assumptions in order to simplify the environment of retirement. First, we suppose that the contribution period is concentrated in the early stage of the working career, with no interruption.

3.1. All the possible scenarios

In the first stage, we create all the possible scenarios by combining 3 main variables :

- Age of retirement;
- Year of retirement;
- Number of years of contribution;

We set the age of retirement to vary between 50 and 70 years. Even if the legal age of retirement was set at 60 years, many retirement types have allowed getting retired starting from the age of 50 for men and 45 for women. Similarly, many workers keep working beyond the age of 60 years and in some cases beyond 65 (Flici and Planchet, 2019). These early retirement types have been removed starting from January 2018.

For the year of getting retired, we preferred to consider the period going from 2015 to 2030.

In concern of the number of years of contribution, we assume it to vary between 15 and 40 years while considering that a max of 32 years is counted to calculate retirement benefits.

In total, we got 8736 scenarios corresponding to the combination of 21 ages of retirement, 16 possibilities with the year of retirement and 26 cases with the years of contribution.

Removal of the impossible combinations

Then, the impossible combinations in the environment of the three variables are removed. To do this, some constraints were introduced:

- The age of getting into the contribution system between goes 18 and 55 years, the ages younger than 18 and those older to 55 are removed.
- The year of getting into the contribution system is between 1977 and 2015;

The first constraint ties to the fact that the mortality surface for Algeria is only available starting from 1977. This information will be used to draw the survival function from the date of the first contribution until the age of retirement. More details about this point will be explained later in this paper.

The second constraint comes from the interaction between the minimal required years of contribution (15 years) and the year of retirement (2015-2030).After the removal of the impossible combinations, we obtain a total number of scenarios equal to 8120.

3.2. Estimation of the IRR for each scenario

The IRR represent simply the discount rate which allows to equalize the sum of expected contributions and the sum of the expected retirement benefits at any time t. If we consider this time t to be the year of retirement under each scenario, and $IRR(i)$ to be the IRR of each scenario 'i' going from 1 to 8120. All payments are supposed to be done at the beginning of the year; the contribution for retirement are paid as a part of the annual wage by applying the CRR at the moment of payment. Wage is supposed to evolve according to age and time, a linear time evolution and a quadratic age function are used to describe this 2-dimensional evolution. The average of the 5 final wages is used to estimate the first retirement benefit. Then, retirement benefits are annually revaluated with a factor (1+rev).

Wages

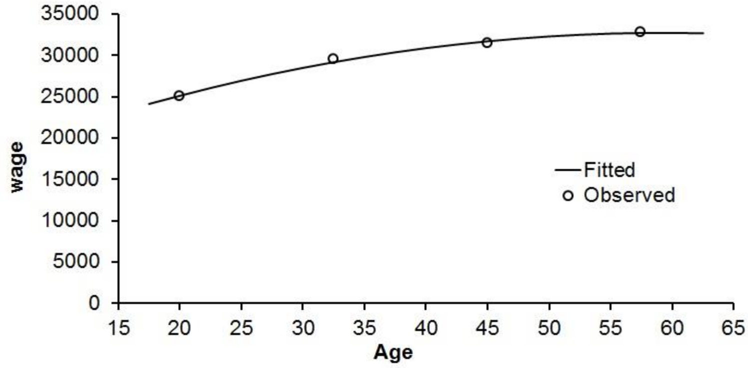
The evolution of wages can be tied to a combination of factors. Essentially, we consider the working experience and time to be the main factors to explain the most important part of wages variability. The working experience or productivity is hugely related to age. On the other hand, wages are affected by general economic growth. Hence, we suppose to explain the age evolution by two factors, age and time. First, we suppose that, in similar conditions, the average wage at the time $(t + 1)$ is equal to the average wage at the time (t) augmented by an evolution factor $(1 + wgr)$ with wgr is “wages growth rate”. We can write:

$$\overline{w}_{(t+1)} = \overline{w}_{(t)} * (1 + wgr)$$

On the other hand, wages vary with age. Flici and Planchet (2019) used the distribution of wages by huge age groups given by the wage survey conducted by the Algerian Office for National Statistics for the year 2011 (ONS, 2014). Figure 1 shows this evolution. Unfortunately, no updated data was published after that date. Here, we suppose that the age distribution of wages remains the same for the whole period of analysis.

In Flici and Planchet (2019b), the crude distribution was fitted with a quadratic function which allowed to estimate the model allowing to write the evolution of wages in regards to single ages going from 18 to 65.

Figure 1: Wage Vs. Age, Algeria (2011)



Source: Flici and Planchet (2019).

Here, we use the same function to extend the distribution until age 70 to suit the working age range in the present paper. Then, instead of modeling $w_{(x,t)}$, we preferred to model the ratio of the wage at age x and time t on the average wage at year t: $\frac{w_{(x,t)}}{\overline{w}_{(t)}}$. Accordingly, we can write:

$$\widehat{w}_{(x,t)} = \overline{w}_{(t)} * (a + b * +c * x^2)$$

If we suppose the first wage to be equal to 1 at age x and time t (we remind that inequalities in salaries are not considered in the present paper. Thus, this assumption doesn't have any effect on the calculation of the IRR), the wage at time $(t+1)$ while the worker is aged $(x+1)$ can be deduced from his wage during the previous years when he was aged x. This relationship can be deduced by combining the two previous formulas. That gives:

$$w_{(x+1,t+1)} = \overline{w}_{(t)} * (1 + \frac{(b + c + 2 * c * x)}{(a + b * x + c * x^2)}) * (1 + wgr)$$

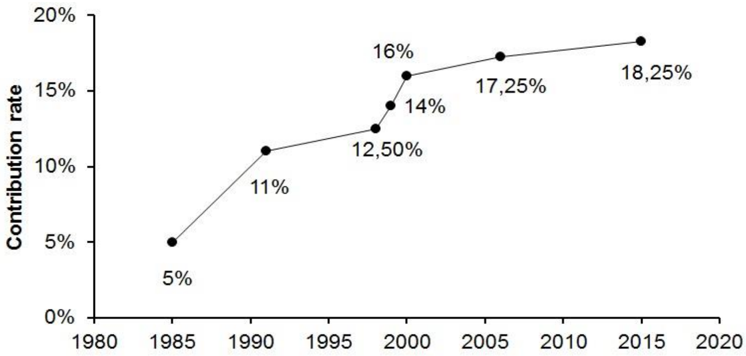
With : $a = 0.5244, b = 0.0196,$ and $c = -0.00016$.

The annual wage growth rate is supposed to keep at 9.5% (for more details see Flici and Planchet, 2019b).

Contributions

Contributions for retirement are paid as a part of the contribution to social security. It represents almost 50% and it's shared between the insured himself, the employer and a part of 0.5% as a public subsidy. Figure 1 shows the historical evolution of the CCR in Algeria.

Figure 2: Evolution of the Contribution Rate for Retirement in Algeria



Source: Flici and Planchet (2019b)

In this paper, contributions are estimated according to the wage at the considered age and year and to the contribution rate during the considered year. For the period [2015,2030], we assume that the CRR will keep at his current level. Thus, the contribution to retirement (CR) can be expressed as follows:

$$CR_{x,t} = w_{x,t} * CRR_t$$

Survival functions

The payment of both contributions and retirement benefits is conditioned by the fact to survive to the payment dates, those of the expected contributions and those of the expected retirement benefits. We assume that longevity will keep improving with different allures for males and females and also for the global population and retirees.

The survival function during the contribution phase is described by the global population coherent forecast conducted by Flici (2016). Due to the absence of a life table specific to the insured population in Algeria, we assume that the mortality of the insured population during the contribution phase is similar to that of the global population with a distinction between males and females. Flici and Planchet (2019) constructed experience prospective life tables for the retirees of CNR, for males and females and for the ages of 50 and over. These life tables are used in the present paper to describe the survival function of retirees from the age of retirement until the surviving age limit. Since the assumed reference year for survivorship during the two phases of retirement is the year of paying the first contribution, and since the experience life tables start at the age of retirement, the survival probability to any age during the retirement age should be taken with respect to the fact to survive to the age of retirement.

If we consider $q_{x,t}^{g}$ to be the age specific mortality rate for an individual from the global population at age x and year t, $q_{x,t}^{e}$ the mortality rate for the retired population, the probability to survive from the first age of contribution (x) to any age of the contribution phase ($x + n$) can be writen as

$$ {}_n p_{x,t} = \prod_{x=x,t=t}^{n,n} (1 - q_{x,t}^{g}) $$

The probability to survive from the first age of contribution (x) to any age ($x + s$) of the retirement phase, giving that $x + r$ is the age of retirement and knowing that $r > s$, can be written as:

$$ {}_s p_{x,t} = {}_s p_{x,t}^{g} \prod_{x=x+s,t=t+s}^{r-s,r-s} (1 - q_{x,t}^{e}) $$

Retirement benefits

The first retirement benefit is calculated as follows:

$$ RB_r = w^* * 2.5\% * n $$

With

r: The age of retirement;

w^* represents the final wage averaged on the last 5 years ($w^* = \frac{1}{5}\sum_{s=r-5}^{r-1} w_s$);

n: the number of years of contribution;

Then, the following retirement benefits are calculated by applying an annual growth rate (br) of 5% (Flici and Planchet, 2019). It gives:

$$ RB_{r+1} = RB_r * (1 + br) $$

Estimating the Internal Return Rate (IRR)

Following the definition of the internal return rate, IRR is the discount/return rate which equalize the sum of the expected contributions (SEC) and the sum of the expected retirement benefits (SERB). If we consider the year of retirement as the reference years for all the financial flows within a retirement contract, we can write:

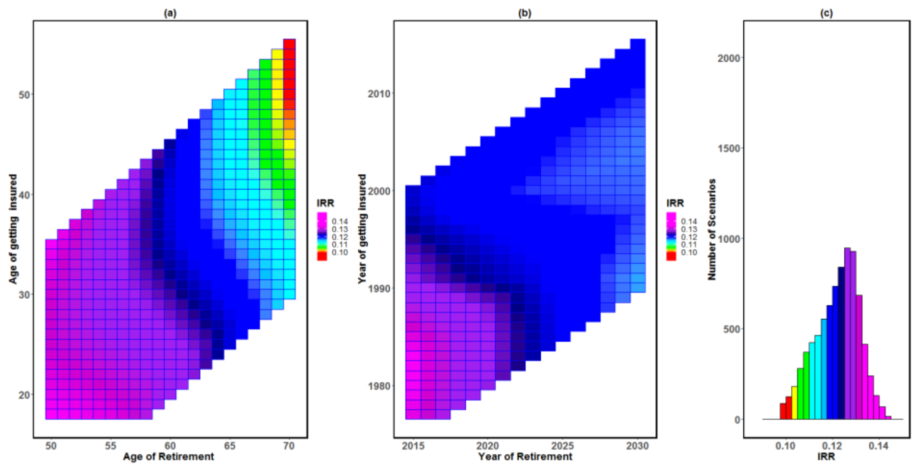
$$ \sum_{m=x,k=t}^{x+r,t+r} CR_{m,k} * {}_{m-x,k-t}p_{x,t}^{g} * (1 + irr)^{(r-k)} - \sum_{l=x+r,v=t+r}^{x+s,t+s} RB_{l,v} * {}_{r,r}p_{x,t}^{g} * {}_{s-r,s-r}p_{x+r,t+r}^{e} * (1 + irr)^{-(v-t-r)} = 0 $$

The IRR of each scenario was estimated through an optimization problem based on the "*optimx*" package under R. The obtained results are shown in Figure 3.

The obtained results show an important variability of the IRR from a scenario to another. Even if the average IRR was estimated at 12.05%, the minimum and maximum were at 9.8% and 14.43% respectively. The standard deviation of the IRR distribution is equal to 0.9%. The lowest IRR is associated with the following scenario: start working (contribution) at age 30 in 1990 and get retired at age 70 in 2030 with a whole contribution period of 40 years. On the other side, we find those who have started working (contributing) at age 18 in 1983 and got retired at age 50 in 2015 after a 32 years career.

This difference of nearly 4.62% can lead to huge disparities in term of total income in the long run. For example, at a pace of 30 years, a return rate of 1% come up with 34% of the initial investment. Our objective in the present paper is to reduce as possible this gap in term of IRR while keeping the calculation method to calculate the first retirement benefit as simple as possible. Perfect equality can only be achieved by making the sum of the expected retirement benefits equal to the sum of the paid contributions while assuming a common actuarial rate for all individuals within the system. However, performing such calculations in an individualized context is very complicated and can not easily be understood by the contractors. It is preferable to keep calculation formulas as simple and understandable as possible.

Figure 3: IRR Vs. Year and age of contribution/retirement



Item (a) refers to the IRR in regard to the age of getting insured and the age of retirement; item (b) draws the same indicators regarding the years of events, (c) shows the density plot of the different scenarios according to the IRR for intervals of 0.25% width from 9% to 15%.

On the other hand, equality goes against equity and risk sharing. One of the objectives of a retirement plan is to ensure solidarity between poor and reach people within the same population.

4. THE PROPOSED MODEL

Our idea can be expressed as follows :

- To assume a common IRR for all the scenarios as a target;
- To estimate the deserved retirement benefit time distribution;
- Giving that retirement benefits are tied to the first retirement benefits, we deduce the value of the optimal first retirement benefit.

- In the second stage, we try to link this latter to the average of the 5 last wages which can be also approximated by a function based on the number of years of contribution using generaled linear models GLM;

This procedure comprises a loss of information related to the fact to replace complicated actuarial formulas by simplified ones with fewer variables. As a result, we must expect that the assumption of the uniqueness of IRR for all scenarios cannot be fully respected. Thus, the final objective is to reduce the gap as possible.

4.1. First pension benefit Vs. Final wage

In the beginning, we have actuarial equality between two quantities: the sum of the expected contribution on one side, and the sum of the expected retirement benefits on the other side.

$$\sum_{m=x,k=t}^{x+r,t+r} CR_{m,k} * {}_{m-x,k-t}p_{x,t}^{g} * (1+irr)^{(r-k)} = \sum_{l=x+r,v=t+r}^{x+s,t+s} RB_{l,v} * {}_{r,r}p_{x,t}^{g} * {}_{s-r}p_{x+r,t+r}^{e} * (1+irr)^{-(v-t-r)}$$

If we try to write the first term of the equation in function of the final wage w*, and the second term in function of the 1st retirement benefit (RB_r), we can write the previous equation as:

$$A * w^* = B * RB_r$$

A and B are the results of dividing the two parts of the previous equation by w^* and RB_r and that we name "coefficient of contributions" and "coefficient of retirement benefits" respectively. w^* can is calculated by averaging the last 5 annual wages during the working career under each scenario. RB_r is defined to be an optimal solution to keep the actuarial neutrality between contributions and retirement benefits. It can simply be obtained by:

$$RB_r = w^* * \frac{A}{B}$$

4.2. Simplifying the Contributions – benefits formula

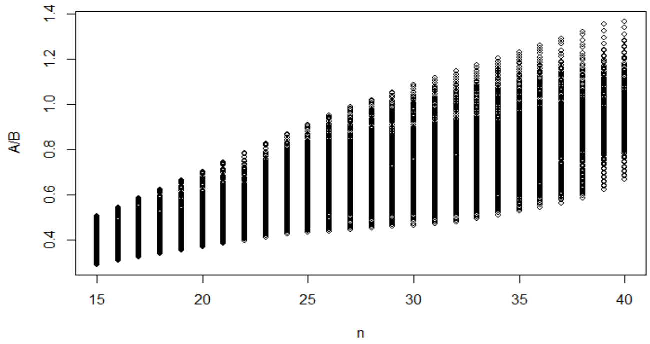
Next, we try to model the first retirement benefit in function of the final wage and the number of years of contribution using a GLM. The idea is to write $\frac{A}{B}$ in function of n. That allows writing:

$$RB_r = w^* * f(n, t, p)$$

Following the actuarial equivalence between the sum of the paid contributions and that of the expected retired benefits, the ratio A/B can be explained mainly by three variables: the years of contribution (n), the age of retirement (t) and the year of retirement (p).

Figure 4 shows the distribution of $\frac{A}{B}$ in function of n.

Figure 4: the ratio A/B in function of the number of years of contribution



In order to keep the formula of calculation of the first retirement benefit simple, we propose to compare three GLM models to fit the ratio A/B:

- 1 variable model based on the years of contribution as an explicative variable: Here, we propose to compare three models: a linear model (L1.1), a log-linear model (L1.2) and a log-log-linear (L1.3) model. These three models can be written as:

$$\frac{\hat{A}}{B} = a_1 + b_1 * n \text{ ………………… (L1.1)}$$

$$exp(\frac{\hat{A}}{B}) = a_2 + b_2 * n \text{ …………... (L1.2)}$$

$$exp(exp(\frac{\hat{A}}{B})) = a_3 + b_3 * n \text{ …… (L1.3)}$$

With a_1, a_2, a_3 and b_1, b_2, b_3 are parameters of the regression models.

- Two variables models: In a first step, we propose to compare a linear and a log-linear formulation based on two variables: the years of contribution n and the age of retirement x.

In the first stage, we propose two models based on n and t. The linear model is named L2.1, while the log-linear model is noted L2.2. The two models can be written as:

$$\frac{\hat{A}}{B} = a_4 + b_4 * n + c_4 * t \text{ ……………………..... (L2.1)}$$

$$exp(\frac{\hat{A}}{B}) = a_5 + b_5 * n + c_5 * t \text{ ………………..... (L2.2)}$$

Then, we base the same models on the variables n and p. The two models are noted L2.3 and L2.4 respectively and can be written as:

$$\frac{\hat{A}}{B} = a_6 + b_6 * n + d_6 * p \text{ ……………………..... (L2.3)}$$

$$exp(\frac{\hat{A}}{B}) = a_7 + b_7 * n + d_7 * p \text{ ………………..... (L2.4)}$$

- Three variables models: In a third stage, we compare two variants of the model based on the three variables n, t and p. The first variant consists of a linear model (noted L3.1) while the second is a log-linear model.

$$\frac{\hat{A}}{B} = a_8 + b_8 * n + c_8 * t \ + d_8 * p \text{ .... } (L3.1)$$

$$exp(\frac{\hat{A}}{B}) = a_9 + b_9 * n + c_9 * t \ + d_9 * p \text{ .... } (L3.2)$$

In all, we have 9 models to be evaluated and compared. From each family, we select the model which allows minimizing the Sum of Squared Errors (SSE) between the observed and the fitted value of A/B.

Figure 6 shows the results obtained with the models L1.1, L1.2 and L1.3.

Figure 5: fitting the ration A / B on n with the one variable GLM models

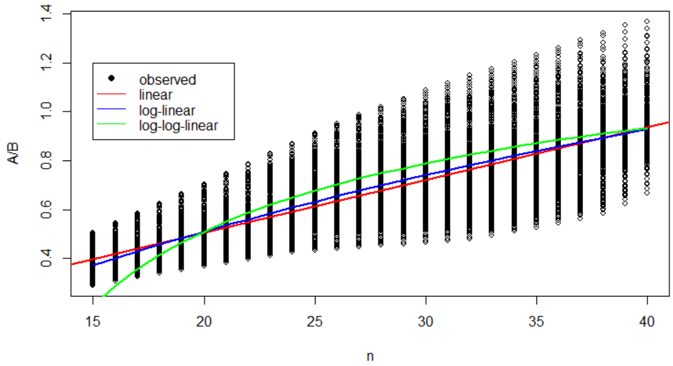
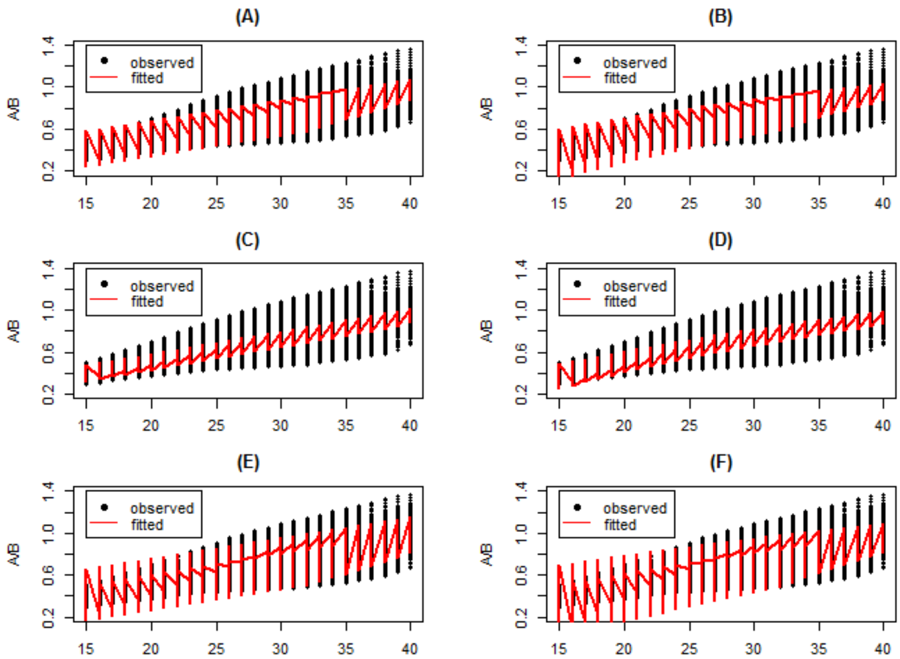


Figure 6 shows the results obtained with the 2 and 3 variables models. The estimated parameters of the 9 models, as well as the Sum of Squared Errors (SSE), are provided in table 2.

Figure 6: fitting the ration A / B on n with the two and three variables models



Items (A), (B), (C) and (D) refers to the two variables models L2.1, L2.2, L2.3, and L2.4 respectively. Items (E) and (F) refers to the three variables models, i.e., L3.1 and L3.2.

Table 1 : Models estimation and comparison

	L1.1	L1.2	L1.3	L2.1	L2.2	L2.3	L2.4	L3.1	L3.2
a(i)	7%	80%	-218%	-91%	-123%	-2064%	-4461%	-2152%	-4641%
b(i)	2%	4%	37%	2%	4%	2%	4%	2%	4%
c(i)	NA	NA	NA	2%	4%	1%	2%	2%	4%
d(i)	NA	NA	NA	NA	NA	NA	NA	1.02%	2%
SSE(i)	122.02	122.5	153.1	41.34	51.9	104.1	109.8	23.5	40.5

The L1.1 model is the best model among the first family of models based on one variable which is the "years of contribution". Also, within the family of models based on two explicative variables, the linear regression model based on the "years of contribution" and the "age of retirement" (noted L2.1) is the one which leads to the smallest SSE. The resulting SEE is equal to 41.34 falls down to 23.5 when the third variable (year of retirement) is added to the model (noted L3.1).

For the following, we will keep working with the three models L1.1, L2.1 and L3.1. The three models are taken according to their simplicity and goodness of fit. The more the goodness-of-fit is better, the less the model is simple.

5. RESULTS DISCUSSION

In what follows, we will estimate the revised IRR based on the different formulas of calculation of the first retirement benefits, and we calculate the distribution of all the scenarios according to the IRR they provide. The more the width of the distribution is narrowed; the degree of equality within the system is highlighted.

The models L1.1, L2.1, and L3.1 lead respectively to the three retirement formulas:

$$RB_r^{l1.1} = w^* * (7.17\% + 2.17\% * n)$$

$$RB_r^{l2.1} = w^* * (41\% + 2\% * (n - 15) + 1\% * (t - 50))$$

$$RB_r^{l3.1} = w^* * (16.25\% + 4\% * (n - 15) + 1.7\% * (t - 50) + 1\% * (p - 2015))$$

The figures 7, 8, and 9 illustrate the distribution of the retirement scenarios in function of the IRR provided by the models L1.1, L2.1, and L3.1 respectively. Compared to the current retirement formula (model 0), the model L1.1 does not provide significant added value regarding the dispersion of the IRR by retirement scenarios. As can be seen in Table 2, the two models have similar distributions.

Table 2: Comparison of the different models in terms of equality

	0	L1.1	L2.1	L3.1	L3.1*
max	14.43%	14.22%	13.45%	14.39%	12.99%
min	9.90%	9.90%	10.66%	8.85%	11.01%
mean	12.18%	12.11%	12.09%	12.06%	12.04%
variance	8.6E-05	8.5E-05	2.8E-05	2.7E-05	1.4E-05
Q10%	10.83%	10.77%	11.44%	11.52%	11.57%
Q25%	11.54%	11.46%	11.71%	11.81%	11.82%
Q50%	12.32%	12.22%	12.02%	12.02%	12.02%
Q75%	12.87%	12.81%	12.46%	12.30%	12.26%
Q90%	13.28%	13.26%	12.83%	12.66%	12.55%

Figure 7: revised IRR Vs. Year and age of contribution/retirement – Model L1.1.

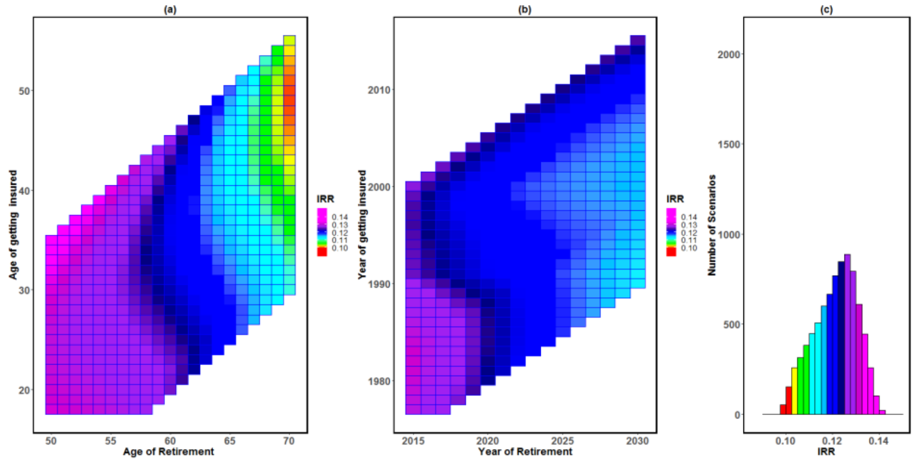


Figure 8: revised IRR Vs. Year and age of contribution/retirement – Model L2.1.

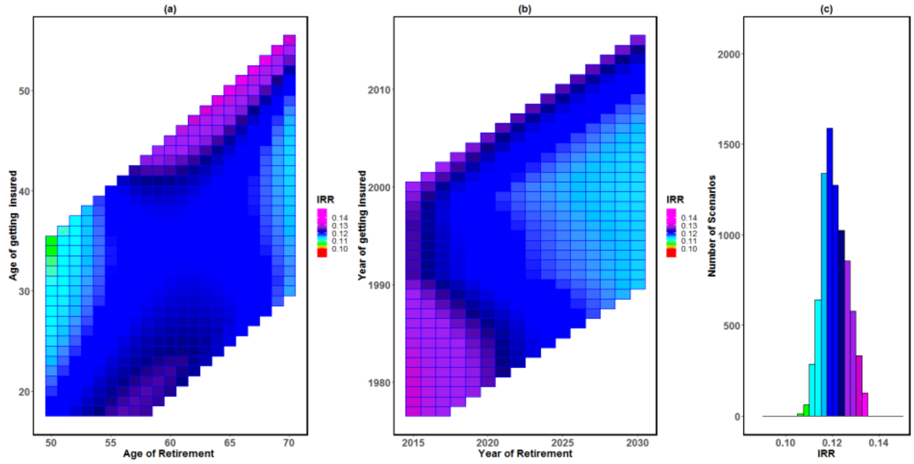
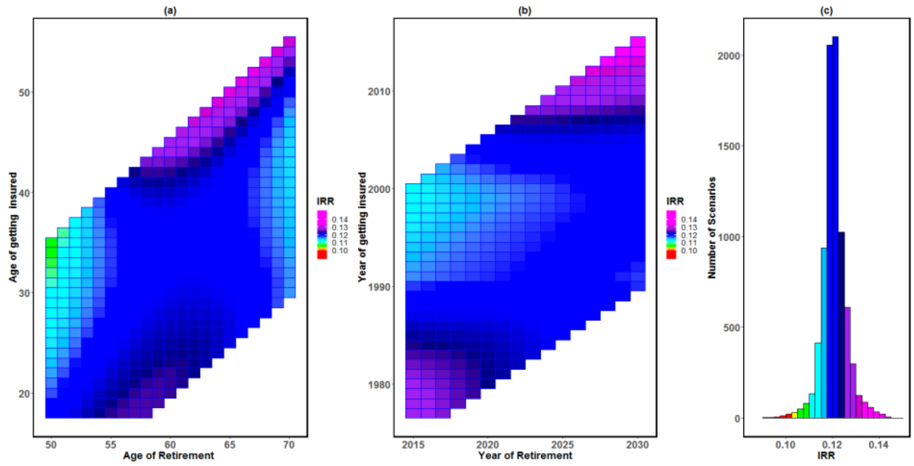


Figure 9: revised IRR Vs. Year and age of contribution/retirement – Model L3.1.

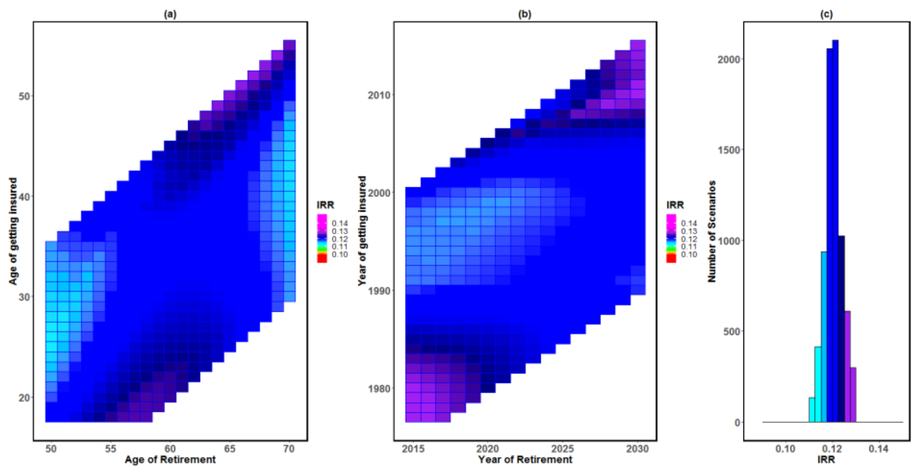


Item (a) refers to the revised IRR in regard to the age of getting insured and the age of retirement; item (b) draws the same indicators regarding the years of events.

In contrary, Model L2.1 allowed to reduce significantly the dispersion of IRR provided by the different retirement scenarios. Even if the average IRR stood nearly the initial average, i.e, from 12.18% to 12.09%, the width of the distribution that it provides narowed significantly by passing from an interval of [9.9%,14.43%] to [10.66%,13.45%]. Model L3.1 allows to concentrate further the distribution of IRRs by scenario. Even if 80% of the scenarios lead, under this last model, to an IRR comprised in the interval [11.52%, 12.66%] which is a little more restircted than under model L2.1. However, when the two models are compared to their distribution min-max, we observe that under Model L3.1 lead to larger distribution width. The minimum and the maximum which were at 10.66% and 13.45% in L2.1 passe to 8.85% to 14.39%, respectively, in L3.1.

Our idea at this point is to enhance the model L3.1 by eleminating the extremes scenarios. To improve the equality within the retirement system, some regulation need to be imposed to disalow people converging towards the extrem working-retirement scearios. In our case, we propose to eliminate the scenarios leading to either an IRR higher than 13% or lower than 11%. The new model is noted L3.1* and the resulting distribution of IRR by retirement scenarios are shown in Figure 10.

Figure 10: Revised distribution of retirement scenarios Vs. IRR



After having eliminated the extreme scenarios, 7573 scenarios are leading to IRR comprised between [11.01%, 12.99%].

The elimination of the extreme scenarios can be achieved by imposing or adjusting the requirements to access to retirement. The main factors that regulation can modify are: the years of contribution and the age of retirement. The evolution of the IRR according to these two variables is shown in Figures 11 and 12 respectively.

From Figure 11, it appears that under model L3.1, it is only starting from an age of retirement of 57 years that retirees can hope to have an IRR higher than 11% however the other retirement parameters are.

Figure 11: IRR Vs. Age of Retirement

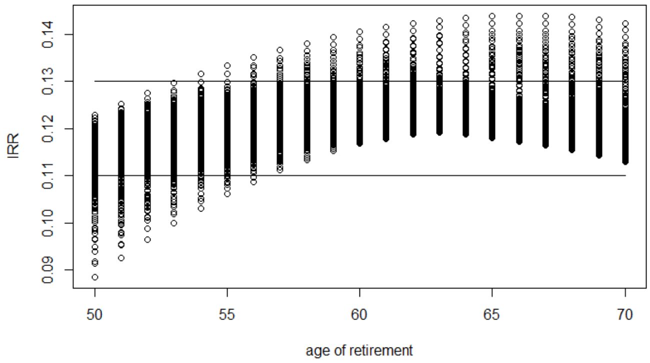
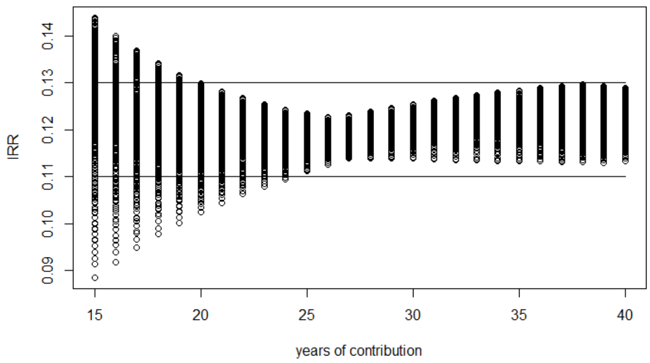


Figure 12: IRR Vs. Years of contribution



The distribution of IRR in function of the years of contribution shows that IRR is situated in the interval [11%, 13%] starting from 25 years of contribution. Setting the minimum years of contribution at 25 years with no limitation in regards to the age of retirement, will allow to improve the equality within the retirement system based on formula L3.1.

6. CONCLUSION

Poverty reducing is the ultimate objective behind maintaining the long run financial sustainability of a retirement system. In Pay-As-You-Go retirement systems, pensions benefits are defined on the basis of the reference wage multiplied by a replacement rate, this last is calculated by associating each contributed year to an annuitization rate. This formulation can be adjusted in order to keep the system sustainable as long as possible but without refereeing to strong formulas.

We have shown in this paper how the classical retirement formula in Pay-As-You-Go system can lead to huge inequalities between the different retirement scenarios taken from the investment return point of view. First, we defined all the possible contribution-retirement scenarios in the Algerian retirement system according to three variables, i.e., the age of retirement, the years of contribution, and the year of retirement. We assumed that: 1) there is an automatic contribution to social security when we start working, and 2) the contribution period is continuous and concentrated to the end of the working career. Also, we assumed that 3) all individuals aged x during the year t earn the same salary.

By assuming wages to keep evolving with an annual rate of 9.5% in the future, the average Internal Return Rate (IRR) is at 12% varying between 9.5% to around 14.5%. Such a huge difference of nearly 5% is supposed to result in a significant gap in wealth accumulation along the working-retirement career.

The main objective of this paper was to propose a retirement formula allowing reducing these inequalities. The idea was to start from a formula based on the actuarial fairness between the paid contributions and the deserved retirement benefits while considering mortality improvement, wages evolution, etc. Then, the expected contributions and the expected benefits are expressed in term of the final wage and the first retirement benefit, respectively. After, the ratio of the two quantities is summarized into a simple formulation using Generalized Linear Models (GLMs). Different GLMs were compared, i.e., a GLM based on the number of years of contributions, a 2-variables GLM based on the number of years of contribution and the age of retirement, a 3-variables GLM including the year of retirement.

The 2 and 3 GLM models allowed reducing significantly the dispersion of the IRR by scenarios. Under the 3-Glm model, over 93% of the scenarios lead to an IRR comprised between 11% and 13%. In the end, it was necessary to delete the 7% of the remaining scenarios by imposing some regulation. In our opinion, it will be necessary to set the minimum years of contribution to be 25 years without any age constraint. That said, a one can retire when having accumulated 25 years of contribution independently from age.

Beside of being an application case for Algeria, the methodology presented here can be used in other countries and contexts in order to adjust retirement formula. Instead of adjusting formulas in an arbitrary way, the presented method can be used to consider the future changes in terms of longevity, wages evolution, etc.,

On the other side, the presented methodology can be used to fix a targeted level of generosity within the system. Then, the retirement formula is set to provide equal Internal Return Rates for the different contribution-retirement scenarios.

In the end, we recall that our application case ignore wages inequalities. Such a point can be addressed with the required attention in a separated paper. Then, the objective can be set to prevent wages inequalities to be reproduced in retirement or simply to reduce the effects of this inequality. Similarly, this paper was based on high inflation case which implies high return rates. Accordingly, the gap between the min and max scenarios is supposed to be relatively important and can lead to important inequalities regarding wealth accumulation for the long run. Such a finding needs to be assessed when wages growth rates are relatively lower. By consequence, the efficiency of the proposed methodology to reduce inequalities need to be assessed on low inflations cases.

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