Revisiting the Macroeconomics of Population Aging

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February 20, 2019

Abstract

We revisit the macroeconomic consequences of population aging. For this, we build a large-scale overlapping generations model with endogenous human capital accumulation and a retirement decision. We calibrate the model to match key moments of US economy and we then perform counterfactual experiments in which demographic variables are changed to their 2050 projected values. Mortality declines stimulate savings, human capital accumulation and labor supply at the extensive margin, while the change of the age structure reduces the supply of production factors. When the contribution rate of the pension system adjusts to balance the budget, the impact of aging on output per capita is negative. A contrario, when the replacement rate of the pension system adjusts, the behavioral responses to the mortality decline fully offset the negative consequences of the age structure change.

Keywords: population aging, economic growth
JEL classification: O41, I15, E13

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†I am grateful to seminar participants at Le Mans Université and François Langot for helpful comments.

1 Introduction

Population in advanced economies is aging. This is the result of fertility rates below the replacement level and mortality declines at old ages. A greater life expectancy and a growing population share of old individuals illustrate this trend. In US, from 2050 individuals will live almost 6 years longer on average and the dependency ratio, hence the ratio of people aged between 20 and 60 to people aged more than 60, will grow from $33,8\%$ to $56,7\%$.[1] There is a renewed academic interest in the economic consequences of these demographic changes. Aging could both explain the persistent low interest rate level (Eggertson et al. (in press)) as well as the growth slowdown (Gordon (2016)) observed in developed economies. In this paper, we revisit the macroeconomic consequences of population aging through the lens of an overlapping generations model (henceforth OLG model).

We extend previous studies by examining the impact of population aging in a framework including all the key decisions related to demographic variables. In our environment, individuals decide how much to save for life cycle and precautionary motives. They decide the fraction of time to be active that they split between working and training to augment their human capital. Individuals also choose the age at which they retire given the constraints of the pension system to which they contribute. We calibrate the stationary equilibrium of this economy to match key moments of US economy given actual demographic data. We then perform a counterfactual experiment in which demographic variables are changed to their 2050-projected values to gauge the magnitude of the various effects at play. The mortality decline has several implications in the individuals' problem, modifying savings, human capital and labor decisions at the intensive and extensive margins. While these behavorial responses have been previously identified, our first contribution is to point out that their interactions create new forces that alter the usual conclusions about the consequences of population aging.

Our second contribution is to argue that the two facets of aging, the mortality decline and the change of the age structure, have *opposite* consequences in terms of economic growth. The rise of survival probabilities stimulates human capital accumulation, labor supply at the extensive margin and the capital stock and thus positively contributes to output per capita. A contrario a greater share of old individuals is unambiguously negative in terms of growth because old individuals hold fewer assets and are less productive. Moreover, in our main experiment, the replacement rate of the pension system is fixed and the tax rate adjusts to balance the budget. Then, the tax rate increase due to the change of the age structure curbs the increase of savings, human capital and labor supply by individuals in response to their

[1] These numbers are obtained from the UN population database.

mortality decline. We strengthen this view by considering an alternative scenario in which the tax rate is fixed to its baseline value while the replacement rate adjusts to balance the budget. In this case, aging is *neutral* in terms of growth because behavorial responses of individuals to mortality declines fully offset the negative effects of the age structure change.

Our rich model is not only useful to assess more precisely the economic consequences of aging but also sheds light on new interactions between mortality rates and individual decisions. First, relative to human capital accumulation. Ludwig et al. (2012) demonstrate quantitatively the importance of taking into account human capital decisions to study the economic consequences of aging. However, in their framework, the retirement age is fixed, which weakens the elasticity of human capital investments to longevity. A contrario, in our framework, individuals delay their retirement as mortality declines occur at old ages. This creates another incentive to augment their human capital through the well-known Ben-Porath effect. Second, relative to savings. Bloom et al. (2003) show theoretically and Carvalho et al. (2016) confirm quantitatively that aging stimulates savings by middle-aged indivduals to finance consumption during retirement. In our environment, this effect is counteracted by declines in savings of young individuals and retirees. Indeed, as young individuals dedicate more time to augment their human capital, they reduce their labor supply. Then their labor earnings and their savings diminish. The increased dissaving of old individuals is a consequence of their bequest motive, which weakens as their mortality declines. A last negative impact of aging on the capital stock is due to the greater population share of old individuals who hold on average fewer assets than middle-aged individuals. We find that the total effect of aging on the capital stock is slightly negative, which contrasts with previous results. Third, relative to the retirement decision. Cooley and Henriksen (2018) study the economic consequences of aging in an OLG model including such a decision. They find that individuals postpone retirement, which mitigates the laborforce decline caused by the rise of the dependency ratio. We argue here that the absence of human capital decision in their model tends to underestimate the positive impact of a delayed retirement. Indeed, Cooley and Henriksen (2018) assume that the individuals'productivity is exogenous and hump-shaped in age. Then, the average productivity of old workers who postpone their retirement is relatively low. Moreover, this low productivity is a disincentive to postpone retirement in response to a longevity increase, which diminishes the elasticity of the retirement age with respect to mortality rates. A contrario, in our framework, the productivity profile (hence human capital) is endogenous and shifts upward as mortality rates decline. This strengthens the role of retirement to curb the laborforce decline due to aging. We confirm these new mechanisms by running the same experiment in simplified environnements in which the human capital or the retirement decision is shut-off.

This paper relates to a vast literature on the economic consequences of population aging. As previously mentioned, we complement the work of Ludwig et al. (2012) and Cooley and Henriksen (2018) by considering a framework including both a human capital and a retirement decision. Boucekkine et al. (2002) also investigates the impact of demographic variables in a general equilibrium model with both decisions, however their results are mainly theoretical. We also complement the results of Aksoy et al. (in press) who focus on the economic impacts of the age structure while we also consider the consequences of mortality declines associated to aging. Moreover, our work differs from theirs in terms of methodology as they use a Panel VAR approach while we use an OLG model. Some authors also more specifically study the impact of aging on the interest rate such as Eggertson et al. (in press), Gagnon et al. (2016) or Krueger and Ludwig (2007). They do so by simulating the transitional dynamics of an OLG model. The novelty of our paper with respect to these papers is to build a model including more decisions related to demographic variables. This is done at the cost of having to focus on comparative static results. We believe that these results are complementary to those obtained by simulating the transitional dynamics as the comparative statics approach allows to quantify the various channels of interaction in a transparent way. Finally, recent contributions by Acemoglu and Restrepo (2018b) and Acemoglu and Restrepo (2017) point out that aging, measured as the change of the age structure, stimulates automation. Our results suggest that the labor scarcity due to aging is less severe than the one implied by the change of the age structure because of the individuals'response to mortality declines.

The rest of the paper is organized as follows. Section 2 presents the model economy used. Section 3 details its calibration. Section 4 presents the results of the counterfactual experiments. Section 5 concludes.

2 Model economy

2.1 Demographics

Individuals start making decisions at age 20. At each age a, they face a survival probability q_a. Their unconditional probability to reach age a is written Q_a. The maximal age is 105 years. The share of individuals aged a is L_a and the total population is normalized to 1.

2.2 Households

They make decisions to maximize their lifetime expected utility:[2]

$$\mathbb{E}_0[\sum_{a=0}^{105} \beta^a Q_{a-1}(q_a u(c_a, \zeta_a 1_{n_a>0} + n_a) + (1 - q_a)v(x_a))] \quad (1)$$

Where β is the discount factor. $u(.,.)$ is the instantaneous utility function, which is defined over good consumption and non-leisure time. $v(.)$ is the warm-glow bequest motive. c_a is the consumption level, n_a is non-leisure time and x_a is the amount of assets hold at age a. ζ_a is a fixed cost of working, which is non-decreasing in age as in Imrhoroglu and Kitao (2012).

For individuals in the workforce, the budget constraint writes:

$$x_{a+1} = (1 + r)x_a + I + (1 - \tau)h_a w n_a(1 - s_a) - c_a - m_a \quad (2)$$

Where r is the interest rate, w the wage rate. I is a lump-sum transfer to redistribute bequests. h_a is the human capital stock of an aged a individual. $1 - s_a$ is the fraction of non-leisure time dedicated to work. τ is the tax rate to finance the government budget. m_a is the exogenous health expenditure level at age a. During retirement period, the budget constraint writes:

$$x_{a+1} = (1 + r)x_a + I + B_a - c_a - m_a \quad (3)$$

Where B_a is the pension level.

2.3 Human capital accumulation

As in Huggett et al. (2011), the human capital of an individual evolves according to the following Ben-Porath technology:

$$h_{a+1} = \Psi_a(h_a + A(h_a n_a s_a)^\theta) \quad (4)$$

Where Ψ_a are independent log-normal shocks to human capital stock: $\log(\Psi_a) \sim \mathcal{N}(\mu, \sigma^2)$. $n_a s_a$ is the total amount of time devoted to acquire skills. $\theta \in (0, 1)$ is the curvature of the human capital production function.

[2]We omit an household index on the individual varaiables to alleviate notations.

2.4 Markets

Markets are incomplete. Individuals cannot purchase annuities to insure against mortality risks and can only partially insure against idiosyncratic human capital shocks by holding assets. They also face a borrowing constraint which imposes them to hold a positive amount of assets.

2.5 Retirement system

Individuals can retire from age 60. This decision is irreversible. If they retire after age 65, their pension level is B, which is proportional to average labor earnings, ρ being the replacement rate. If individuals retire before age 65, their pension is diminished by a malus until age 65. Individuals are forced to retire before age 80. We also assume that part (65%) of health expenditures of individuals aged more than 65 are publicly financed. The tax rate τ adjusts to balance the government budget.

2.6 Production

The final good is produced by a representative firm with Cobb-Douglas technology:

$$Y = K^{\alpha} H^{1-\alpha} \tag{5}$$

Where Y is output, K is the capital stock and H is the labor input. $\alpha \in (0,1)$ is the capital share. Capital depreciates at rate δ. Production factors are paid at their marginal productivity. Noting $k = \frac{K}{H}$, the capital to labor ratio, we obtain:

$$w = (1-\alpha)k^{\alpha} \tag{6}$$

$$r = \alpha k^{\alpha-1} - \delta \tag{7}$$

2.7 Equilibrium

Each individual is caracterized by a state $\Omega = (a, x, h, R)$ where a denotes age, x the amount of assets held, h the human capital stock. R denotes the retirement status of the individual. It is equal to 0 for a retiree and 1 for a worker. Solving the individual problem amounts to compute the decision rules associated to each state. We detail the algorithm to achieve this in Appendix A. For a variable y of the individual problem, we note $y(\Omega)$ the optimal value of y in state Ω. We also note μ_a the distribution of states conditional on age a. We define a stationary competitive equilibrium as follows:

1) Individuals maximize their objective (1) subject to the budget constraints (2) and (3) and the law of human capital accumulation (4).

2) The representative firm chooses capital stock K and labor input H to maximize profits given the production function (5).

3) The lump-sum transfer to redistribute bequests is given by:

$$I = \sum_{a=20}^{105} L_a \int (1 - q_a) x(\Omega) \mu_a(d\Omega) \tag{8}$$

4) Capital and labor markets clear:

$$K = \sum_{a=20}^{105} L_a \int x(\Omega) \mu_a(d\Omega) \tag{9}$$

$$H = \sum_{a=20}^{105} L_a \int h(\Omega) n(\Omega) (1 - s(\Omega) \mu_a(d\Omega) \tag{10}$$

5) The budget of the government is balanced:

$$\tau w \sum_{a=20}^{80} L_a \int h(\Omega) n(\Omega) (1 - s(\Omega) \mu_a(d\Omega) = \sum_{a=60}^{105} L_a B_a \int (1 - R(\Omega)) \mu_a(d\Omega) + 0,65 \sum_{a=65}^{105} L_a m_a \tag{11}$$

6) The distribution $(\mu_a)_{20 \leq a \leq 105}$ is stationary.

3 Calibration

We calibrate the model for the stationary equilibrium to replicate key moments of the US economy in 2015. Table 1 summarizes the values and the sources of the different parameters. We solve the model at an annual frequency.

Demographic data: The population shares by age $(L_a)_{20 \leq a \leq 105}$ are obtained in the UN population database. This database also provides the life expectancy at birth for both sexes. We then follow the procedure of Henriksen (2015) to compute the survival curve $(q_a)_{20 \leq a \leq 105}$ associated to this life expectancy.

Production: The capital share is set to the standard value of 0.35 and the depreciation rate is equal to 0.06.

Table 1: Model parameters

Parameters	**Symbol**	**Value**
Leisure weight	χ	1,1
Utility exponent	ϕ	5
Discount factor	β	0.9725
Human capital production function	A	1.28
Exponent Human production function	θ	0.62
Shock distribution	(μ, σ^2)	$(-0.029, 0.111)$
Capital share	α	0.35
Depreciation rate	δ	0.06
Bequest weight	B_{eq}	0.4
Fixed cost of working	$(\kappa_1, \kappa_2, \kappa_3)$	(0.043, 0.000013, 1.51)
Replacement rate	ρ	0.4

Preferences: We use the following instantaneous utility function:

$$u(c,n) = \log(c) + \chi \frac{(1-n)^{1-\phi}}{1-\phi} \tag{12}$$

Where the value of ϕ is set to 5 as in Guvenen et al. (2014). This implies that the Frisch-elasticity of an individual working the average amount of hours is equal to $0,3$. The functional form for the warm-glow bequest motive is $v(x) = B_{eq} \log(x)$.

Health expenditures: Dalgaard and Strulik (2014) provide evidence that health expenditures grow at the annual rate of 2% over the life cycle in advanced economies. Thus we assume that $m_a = M(1+g)^a$ with $g = 2\%$. We compute M for the aggregate health expenditure to GDP ratio (hence $\frac{\sum_{a=20}^{105} L_a m_a}{Y}$) to equal $17,7\%$ (BEA). Finally, as suggested by Nardi et al. (2016), 65% of health expenditures of old individuals are publicly financed.

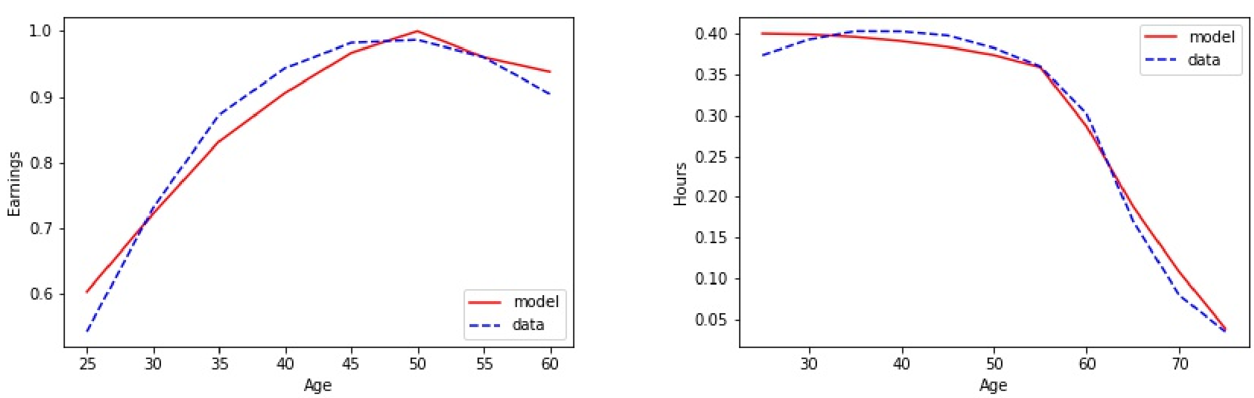
Other parameters: We set the parameter θ to 0.65. The mean and the variance of the idiosyncratic shocks are borrowed from Huggett et al. (2011). The replacement rate is fixed to 0.4 as in Aguiar and Hurst (2013). Regarding the fixed cost of working, we consider the following functional form: $\zeta_a = \kappa_1 + \kappa_2 a^{\kappa_3}$.

This leaves us with 7 parameters $\Theta = (A, \chi, \beta, B_{eq}, \kappa_1, \kappa_2, \kappa_3)$ to determinate. We choose Θ for our model to reproduce several moments of the US economy. Our choice of moments is motivated by the key behavorial responses of individuals to an increase of their survival probabilities. More specifically, to discpline the process of human capital accumulation we target a life cycle profile of labor earnings, to discipline the retirement decision we target a life cycle profile of hours worked and to discipline the savings decision we target the US capital to output ratio.

To build the two life cycle profiles, we use the PSID dataset, more precisely the waves from 1980-2015. For the labor earnings profile, we use the consumer price index to compute real earnings. We keep individuals working between 260 and 5820 hours and earning at least $5,000 in 2015 dollars. We also restrict observations to individuals aged less than 60. This is because the number of observed workers aged more than 60 is limited in the database by their exit of the workforce. To obtain the age profile, we control for year effects. To build the hours profile, we follow the same steps except that we keep all individuals working less than 5820 hours in order to take into the individuals who retire. The capital to output ratio we target is equal to 3.22. We constraint the model to match exactly this moment by iterating over the discount factor β. The other components of Θ are obtained by minimizing the squared percentage deviation between the empirical moments and their model counterparts.

Backfitting: We first examine the performance of the model to reproduce the targeted moments, hence the life cycle profile of labor earnings and hours worked. Figure 1 shows that the model produces life cycle profile for earnings and hours worked in line with the data.

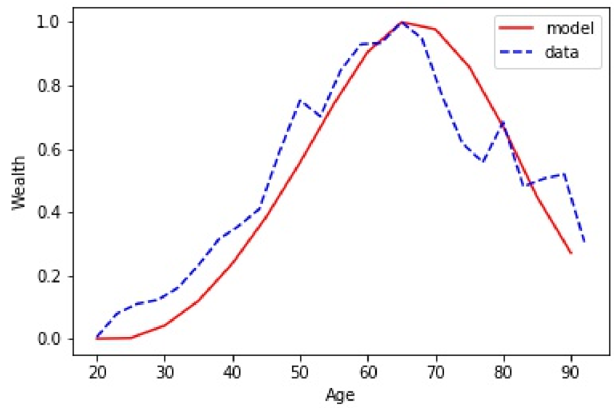
Figure 1: Life cycle profiles in the data and in the baseline economy



Notes: the figure displays the life cycle profile of average labor earnings and average hours worked in the model and in the data. Labor earnings are normalized by the maximal value. Hours worked are expressed as a fraction of an annual time endowment equal to 5,110. The data source is the PSID.

To verify that the savings behavior of the model is realistic, we also compute a wealth life cycle profile from the Survey of Consumer Finance (2013). Then we compare this profile to the one generated by the model. Figure 2 reveals that this untargeted moment is also well reproduced by our model. This makes us confident that our model is a good laboratory to examine the impact of population aging.

Figure 2: Wealth profile in the data and in the baseline economy



Notes: the figure displays the life cycle profile of average wealth in the model and in the data. Wealth is normalized by the maximal value. The data source is the Survey of Consumer Finance.

4 Computational experiment

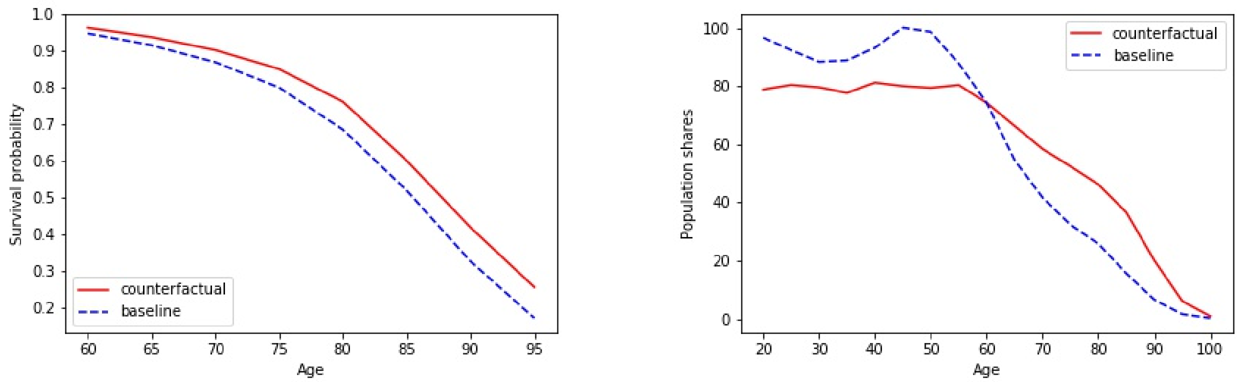
In this section we present the results of several experiments to understand and quantify the economic consequences of aging. In our main experiment, we change the survival probabilities $(q_a)_{20\leq a\leq 105}$ and the population shares $(L_a)_{20\leq a\leq 105}$ to their 2050 projected values. These projections are obtained from the same source as the 2015 values. Figure 3 displays the demographic values used in the baseline economy and in the counterfactual economy. These demographic changes result in an increase of the dependency ratio from $33,8\%$ to $56,7\%$. In Table 2, we report the change of the main variables in the counterfactual economy. This is the main result of the paper.

Table 2: Results of the main experiment

Variable	r	w	K	H	Y	τ
Percentage change	$-7,5$	+1,7	$-1,9$	$-6,4$	$-4,8$	+52,5

The table reports the percentage change of various variables when demographic variables are set to their 2050-projected values. The replacement rate is fixed to its baseline value.

Figure 3: Demographic variables

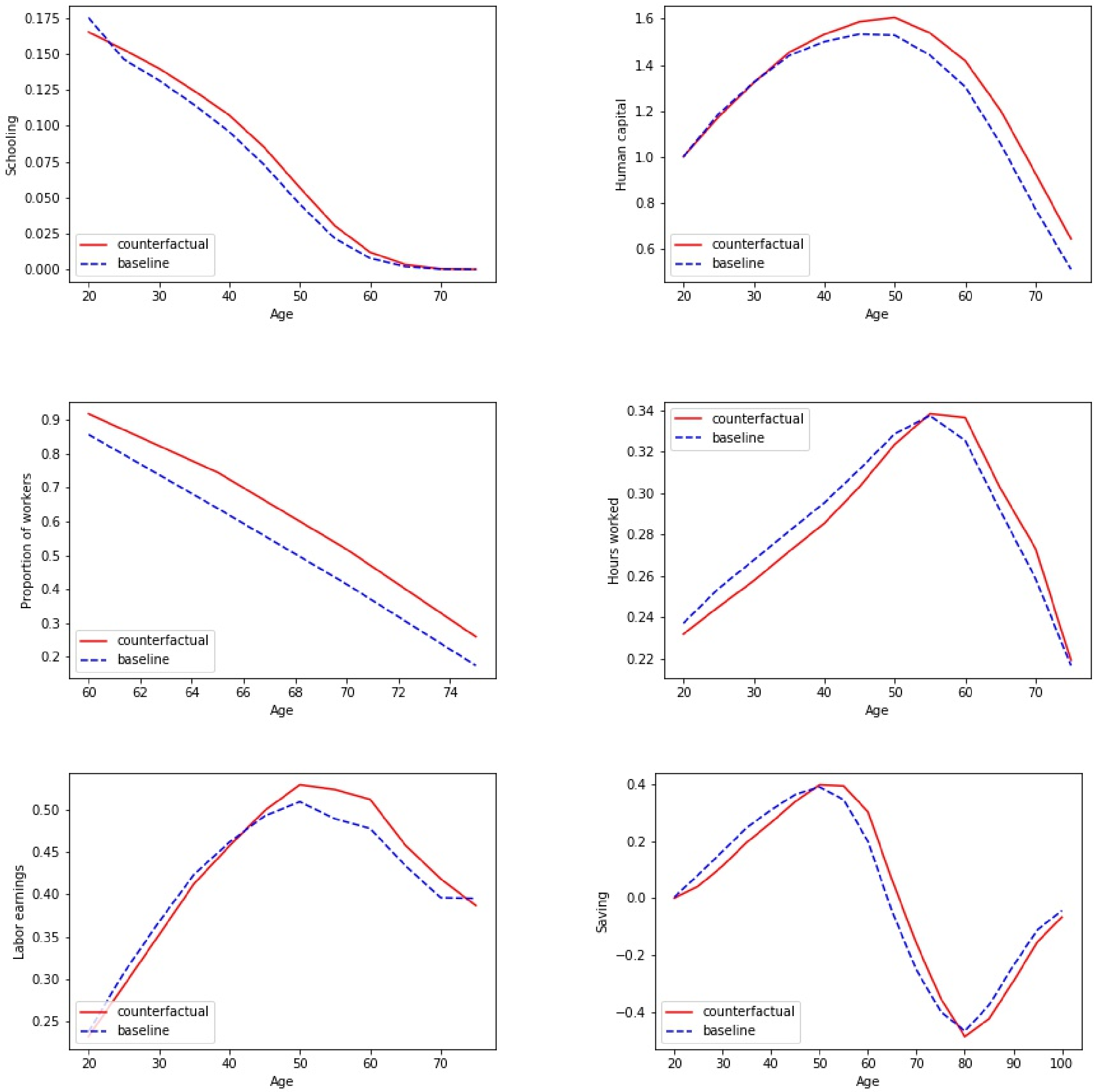


Notes: the figure displays the survival curve and the population shares used in the baseline and in the counterfactual economy. The population shares are obtained from the UN population database. See the text for the source of the survival curves.

We find that capital stock, labor input and output per capita all decline. The most surprising result is the decline of the capital stock, which contrasts with previous literature results. We first comment on the laborforce decline. The main effect of aging is the fall of the number of working-age individuals. As already mentioned, this is captured by the rise of the dependency ratio from 33, 8% in the baseline economy to 56, 7% in the counterfactual economy. This force is mitigated by several behavorial responses. First, individuals postpone retirement. Average retirement increases from 65, 2 to 67 between the baseline and the counterfactual economy. While the fraction of individuals aged less than 65 decreases by 13, 3%, the fraction of workers in the population only falls by 7, 4%. Second, individuals invest more time to augment their human capital. Figure 4 shows the upward shift of the human capital life cycle profile after demographic changes. Aging also changes the life cycle profile of hours worked, even though its total effect on the laborforce is quite neutral. As young individuals spend more time to study, their labor supply is reduced compared to that of the baseline economy. The opposite holds for older individuals. Indeed, their greater human capital spurs them to work more.

The capital stock decline is also the result of several opposite forces. The first effect is due to the composition change of the population. From Figure 2, old individuals hold fewer assets than middle-aged individuals. Thus, a greater share of old individuals reduces aggregate wealth. The other effects are due to the change of savings behavior by individuals along the life cycle. Figure 4 plots the life cycle profile of savings in the baseline and in the counterfactual economy. We observe that aging stimulates savings of middle-aged individuals. This is the usual horizon effect highlighted by Bloom et al. (2003). Figure 4 also reveals that both young and old individuals diminish their savings in response to aging.

Figure 4: Life cycle profiles in the baseline and in the counterfactual economy.



Notes: the figure displays the life cycle profile of various variables in the baseline and in the counterfactual economies.

As young individuals supply less labor to dedicate more time to training, their labor earnings and then their savings decrease. The savings decline of old individuals is a consequence of their bequest motive, which weakens as their survival chances improve. The total effect of aging on the capital stock is negative. As the two production factors decline, output per capita of the counterfactual economy is lower than in the baseline economy. The decline is equal to $4,8\%$. Put differently, from 2050, demographic changes reduce the annual growth rate by $0,14$. This decline is lower than that reported by Nardi et al. (1999) or Cooley and Henriksen (2018) because our model includes human capital and labor decisions that mitigate the negative impact of aging.

We now conduct several other experiments to further examine the mechanisms at play. First, we emphasize that the two facets of aging, the mortality decline and the age structure change, have opposite consequences in terms of growth. For this, we set the survival curve to its 2050-projected value, while we keep the age structure to its 2015-value. Table 3, reports the result of this experiment. Capital stock, labor input and output per capita all *increase* with respect to the baseline economy. These effects are entirely due to behavorial responses to mortality declines. These ones imply that individuals accumulate more human capital and retire later, which explains the labor input increase. The capital stock increase is less obvious. In the main experiment we noted that the mortality decline effect on savings is negative for young and old individuals, while it is positive for middle-aged individuals. This implied that the total increase of savings was small. Here the total increase of savings is greater than in the main experiment. This relates to the tax rate change in the two experiments. This tax rate substantially increases in the main experiment, while it diminishes when the age structure is unchanged. A tax rate increase reduces savings for all workers. Then, the diffferent tax change between the two experiments explain that the increase in savings is larger when the age structure is unchanged.

Table 3: Survival curve change only

Variable	r	w	K	H	Y	τ
Percentage change	$-2,9$	+0,5	$+4,8$	$+2,4$	$+3,2$	-4,3

The table reports the percentage change of various variables when survival probabilities are set to their 2050-projected values. The replacement rate is fixed to its baseline value.

Table 4: Age structure change only

Variable	r	w	K	H	Y	τ
Percentage change	$-4,35$	$+1$	$-6,5$	$-8,9$	-8	$+56$

The table reports the percentage change of various variables when population shares are set to their 2050-projected values. The replacement rate is fixed to its baseline value.

We now keep the survival curve to its 2015 value, while we set the age structure to its 2050-projected value. Table 4 reports that capital stock, labor input and output per capita all *decline* with respect to the baseline economy. As old individuals hold fewer assets than middle-aged individuals, the age structure change reduces the capital stock. Also, as old individuals are less productive and less likely to work, the age structure change reduces labor input. Moreover there is a large tax rate increase to balance the budget in response to the greater number of retirees. This also diminishes the savings and the human capital of individuals which further reduces capital stock and labor input. From these two last experiments, a clear picture of the growth consequences of aging emerges. Mortality declines are growth enhancing, while age structure changes are harmful for growth. These two opposite forces imply that the total impact of aging lies between the one of these two experiments.

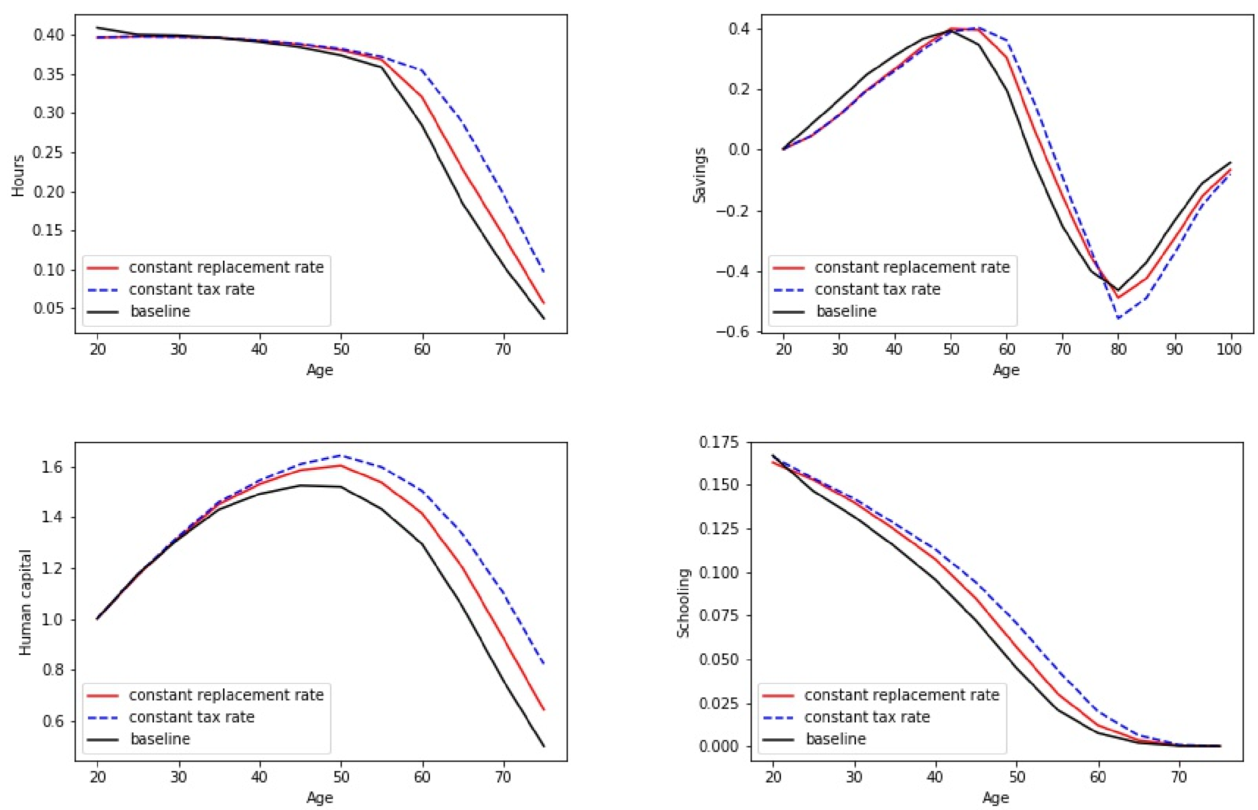
Table 5: Constant tax rate experiment

Variable	r	w	K	H	Y	τ
Percentage change	$-7,4$	+1,6	$+2,3$	$-2,2$	-0.6	0

The table reports the percentage change of various variables when demographic variables are set to their 2050-projected values. The tax rate is fixed to its baseline value.

These experiments also point out a mechanism that we have neglected so far, namely the effect of the tax increase due to the age structure change. This tax increase curbs the positive impacts of mortality declines on savings, human capital and labor supply. This suggests to examine the consequences of aging under an alternative scenario in which the tax rate is fixed to its baseline value while the replacement rate adjusts to balance the budget. Table 5 reports the results. In this case, population aging is close to be *neutral* in terms of output. The capital stock now increases, while the laborforce decline is much less severe than in the main experiment. This small laborforce decline is largely due to the increase of the retirement age from 65,2 to 68,5, compared to 67 in the main experiment. This higher elasticity of the retirement age is due to three effects. First, in this experiment, the tax rate is unchanged by assumption contrary to the main experiment in which it increases substantially. Second, in this experiment, the replacement rate adjusts downards to balance the budget. This diminishes the pension level which creates an incentive to delay retirement. Third, the tax rate difference between this experiment and the main experiment also affects human capital accumulation. A higher tax rate discourages human capital accumulation as shown by Figure 5. The higher human capital has a direct positive effect on the labor input and also creates an incentive to delay retirement. The capital increase in the constant replacement rate scenario is the result of two opposite forces. First, the lower tax rate than in the main experiment increases savings of workers. This effect is particularly visible for workers aged between 50 and 80. Before 50, individuals supply less labor than in the main experiment so as to increase their human capital. This implies that their savings are similar in the two experiments. Second, individuals aged more than 80 dissave more than in the main experiment. This is due to the lower pension level. The first effect dominates the second effect because there are more individuals aged between 50 and 80 than individuals aged more than 80. This explains why the capital stock increases in the constant replacement rate scenario, while it decreases in the main experiment.

Figure 5: Life cycle profiles in the baseline and in the counterfactual experiments.



Notes: the figure displays the life cycle profile of various variables in the baseline economy, in the constant tax rate scenario and in the constant replacement rate scenario.

5 Conclusion

Population in developed economies is aging. In this paper, we have assessed the economic consequences of this demographic phenomenon. To achieve this, we have first built a rich OLG model with both human capital accumulation and a retirement decision. We have then calibrated the model to reproduce key moments of US economy given actual demographic data. Finally, we have used the model to perform counterfactual experiments on demographic variables. In our main experiment, when tax rates adjust to balance the social security budget, capital stock, laborforce and output per capital all decline. Behavorial response to mortality declines in terms of human capital, savings and labor supply do not suffice to compensate the negative impact of the change of the age structure. These behavorial responses are curbed by the necessary tax increase to finance the greater number of retirees. In an alternative scenario, we investigate the consequences of aging when the replacement rate adjusts to balance the pension system budget. In this case, the behavorial responses to the mortality decline are not mitigated by a tax increase and fully offset the negative consequences of aging in terms of growth.

Contrary to previous literature, we find a negative effect of aging on capital stock and a less negative impact on the laborforce. Hence we find a smaller labor scarcity due to aging. Acemoglu and Restrepo (2018b) argue that this labor scarcity is a source of technological progress and particularly of automation. This could be assessed in a future work by coupling our model with an endogenous automation set-up as that proposed by Acemoglu and Restrepo (2018a).

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6 Appendix A

In this section, we explain how we solve the individuals' problem. During the retirement period, the Bellman equation writes:

$$v_a(x) = \max_{0 \le c, x'} (u(c,0) + \beta q_a v_{a+1}((1+r)x + I_a + B_a - c) + \beta(1-q_a)v((1+r)x + I + B_a - c)) \quad (13)$$

With $I_a = I - m_a$. The Inada condition on $v(.)$ implies that the constraint $0 \le x'$ never binds. Then, the first-order condition on c writes:

$$\frac{1}{c} = \beta(q_a \frac{\partial v_{a+1}}{\partial x'}(x') + (1-q_a)v'(x')) \quad (14)$$

This writes:

$$c = \frac{1}{\beta(q_a \frac{\partial v_{a+1}}{\partial x'}(x') + (1-q_a)v'(x'))} \quad (15)$$

We then apply an endogenous grid method (Carroll (2006)). We work with a grid over the post-decision asset level x': $(x'(n))_{n\in N'}$. For each gridpoint $x'(n)$, the RHS of (15) is known and we can compute the consumption level $c_a(n)$. Then, the pre-decision asset level is given by: $x_a(n) = \frac{x'(n)+c_a(n)-I_a-B_a}{1+r}$. Then, we interpolate on the pre-decision grid $(x_a(n))_{n\in N}$ to determine the optimal consumption level for any asset level.

For ages between 60 and 80, there are two cases to consider. If the individual is retired, hence $R = 0$ then the Bellman equation is identical to that of the retirement period. If the individual is not retired, hence $R = 1$, then the Bellman equation writes:

$$\begin{aligned} v_a(x, h, R = 1) = \max_{0\leq c,x',s,n} &(u(c, 1_{n>0}\zeta_a + n) \\ &+ \beta q_a E_a[v_{a+1}((1 + r)x + I_a + B_a(1 - 1_{n>0}) + whn(1 - s)1_{n>0} - c, e^{\zeta_a}(h + A(hns)^{\theta}), R')] \\ &+ \beta(1 - q_a)v((1 + r)x + I_a + B_a(1 - 1_{n>0}) + whn(1 - s)1_{n>0} - c)) \end{aligned} \quad (16)$$

The fixed-cost of working and the pension benefit imply that the RHS of (16) has kinks. To remedy this, we first compute the value function of an individual who decides to retire. Then, we compute the value function of an individual who is constrained not to retire. The value function (16) is then obtained by taking the maximum of these two quantities. The value function of an individual who decides to retire is given by:

$$\begin{aligned} \max_{0\leq c,x'}&(u(c, 0) + \beta q_a E_a[v_{a+1}((1 + r)x + I_a + B_a - c, e^{\zeta_a}h, 0)] \\ &+ \beta(1 - q_a)v((1 + r)x + I_a + B_a - c)) \end{aligned} \quad (17)$$

The first order condition writes:

$$c = \frac{1}{\beta(q_a E_a[\frac{\partial v_{a+1}}{\partial x'}(x', e^{\zeta_a}h, 0)] + (1 - q_a)v'(x'))} \quad (18)$$

We now consider another grid for human capital stock: $(h(p))_p$. We compute the consumption level of an individual with human capital $h(p)$ for any asset level. For this, we use our post-decision grid for assets. We compute the consumption level of an individual with human capital $h(p)$ and post-decision asset level $x'(n)$ using (18). We obtain the pre-decision asset level: $x_a(n) = \frac{x'(n)+c_a(n)-I_a-B_a}{1+r}$. By interpolation on the grid $(x_a(n))_n$, we can compute the consumption level of an individual with human capital $h(p)$ and pre-decision asset level $x(n)$. This gives the value function of individual who decides to retire for any point on the pre-decision grid $(h(p), x(n))_{p,n}$.

We now consider the value function of an individual who is constrained not to retire:

$$\begin{aligned} \max_{0\leq c,x',s,n}&(u(c, \zeta_a + n) + \beta q_a E_a[v_{a+1}((1 + r)x + I_a + whn(1 - s) - c, e^{\zeta_a}(h + A(hns)^{\theta}), 1)] \\ &+ \beta(1 - q_a)v((1 + r)x + I_a + whn(1 - s) - c)) + \mu(1 - s) \end{aligned} \quad (19)$$

Where μ is the Lagrange multiplier associated to the constrained $s \leq 1$. We now consider a post-decision grid $f(m)$ for the quantity $h + A(hns)^{\theta}$. We compute the decision rules associated to the post-decision state $(x'(n), f(m))_{n,m}$. For this, we use the FOCs associated to (19):

$$c = \frac{1}{\beta(q_a E_a[\frac{\partial v_{a+1}}{\partial x'}(x'(n), e^{\zeta_a} f(m), 1)] + (1-q_a)v'(x'(n)))} \tag{20}$$

$$\begin{aligned}\chi(1-n)^{-\phi} = wh(1-s)(\beta(q_a E_a[\frac{\partial v_{a+1}}{\partial x'}(x'(n), e^{\zeta_a} f(m), 1)] + (1-q_a)v'(x'(n)))) \\ + \beta q_a A\theta(hs)^\theta n^{\theta-1} E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)]\end{aligned} \tag{21}$$

$$\begin{aligned}A\theta(hn)^\theta s^{\theta-1}\beta q_a E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)] = whn(\beta(q_a E_a[\frac{\partial v_{a+1}}{\partial x'}(x'(n), e^{\zeta_a} f(m), 1)] \\ + (1-q_a)v'(x'(n)))) + \mu\end{aligned} \tag{22}$$

Note that the consumption level associated to the post-decision state $(x'(n), f(m))_{n,m}$ is given by (20). To compute hours worked and schooling time, it is convenient to define $Z(n,m) = \beta(q_a E_a[e^{\zeta_a} \frac{\partial v_a}{\partial h'}(x'(n), e^{\zeta_a} f(m))] + (1-q_a)v'(x'(n)))$. We first consider an interior solution for s. Then, $\mu = 0$ and equation (22) implies:

$$hns = (\frac{\beta q_a A\theta E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)]}{wZ(n,m)})^{\frac{1}{1-\theta}} \tag{23}$$

Using the law of human capital accumulation, we obtain the pre-decision level of human capital:

$$h(n,m) = f(m) - A(\frac{\beta q_a A\theta E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)]}{wZ(n,m)})^{\frac{\theta}{1-\theta}} \tag{24}$$

Note that the RHS of (21) is equal to $wh(m,n)Z(m,n)$. Then, (21) writes:

$$\chi(1-n)^{-\phi} = wh(m,n)Z(m,n) \tag{25}$$

We can solve this equation to obtain the hours associated to the post-decision state $(x'(n), f(m))_{n,m}$:

$$n(m,n) = 1 - (\frac{\chi}{wh(m,n)Z(m,n)})^{\frac{1}{\phi}} \tag{26}$$

And the time dedicated to school: $s(m,n) = (\frac{\beta q_a A\theta E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)]}{wZ(n,m)})^{\frac{1}{1-\theta}} \frac{1}{n(n,m)h(n,m)}$. This solution is admissible only if $n(m,n) > 0$, hence if $1 - (\frac{\chi}{wh(m,n)Z(m,n)})^{\frac{1}{\phi}} > 0$. If this condition holds and if $h(n,m) > 0$ and $s(n,m) < 1$, then $c(n,m), n(n,m), s(n,m)$ satisfy the FOCs associated to the post-decision states $x'(n)$ and $f(m)$ and the pre-decision states $x(n,m)$ and $h(n,m)$. We now examine a possible corner solution for the time dedicated to school, hence $s = 1$. In this case, (21) writes:

$$\chi(1-n)^{-\phi} = \beta q_a A\theta h^\theta n^{\theta-1} E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m))] \tag{27}$$

We define $C_1 = \frac{\chi}{\beta q_a A\theta E_a[\frac{\partial v_{a+1}}{\partial h'}(x'(n), e^{\zeta_a} f(m), 1)]}$. Then, the previous equation writes:

$$h = C_1^{\frac{1}{\theta}} \frac{n^{\frac{1-\theta}{\theta}}}{(1-n)^{\frac{phi}{\theta}}} \tag{28}$$

We use this relationship with the law of accumulation of human capital to obtain the following equation for n:

$$f(m) = C_1^{\frac{1}{\theta}} \frac{n^{\frac{1-\theta}{\theta}}}{(1-n)^{\frac{\phi}{\theta}}} + AC_1 \frac{n^{1-\theta}}{(1-n)^{\phi}} \tag{29}$$

We rootfind the previous equation to get $n(m, n)$, then $h(m, n)$ is obtained by equation (28). These quantities can define a solution only if the Lagrange multiplier μ, defined by (22) is non-negative.

To sum up, for each couple $(x'(n), f(m))$, we first check if the necessary conditions for an interior solution to exist are satisfied. If this is the case, we compute the decision rules as explained above. Then, we rootfind (29) and check whether the associated Lagrange multiplier is non-negative. If this is the case, we also store this decision rule. Note that it is possible that two decision rules are associated to one post-decision state. This possibly happens because our objective function is not necessarily concave, meaning that the FOCs are only necessary. The rest of the procedure consists of computing the decision rule for each state in the pre-decision grid. For this, we follow Druedahl and Jorgensen (2017). We consider triangles $((x'(n), f(m)), (x'(n+1), f(m)), (x'(n+1), f(m+1)))$ in the post-decision state space, for which we compute the associated triangles $((x(n, m), h(n, m)), (x(n+1, m), h(n+1, m)), (x(n+1, m+1), h(n+1, m+1)))$ in the pre-decision state. We look for points of our regular grid that lie in these triangles. For such points, we interpolate the decisions rules. If a point is associated to two decision rules, then we keep the one for which the value function is greater. Pratically, at the end of this procedure, the decision rules are not computed for 2% of the points of our regular grid. For these points, we use a global maximizer to directly solve the Bellman equation. Once we have computed the decision rules at each point of the regular grid, we can compute the value function at these points of an individual who is constrained not to retire. The last step of the procedure is to compare this value function to the value function of an individual who decides to retire. The decision rules are then the ones associated to the highest value function of the two possibilities. Finally, we discretize the shocks according to Rouwenhorst method to compute expectations.

7 Appendix B

In this section we explain how we solve the stationary equilibrium of the model. We do this by constraining the capital to output ratio to be equal to 3.22 as in the data. From (7):

$$r = \alpha k^{\alpha-1} - \delta = \alpha \frac{Y}{K} - \delta \tag{30}$$

This implies that the value of k is determined by the value of the capital to output ratio, *ie* $(\frac{K}{Y})^{1-\alpha}$. We can then compute the interest rate and the wage level. We also simulate the shocks history of 5,000 individuals. We keep these shocks constant across all resolutions. We then repeat the following steps:

1) We make a guess on the discount factor β, the pension level B, the transfer level I and the tax rate τ.

2) The guess on the pension level gives a value for the efficient labor input: $H = \frac{B}{\rho w} \sum_{a=20}^{80} L_a$ and then a value for output: $Y = Ak^{\alpha}H$. This allows to compute the constant M and then the exogenous health expenditure profile $(m_a)_a$.

3) We solve the individuals' problem according to the algorithm detailed in Appendix A.

4) With the policy functions and the history of shocks, we compute aggegrate capital, aggregate labor and the number of retirees.

5) We update our guesses on the pension level, the transfer level and the tax rate by repeating steps 2) to 5).

6) We update our guess of β by repeating steps 2) to 6) until the capital to output ratio is equal to $\frac{K}{Y}^{1-\alpha}$.