Fiscal policy and the unbalanced pension system[1]

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Abstract

The ageing of the population forces the government to carry out pension reforms in order to insure the future sustainability of the pension system. Due to the recently stressed public finances, the reforming of the social security systems is becoming even more urgent as governments will not be able to cover all the deficit of the pension fund with transfers from federal budgets. We suggest that the reforms of the pension systems may be considered as the measures of fiscal consolidation. In order to define the optimal policy mix of measures we consider how optimal tax policy varies with changes in the retirement age, life expectancy and productivity on the basis of the OLG model. Our welfare analysis shows that higher income tax is socially optimal in financing pensions when the deficit of the pension fund is covered out of the state budget. This illustrates that income tax and social contributions can be considered as imperfect substitutes.

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1 Introduction

Many countries are launching social security reforms in order to secure the sustainability of the social security system in the future, providing increasing expenditures on social payments. The main drivers of this dynamics are demographic changes: fertility rates below the replacement level and higher life expectancy.

Although several reforms of social security systems have been implemented, federal transfers remain one of the key sources of balancing the budget of pension systems. Moreover, their share is expected to rise steadily in the future: according to OECD estimates, this part of fiscal expenditures will increase from 9.3% of the GDP in 2010 to 11.7% of the GDP in 2050.[1] In Russia, the deficit of the pension fund is also covered by transfer from the federal budget. The transfer amounted to 4.3% of the GDP in 2013 and 3.4% in 2014.[2] However, financing pension fund deficits out of federal budgets has become even more complicated after the financial crisis of 2008-2009 and the European debt crisis, which started in 2010. Pension reforms (introduction of the higher retirement age, higher social deductions, lower pensions) can be considered as an alternative to the traditional measures of the fiscal consolidation.

The aim of this research is to define the optimal combination of two fiscal instruments (rate of social contributions and income tax) chosen by the social planner and to specify how this policy mix changes with the retirement age, life expectancy, productivity and depending on the type of the pension system (balanced or unbalanced). The analysis is based on the overlapping generations model (OLG) initially developed by Yaari (1965) and Blanchard (1985) and extended further by Buiter (1988), Giovannini (1988), Weil (1989) and Bovenberg (1993). In order to investigate the optimal policy mix we extend the model of Heijdra and Bettendorf (2006), who analyzed the economic consequences of lower pensions and a higher retirement age in an open economy with traded and non-traded sectors. They, however, consider an exogenous interest rate along with a rudimentary pension system, which allows to analyze intergenerational redistribution yet is assumed to be balanced. We extend their model to investigate the unbalanced budget of a pension fund in a closed economy with an endogenous interest rate, which allows us to account for the effect of different economic environment (retirement age, life expectancy, labor productivity) on the capital accumulation. Moreover, while Heijdra and Bettendorf (2006) consider the consequences of shocks to the welfare of each generation, we investigate the optimal subset of measures conducted by the benevolent government, which maximize the social welfare function.

Nickel et al. (2008) is the other paper which is close to the current research. They extend the framework of Nielsen (1994) and Heijdra and Bettendorf (2006) by

[1] OECD, Pensions at glance 2013

[2] Transfer has decreased due to the freeze of the accumulation part of the pension savings.

considering an unbalanced pension system and assuming that firms issue equities and face adjustment costs in investment. Nickel et al. (2008) consider three fiscal scenarios in the economy with decreasing population: suspension of the public pension system and a decrease in lump-sum labor tax, suspension of the public pension system and a decrease in distortionary corporate tax and an increase in the retirement age. The main results of their research suggest that the adverse consequences of the pension reforms can be decreased by appropriate taxation policies. The main difference with the current research is that Nickel et al. (2008) consider government as non maximizing entity and investigate how the predetermined changes in policy instruments would affect the transition of the main macroeconomic variables to the new equilibrium in an open economy, while we define socially optimal fiscal policy (social contributions and income tax) and compare the optimal set of policy instruments in equilibrium with both increasing and decreasing population in the closed economy.

First, we consider how the optimal tax rates differ in the equilibrium with different retirement ages. The results show that the optimal set of measures depends on the structural characteristics of the economy (birth and death rates, productivity, type of the pension system). When the income tax is fixed, chosen optimally rate of social contributions decreased with the higher retirement age. However, when both, the share of the social contributions income tax are chosen optimally, pensions are financed by the income tax which covers the deficit of the pension fund. At the same time social contributions are at zero. Moreover, under a higher retirement age with optimally chosen income tax the deficit of the pension fund is lower coupled with lower pensions.

When the longer life expectancy is considered, quantitative results suggest that the optimal income tax remains unchanged. However, if balanced and unbalanced pension systems are considered, in the former case optimal rate of the social contributions is rather high coupled with constant income tax for different growth rates, while in the latter case social contributions are at zero and the income tax changes with the population growth rates.

It was also shown that the optimal income tax is not sensitive to the changes in labor productivity, while public debt and the deficit of the pension fund are lower in the equilibrium with higher productivity due to higher income revenues which cover the deficit of the pension fund.

The paper is organized as follows. Section 2 presents an extended OLG model of Heijdra and Bettendorf (2006) with the unbalanced pension system. In Section 3 the results of the comparative analysis are presented (varying retirement age, different life expectancy for both two types of pension systems and different labor productivity). Section 4 summarizes the results with details of the results in the appendix.

2 The model

We extend the model of Heijdra and Bettendorf (2006) by introducing an unbalanced pension system in a closed economy with an endogenous interest rate. The deficit of the pension system is considered as a liability of a benevolent government, which conducts fiscal policy to maximize the social welfare. In order to investigate fiscal policy we include government expenditures in the utility function, so that public and personal consumption are imperfect substitutes.

2.1 Households

Individual households

The representative consumer born in the period υ maximizes an expected present value of instantaneous utility, which is additively separable with respect to the personal consumption and government expenditures. Individual consumption and public good are assumed to be imperfect substitutes, with the elasticity of substitution $\kappa > 0$.

$$U(\upsilon, t) = \int_t^\infty \left[(1-\kappa)\ln \bar{c}(\upsilon, t) + \kappa \ln g(\upsilon, t)\right] \mathrm{e}^{(\rho+\beta)(t-\tau)} d\tau, \tag{1}$$

where $\bar{c}$ is personal consumption, g is the per capita government expenditures, $\rho > 0$ is the rate of time preference and $\beta \geq 0$ is the probability of death.

The households receive an interest rate $r(\tau)$ on the financial wealth, $\bar{a}(\upsilon, \tau)$, and have a non-interest income, net of lump-sum taxes or transfers, $WI(\upsilon, \tau)$. The payment $\beta a(\upsilon, \tau)$ is the actuarially fair annuity paid by the life insurance company.[1] Interest and non-interest net labor income are spent on consumption and saving. Household financial wealth consists of capital goods, $(\bar{k})$, and government bonds, $(\bar{a}^G)$, both denominated in terms of consumer goods.

The household budget constraint in terms of the consumer good is:[2]

$$\dot{\bar{a}}(\upsilon, \tau) = (r(\tau) + \beta)\bar{a}(\upsilon, \tau) + WI(\upsilon, \tau) - \bar{c}(\upsilon, \tau) \tag{2}$$

$$\bar{a}(\upsilon, \tau) = \bar{k}(\upsilon, \tau) + \bar{a}^G(\upsilon, \tau) \tag{3}$$

Following Bettendorf and Heijdra (2006) we use the PAYG pension scheme, introduced by Nielsen (1994). We assume that the young individuals, aged from zero to π, pay a lump-sum tax t_W. After the age threshold of π the households start to receive

[1] See Yaari (1965), Blanchard (1985)

[2] A dot above the variable stands for the variable's time derivative (change in time), thus, $\dot{\bar{a}}(\upsilon, \tau) = d\bar{a}(\upsilon, \tau)/d\tau$.

a lump-sum transfer z. Net labor income takes the following form:

$$WI(\upsilon,\tau) = \begin{cases} (1-t_L)W^N(\upsilon,\tau) - t_W & \text{for } \tau-\upsilon \leq \pi, \\ (1-t_L)W^N(\upsilon,\tau) + z & \text{for } \tau-\upsilon > \pi. \end{cases} \tag{4}$$

where $W^N(\upsilon,\tau)$ is the wage of the worker born in period υ at the time τ. Labor productivity is assumed to depend on the age of the worker. The worker of the generation υ at time τ supplies $n(\upsilon,\tau)$ efficiency units of labor:

$$n(\upsilon,\tau) = E(\tau-\upsilon)\bar{l}(\upsilon,\tau), \tag{5}$$

where $\bar{l}(\upsilon,\tau) = 1$ is the labor hours and following Blanchard (1985) $E(\tau-\upsilon)$ is the efficiency index, which falls exponentially with the worker's age:

$$E(\tau-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(\tau-\upsilon)}, \tag{6}$$

where ω_0 is a positive constant and $\alpha > 0$ specifies the speed at which the efficiency falls with age.

At each period t the household chooses the paths of consumption and financial assets so to maximize the present value of the lifetime utility (1) subject to the budget constraint (2) and a transversality condition. The initial value of the financial assets $a(\upsilon,t)$ and the government consumption per household are taken as given.

The optimal path of the household consumption is defined by Euler condition:

$$\frac{\dot{\bar{c}}(\upsilon,t)}{\bar{c}(\upsilon,t)} = r(t) - \rho \tag{7}$$

Consumption in each period is proportional to the total wealth:

$$\bar{c}(\upsilon,t) = (\rho+\beta)(\bar{a}(\upsilon,t) + \bar{a}^H(\upsilon,t)), \tag{8}$$

where $\bar{a}^H$ is a human wealth defined as the present value of the after-tax labor income:

$$\bar{a}^H(\upsilon,t) = \int_t^\infty WI(\upsilon,\tau)\mathrm{e}^{(\rho+\beta)(t-\tau)}d\tau \tag{9}$$

Demography

Following Bettendorf and Heijdra (2006) we model the framework that allows us to consider a non-zero population growth, by distinguishing the probability of death $\beta \geq 0$, and the probability of birth, $\eta > 0$.[3] The population size $L(t)$ grows with net growth

[3]This framework was developed by Buiter (1988)

rate n_L:

$$\frac{\dot{L}(t)}{L(t)} = \eta - \beta = n_L \tag{10}$$

Taking into account the initial condition $L(0) = 1$, the population size is:

$$L(t) = \mathrm{e}^{n_L t} \tag{11}$$

The size of the generation born in the current period is assumed to be proportional to the size of the population in this period:

$$L(\upsilon, \upsilon) = \eta L(\upsilon) \tag{12}$$

The size of each generation falls exponentially with the probability of death β:

$$L(\upsilon, t) = \mathrm{e}^{\beta(\upsilon - t)} L(\upsilon, \upsilon), t \geq \upsilon \tag{13}$$

The current size of the generation born at time υ can be obtained by substituting (11) and (12) into (13):

$$L(\upsilon, t) = \eta \mathrm{e}^{\eta\upsilon} \mathrm{e}^{-\beta t} \tag{14}$$

Aggregate household sector

The aggregate variables are the integral of the variable values, specific for each living generation, weighted by the size of that generation. Aggregate consumption, for example, can be defined as follows:

$$C(t) = \int_{-\infty}^{t} L(\upsilon, t)\bar{c}(\upsilon, t) d\upsilon, \tag{15}$$

where $L(\upsilon, t)$ and $\bar{c}(\upsilon, t)$ are given by (14) and (8), respectively.

Aggregate consumption is proportional to the household's wealth, where $A(t)$ is aggregate financial wealth and $A^H(t)$ is aggregate human wealth:

$$C(t) = (\rho + \beta) \left[A(t) + A^H(t) \right] \tag{16}$$

The change in the aggregate consumption is obtained by differentiating (15) with respect to time and taking into account (14):

$$\dot{C}(t) = \int_{-\infty}^{t} L(\upsilon, t)\bar{c}(\upsilon, t) d\upsilon + \eta L(t)\bar{c}(t, t) - \beta C(t) \tag{17}$$

The growth rate of the aggregate consumption is obtained by substituting (7) into

(17) and dividing by $C(t)$:

$$\frac{\dot{C}(t)}{C(t)} = [r(t) - \rho] + \frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} \tag{18}$$

The first item on the right-hand side is the growth of individual consumption, while the second term represents the so-called generational turnover (Bettendorf and Heijdra, 2006), which depends on the demographic parameters. Aggregate consumption increases with the arrival of new agents and decreases with the death of the older generation.

The growth rate of the aggregate consumption can be simplified to:[4]

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \alpha + n^L - (\rho + \beta)\frac{\eta\gamma L(t) + (\alpha + \eta)A(t)}{C(t)}, \tag{19}$$

$$\gamma(t) = \frac{d(t)}{r(t) + \beta} + (r(t) + \alpha + \beta)\left(\frac{\mathrm{e}^{-\beta\pi}}{1 - \mathrm{e}^{-\eta\pi}}\right)\left(\frac{z + d(t)}{r(t) + \beta}\right)\left(\frac{\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r(t)}\right) \tag{20}$$

The aggregate consumption growth, therefore, exceeds the growth of individual consumption if the net population growth is positive ($n_L > 0$), the labor productivity decreases over time ($\alpha > 0$). It can be lower if newborns consume less or due to the redistribution from the young to the old through the pension system. In contrast to Bettendorf and Heijdra (2006) γ depends on the surplus (deficit) of the pension fund.

Aggregate financial wealth is defined as follows:

$$A(t) = \int_{-\infty}^{t} L(v,t)\bar{a}(v,t)dv \tag{21}$$

The definition of the aggregate savings can be found by differentiating an equation (21) for the aggregate financial wealth with respect to time and taking into account that the newborn generation does not have any financial wealth, $\bar{a}(t,t) = 0$:

$$\dot{A}(t) = -\beta A(t) + \int_{-\infty}^{t} L(v,t)\dot{\bar{a}}(v,t)dv \tag{22}$$

By substituting (2) in (22) we get:[5]

$$\dot{A}(t) = r(t)A(t) + WI(t) - C(t), \tag{23}$$

$$WI(t) = \frac{\eta\omega_0}{\alpha + \eta}(1 - t_L)F_N(k_N(t),1))L(t) - D(t), \tag{24}$$

where $F_N(k_N(t),1)$ is the marginal product of labor and $D(t)$ is the surplus of the pension system.

The aggregate labor supply in period t measured in efficiency units is proportional

[4]For greater detail see Appendix 1

[5]for grater details see Appendix 2

to the population size in the corresponding period and is obtained from (5), (6), (11) and (14):

$$N(t) = \int_{-\infty}^{t} L(v,t)\bar{n}(v,t)dv = \frac{\eta\omega_0}{\alpha+\eta}L(t) \tag{25}$$

2.2 Firms

As opposed to Bettendorf and Heijdra (2006) we consider a closed economy with one domestic production sector. The output is produced according to the Cobb-Douglas technology $Y = F(K,N) = K^{\varepsilon}N^{1-\varepsilon}$, where K and N represent capital and efficiency labor units. The production function is characterized by the constant returns to scale, the positive and diminishing marginal products of both factors and unitary substitution elasticity. Efficiency units of labor are:

$$N(t) = \int_{-\infty}^{t} E(\tau - v)L(v,t)dv \tag{26}$$

Producers maximize the profit, choosing the optimal level of capital and labor:

$$\Pi(t) = Y(t) - \int_{-\infty}^{t} W^N(v,t)L(v,t)dv - W^K(t)K(t), \tag{27}$$

where $W^K(t)$ is a capital rent and $W^N(v,t)$ is the wage of the worker of generation v at time t.

The first order conditions are:

$$W^K(t) = F_K(k_N(t),1) \tag{28}$$

$$W^N(t) \equiv \frac{W^N(v,t)}{E(\tau - v)} = F_N(k_N(t),1), \tag{29}$$

where $F_K = \partial F/\partial K_N$ and $F_N = \partial F/\partial N$. $W^N(t)$ is the wage per efficiency unit of labor and $k_N(t) = K(t)/N(t)$ is the capital per efficiency unit of labor.

The produced output is allocated to private consumption, investment and government expenditures.

$$Y(t) = C(t) + I(t) + G(t) \tag{30}$$

2.3 Portfolio investments

The optimal investment decision is based on the maximization of the net present value of cash flows from the investor's capital stock subject to the capital accumulation identity:

$$V(t) = \int_{t}^{\infty} \left[W^K(\tau)K(\tau) - I(\tau)\right] \mathrm{e}^{-R(t,\tau)}d\tau \tag{31}$$

$$s.t.\dot{K}(\tau) = I(\tau) - \delta K(\tau) \tag{32}$$

where I represents the gross investment and $R(t,\tau) = \int\limits_t^\tau r(s)ds$ is a discount factor.

Thus, (33), the first order condition for the problem, specifies that the rental rate W^K equals the return on the capital $r(t)$ taking into account the amortization rate δ.

$$W^K = r(t) - \delta \tag{33}$$

In the presented model, as opposed to Bettendorf and Heijdra (2006), interest rate $r(t)$ is endogenous, and is defined by the equilibrium level of capital.

2.4 Public sector and the benevolent government

Government budget identity defines the accumulation path of public debt A^G, which depends on the current government expenditures $G(t)$, revenues from the labor tax and we introduce an additional income (or expenditure) coming from the surplus (or deficit) of the pension fund.

$$\dot{A^G}(t) = r(t)A^G(t) + G(t) - t_L W^N(t)N(t) - D(t) \tag{34}$$

Taking into account the transversality condition:

$$\lim_{\tau \longrightarrow \infty} A^G(t)\mathrm{e}^{R(t,\tau)} = 0 \tag{35}$$

Public debt is:

$$A^G(t) = \int\limits_{-\infty}^{\tau} \left[t_L W^N(\tau)N(\tau) - G(\tau) + D(\tau)\right] \mathrm{e}^{-r(t,\tau)}d\tau, \tag{36}$$

where ψ is the share of the social contributions in the median wage.

$$t_W = \psi\omega_0 F_N(k(t),1)\mathrm{e}^{-\alpha\tau}; \tau = \pi - \frac{1}{\alpha}\ln\left(\frac{1+\mathrm{e}^{\alpha\pi}}{2}\right) \tag{37}$$

We define the social welfare function as the present value of the utility of all currently living and future generations taking into account their share in the population. The first term in (38) represents the welfare of retirees, while the second is the welfare of the young.

$$SW(\tau) = \int\limits_t^\infty \int\limits_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)\ln\bar{c}(\upsilon,\tau) + \kappa\ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau + \tag{38}$$

$$+ \int\limits_t^\infty \int\limits_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)\ln\bar{c}(\upsilon,\tau) + \kappa\ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau$$

In order to derive social welfare as a function of the steady-state value of k^* we express the individual consumption for young and elder generation as a function of their individual human wealth (a_y^H and a_o^H, respectively).[6]

$$SW(t) = \chi e^{\eta\pi}\left[(1-\kappa)(\ln((\rho+\beta)a_o^H) + (r-\rho)\frac{\pi\eta+1}{\eta}) + \kappa \ln g\right] - \quad (39)$$

$$-\chi\left[(1-e^{-\eta\pi})((1-\kappa)\ln((\rho+\beta)a_y^H) - \kappa\ln g) - (1-\kappa)(r-\rho)(1-e^{\eta\pi} - \eta\pi e^{-\eta\pi})\eta^{-1})\right],$$

$$\chi = \frac{e^{n_L t}}{n_L - \rho - \beta}.$$

2.5 Pension system

The key difference with the paper of Bettendorf and Heijdra (2006) is the assumption that the PAYG pension system can be run on an unbalanced-budget basis, with a surplus $D(t) > 0$ or deficit, $D(t) < 0$.

$$t_W(1-e^{-\eta\pi})L(t) = z e^{-\eta\pi}L(t) + D(t) \quad (40)$$

The left-hand side of (40) represents the total social contributions paid by the young, while on the right-hand side are total pensions paid to the old and the surplus (or deficit) of the pension fund if the sum of social contributions and pensions do not match.

2.6 Model summary

The key equations in per capita terms are presented in Table 1 below. The endogenous variables are k, y, c, a, a^G, r, W^N,W^K, γ, n. The exogenous variables are β, α, η, π, ρ, z, t_W, t_L.

Table 1. *Summary of the Log-linear Model*

Description	Analytical representation	
Dynamic equations:		
Capital	$\dot{k}(t) = ny(t) - c(t) - g(t) - (n_L+\delta)k(t)$	$(T1.1)$
Private consumption	$\dot{c}(t) = (r(t)-\rho+\alpha)c(t) - (\rho+\beta)(\eta\gamma(t) + (\alpha+\eta)a(t))$	$(T1.2)$
Public debt	$\dot{a}^G(t) = (r(t)-n_L)a^G(t) + g(t) - nt_L W^N(t) + d(t)$	$(T1.3)$
Private savings	$\dot{a}(t) = (r(t)-n_L)a(t) - c(t) + n(1-t_L)W^N(t) + d(t)$	$(T1.4)$
Static equations:		
	$\gamma(t) = \frac{-d}{r(t)+\beta} + \left(\frac{e^{-\beta\pi}}{1-e^{-\eta\pi}}\right)\left(\frac{z-d}{r(t)+\beta}\right)(r(t)+\alpha+\beta)\left(\frac{e^{-r(t)\pi} - e^{-n_L\pi}}{n^L - r(t)}\right)$	$(T1.5)$
Pension fund	$t_W(1-e^{-\eta\pi}) = ze^{-\eta\pi} - d(t)$	$(T1.6)$
Rental rate	$W^K(t) = \varepsilon k_N(t)^{\varepsilon-1} = \varepsilon\left(\frac{k(t)}{n}\right)^{\varepsilon-1}$	$(T1.7)$
Interest rate	$r(t) = W^K(t) - \delta$	$(T1.8)$
Wage	$W^N(t) = (1-\varepsilon)y(t)$	$(T1.9)$
Output	$y(t) = k_N^{\varepsilon}(t) = \left(\frac{k(t)}{n}\right)^{\varepsilon}$	$(T1.10)$
Supplied efficiency units	$n = \frac{\eta\omega_0}{\alpha+\eta}$	$(T1.11)$
	$a(t) = k(t) + a^G(t)$	$(T1.12)$

[6]For greater detail see Appendix 3

The Eq.T1 corresponds to the accumulation of capital per capita, and it is obtained by combining (30) and (32). Eq.T2 stands for the optimal path of per capita consumption, obtained from (19) in per capita terms. Eq.T3 is the government budget constraint expressed in per capita terms, derived from the government budget constraint (34).The last dynamic equation, Eq.T4, represents the accumulation of per capita assets and is obtained from (23), taking into account (24) and (40).

Definition 1. *Given the set of policy variables* $\left\{g_t, a_t^G, t_L, t_W, z, \pi\right\}$ *that satisfy the government budget constraint the set* $\left\{W_t^K, r_t, y_t, c_t, k_t, a_t, d_t\right\}$ *defines equilibrium, if it satisfies the optimal conditions of households and firms, and the equilibrium conditions for the goods market and labor market.*

$$y(t) = c(t) + i(t) + g(t) \tag{41}$$

$$a(t) = k(t) + a^G(t) \tag{42}$$

In the long run the economy reaches its equilibrium. In the steady state $\dot{k}(t) = 0$, $\dot{c}(t) = 0$ and $\dot{a}^G(t) = 0$.

As the system of dynamic equations is non-linear and cannot be solve analytically we are solving numerically the system of equations T1.1-T1.3, taking into account T1.4-T1.2.[7] After restricting $\dot{k}(t) = 0$, and $\dot{a}^G(t) = 0$ we use a root-finding method (the bisection method) to define the level of capital per capita to bring the growth of consumption per capita to zero, $\dot{c}(t) = 0$. All the possible combinations of fixed and variable parameters on the initially set intervals are considered to determine the steady-state level of k^* and the corresponding combinations of parameters which brings $\dot{c}(t) = 0$.

To distinguish the socially optimal policy mix of measures we check if the resulting set of possible equilibria satisfies the stability condition of the equilibrium and the condition on the limit of public debt.[8]

3 Comparative analysis

3.1 Calibration

To specify socially optimal income tax and the share of social contributions some of the parameters were fixed, allowing us to focus on the fiscal instruments subject to the characteristics of the pension system and demographic conditions. The parameters used for baseline calibration are presented in Table 2. Capital depreciation rate δ is set to 3%, output elasticity of capital ε to 0.33, rate of time preference ρ to 1.5%. The share of the government expenditures in the utility function, κ, is set to 0.5, while the value of

[7] All calculations were conducted using the Matlab language

[8] The stability condition insures that the determinant of the Jacobian matrix of the log-linearized dynamic system of equations T1.1, T1.2 and T1.4 is less than zero. In this case model is locally saddle-point stable.

government expenditures is fixed at 25% of the GDP, which is a common value for OECD countries. Moreover, the productivity declines with age in the speed, α, which equals 1.25%, meaning that the worker is half as productive at the retirement age.

Table 2. *Calibrated Parameter Values*

Variable	Symbol	Value
Rate of time preference	ρ	1.5
Birth rate	η	1
Probability of death	β	1
Share of the government expenditures in the welfare function	κ	0.5
Positive constant in the efficiency index	ω	1
Output elasticity of capital	ε	0.33
Speed of decline in the labor efficiency	α	1.25%
Capital depreciation rate	δ	3%
Retirement age	π	60

The share of pensions is fixed at 30% of the median life-time wage, while the optimal size of the mandatory social contributions, ψ, as a share of the median wage, as well as income tax are chosen optimally from the maximization of the social welfare.[1] We fix the value of pensions as in the case where both the share of pensions and social contributions are chosen optimally, benevolent government would choose to abolish the pension system at all, since rational agents can smooth their consumption via voluntary savings.[2]

3.2 Different retirement ages

Optimal social contributions

First, we consider how optimal social contributions and income tax differ depending on the different retirement age. For the results below the death rate β equals 1.25%, bringing the life expectancy to 80 years. The birth rate η varies from 1% to 3%. This range allows us to analyze both negative and positive population growth. The results for the steady-state of the economy under the different growth rates are presented in Table 3 below.

Table 3. *Optimal social contributions under different retirement age*

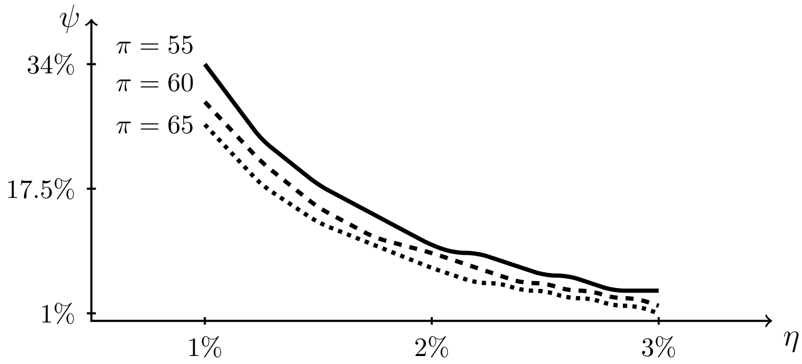
n_L	**-0.25%**	**0.00%**	**0.25%**	**0.50%**	**0.75%**	**1.25%**	**1.75%**
$\pi = 60$							
ψ	29%	21%	15%	11%	9%	5%	2%
c_y	54.5%	53.2%	51.8%	50.4%	49.4%	46.9%	44.6%
d_y	-1.66%	-1.52%	-1.62%	-1.64%	-1.35%	-1.4%	-1.61%
$\pi = 65$							
ψ	26%	18%	13%	10%	7%	4%	1%
c_y	54.5%	52.9%	51.8%	50.4%	49.3%	46.9%	44.5%
d_y	-1.56%	-1.59%	-1.55%	-1.36%	-1.48%	-1.3%	-1.64%
$\tilde{z}$	-1.9%	-1.3%	-1.9%	-1.9%	-1.8%	-1.9%	-1.7%
$\tilde{t}_W$	-12.1%	-15.4%	-15.0%	-10.9%	-23.6%	-21.6%	-50.9%
$\tilde{d}$	6.2%	-5.4%	4.2%	17.2%	-10.3%	5.9%	-1.9%

[1]The results are robust to the change in the share of pensions.

[2]In the further research we plan to introduce the assumptions under the existence of pension system would be optimal.

To focus on how the optimal social contributions depend on the retirement age we, first, consider the case with a fixed income tax. Quantitative results suggest that a higher retirement age leads to the lower pensions for the considered birth rates. This dynamics can be explained by the lower median wage due to the extended working period. At the same time the social contributions are decreasing with the higher retirement age. The reason for this result is twofold: first, the lower median wage corresponds to the longer working period; second, the optimal share of contributions, ψ is lower as lower pensions are paid to retirees. As it is illustrated by Fig.1, optimal share of the social contributions, ψ, is decreasing with the higher retirement age, since less resources are needed to finance pensions.

Figure 1. *Optimal share of the social contributions under different retirement age*



Optimal tax policy

Next we consider the case where the income tax, t_L, was chosen optimally on the interval $[0; 60]$ to maximize of the social welfare function. The results for $\pi = 60$ and $\pi = 65$ are presented in Table 4 below.[3]

Table 4. *Optimal social contributions and income tax under different retirement age*

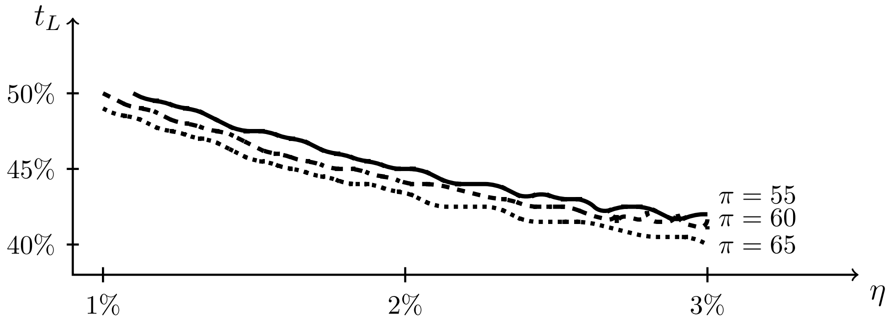
n_L	-0.25%	0.00%	0.25%	0.50%	0.75%	1.25%	1.75%
$\pi = 60$							
ψ	0%	0%	0%	0.5%	0.5%	0%	0.5%
t_L	50%	48%	46.5%	45%	44%	42.5%	41%
c_y	53.7%	52.3%	50.8%	49.6%	48.6%	46.4%	44.6%
d_y	-8.1%	-7.0%	-6.0%	-5.0%	-4.3%	-3.3%	-2.2%
$\pi = 65$							
ψ	0%	0%	0.5%	0%	0%	0.5%	1%
t_L	49%	47.5%	45.5%	44.5%	43.5%	41.5%	40%
c_y	53.7%	52.3%	50.8%	49.6%	48.5%	46.4%	44.5%
d_y	-7.5%	-6.4%	-5.3%	-4.6%	-3.9%	-2.6%	-1.6%
$\tilde{z}$	-1.9%	-1.9%	-1.9%	-1.9%	-1.9%	-1.9%	-1.8%
$\tilde{d}$	-6.7%	-7.9%	-11.5%	-7.3%	-7.7%	-19.4%	-26.2%

The pension system in this case is run with a deficit as well. It varies from 8.2% to 1.6% of the GDP, depending on the growth rate. The deficit of the pension fund is

[3]For greater details see the Appendix 4

generally decreasing for the considered growth rates. Pensions equal 30% of the median wage which is changing alone with the working period, so pensions work as an automatic stabilizer. In this case the value of ψ is less informative since most of the adjustment falls on the income tax. For most growth rates, the optimal level of ψ is chosen at zero or close to it. This illustrates that income tax and social contribution rates can act as imperfect substitutes in the equilibrium. The relationship of the optimal income tax and the birth rate is presented on Fig.2.[4] Optimal income tax exhibits the same inverse relationship we saw in the previous case, when the share of social contributions was changing along with the population growth. In this case social contributions are zero and income tax rate is decreasing with the birth rate since the deficit and pensions can be financed with lower income taxes when the share of young population is high.

Figure 2. *Optimal income tax under different retirement age*



3.3 Different life expectancy

Next we compare the steady state values for the different levels of the life expectancy, namely 70 and 80 years under the unbalanced pension system. Different life expectancy corresponds to β varying from $\beta = 1.43\%$ to $\beta = 1.25\%$. Income tax and social contribution rates are chosen optimally to maximize the social welfare.[5]

We have considered the fixed retirement age so that equilibria under the same birth rates were comparable. The results for the optimal income tax and share of social contributions are presented in Table 5.

Under the unbalanced pension system higher life expectancy leads to the lower private consumption, higher output and, as the result, higher government expenditures in the steady state. Public debt is higher in the steady state under a higher life expectancy due to the higher government expenditures and lower interest rate, while optimal income tax rate remains virtually unchanged with different β.

At the same time pension system is run with deficit. Its share to the GDP is almost the same under different β because the optimal share of social contributions, ψ, is zero or close to it for most cases, while the share of pensions in the GDP is constant as well. Although the absolute value of pensions is increasing with the higher life expectancy due

[4]The death rate β equals 1.25%.

[5]In order to check the robustness of the results the change in β was considered for different retirement ages, for π from 55 to 70. Since the results for different retirement ages are similar, we provide here the results for $\pi = 60$.

to the higher median wage, its change is proportional to the change in the output (as both output and median wage are functions of capital).

Table 5. *Optimal social contributions and income tax under different life expectancy*

η	**1%**	**1.25%**	**1.5%**	**1.75%**	**2%**	**2.5%**	**3%**
$\beta = 1.43\%$							
n_L	-0.43%	-0.18%	0.07%	0.32%	0.57%	1.07%	1.57%
ψ	0%	0%	0%	0.5%	0.5%	0%	0.5%
t_L	50%	48%	46.5%	45%	44%	42.5%	41%
c_y	54.8%	53.6%	52.3%	51.2%	50%	42.1%	45.8%
d_y	-8.1%	-7%	-6%	-5%	-4.3%	-3.3%	-2.2%
z_y	14.72%	14.72%	14.72%	14.72%	14.72%	14.72%	14.72%
$\beta = 1.25\%$							
n_L	-0.25%	0%	0.25%	0.5%	0.75%	1.25%	1.75%
ψ	0%	0%	0%	0.5%	0.5%	0%	0.5%
t_L	50%	48%	46.5%	45%	44%	42.5%	41%
c_y	53.7%	52.3%	50.8%	49.6%	48.6%	46.4%	44.6%
d_y	-8.1%	-7%	-6%	-5%	-4.3%	-3.3%	-2.2%
z_y	14.72%	14.72%	14.72%	14.72%	14.72%	14.72%	14.72%

If two balanced pension systems under different life expectancy are considered, the optimal share of social contributions in the median wage is constant as well as the optimal tax income for the corresponding growth rates. In this case, pensions, income tax rate and contributions rate work as automatic stabilizers, where the optimal rates are unchanged, yet the values paid adjust to the higher level of the median wage due to the higher level of capital.

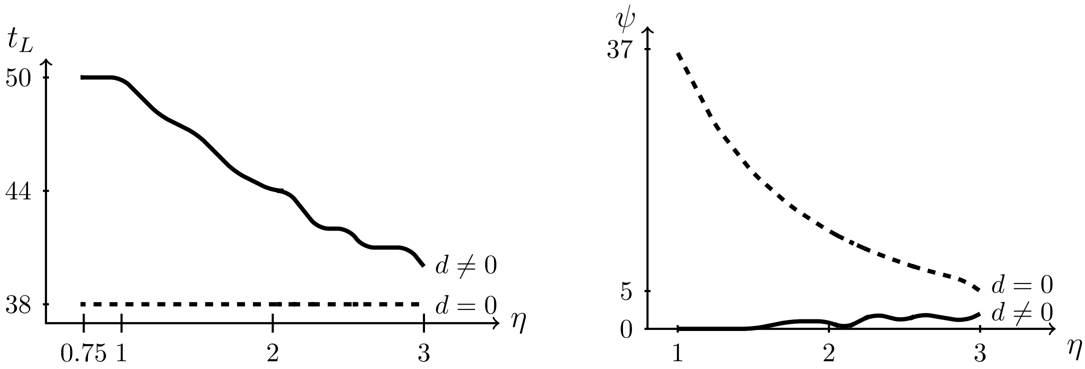
3.4 Balanced and unbalanced pension system

Under the balanced pension system the optimal income tax is lower than in the case of the unbalanced pension system for all the growth rates, because in this case pensions are financed by social contributions and not out of the government budget. The optimal level of the income tax is constant for the considered growth rates, while the share of social contributions is decreasing with the population growth. Higher birth rate increases the value of the social contributions to the pension fund because in this case more young agents make contributions. Therefore, the lower social contributions rate is needed to keep the pension fund balanced. At the same time, public debt in the equilibrium with the balanced pension system is higher, due to the lower tax revenues (both the rate and the aggregate wage is lower due to the fall in capital).

Figure 3 illustrates the results for the optimal level of t_L and ψ for the case of the balanced and unbalanced pension system.[6]

[6] The picture presents the results for $\beta = 1.25\%$ and $\pi = 60$

Figure 3. *Optimal rate of the social contributions and income tax under balanced and unbalanced pension system*



Therefore, depending on the type of pension system, optimal income tax or social contributions are changing with the growth rate with the other remaining at its optimal level.

3.5 Different labor productivity

Higher productivity is modeled as the lower speed, α, with which the product of labor is falling with age. The results are presented in Table 6 for α equals 1.25%, 1.2% and $\alpha = 1.1\%$.

Table 6. *Optimal social contributions and income tax under different labor productivity*[7]

n_L	-0.25%	0%	0.25%	0.5%	0.75%	1.25%	1.75%
$\alpha = 1.25\%$							
ψ	0%	0%	0%	0.5%	0.5%	0%	0.5%
t_L	50%	48%	46.5%	45%	44%	42.5%	41%
c_y	53.7%	52.3%	50.8%	49.6%	48.6%	46.4%	44.6%
d_y	-8.1%	-7%	-6%	-5%	-4.3%	-3.3%	-2.2%
z_y	14.7%	14.7%	14.7%	14.7%	14.7%	14.7%	14.7%
$\alpha = 1.2\%$							
ψ	0%	0%	0%	0%	0.5%	0%	0.5%
t_L	50%	48.5%	47%	45.5%	44%	42.5%	41%
c_y	53.7%	52.3%	51.1%	49.6%	48.6%	46.4%	44.6%
d_y	-8.2%	-7%	-6%	-5.2%	-4.3%	-3.3%	-2.3%
z_y	14.9%	14.9%	14.9%	14.9%	14.9%	14.9%	14.9%
$\alpha = 1.1\%$							
ψ	0%	0%	0%	0%	0%	1%	0.5%
t_L	50%	48.5%	47%	45.5%	44.5%	42%	41%
c_y	53.7%	52.3%	51.1%	50.0%	49.0%	46.8%	44.7%
d_y	-8.3%	-7.2%	-6.2%	-5.3%	-4.6%	-3.0%	-2.3%
z_y	15.2%	15.2%	15.2%	15.2%	15.2%	15.2%	15.2%

Higher labor productivity results in the higher capital in the steady state and, therefore, output per capita as well as government expenditures and private consumption, although the share of private consumption remains the same. Higher productivity leads

[7] The results are presented for $\pi = 60$ and $\beta = 1.25\%$.

to the higher wage, increasing the level of pensions, that equal 30% of the median wage. It puts the higher pressure on the pension system and, thus, the government budget. However, optimal income tax remains virtually unchanged in the equilibrium with different labor productivity, varying within $40-50\%$ for the considered growth rates. However, despite the fact that the deficit of the pension fund is higher under higher labor productivity, public debt is lower in the equilibrium with the higher productivity due to the higher income tax payments (driven by the higher wage).

4 Conclusion

We extend the OLG model with the infinitely living households developed by Bettendorf and Heijdra (2006) by introducing an unbalanced pension system, where the deficit of the pension fund is covered by the transfer from the government budget. This assumption makes income tax and characteristics of the pension system interact as imperfect substitutes to insure the stability of the public debt in the equilibrium.

The developed framework can be used to analyze further the optimal set of the reforms of the pension system and fiscal policy measures, increasing the number of fiscal instruments, that can be chosen optimally.

The results of this research suggest that the financing of the pension fund deficit through the income tax is optimal, when the deficit is covered by the transfer from the state budget. Moreover, the reforms of the pension system such as higher pension age can lower the value of the public debt, and, therefore, can be used together with the traditional measures of the fiscal consolidation.

The results of the research allow to extend the research of the fiscal measures of consolidation and define the welfare optimal measures of fiscal consolidation and pension reforms. These results can be useful in the analysis of consequences of demographic changes for the public finances and in the development of the optimal consolidation measures.

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Appendix

1. Derivation of the aggregate Euler equation

The equation (18) can simplified as follows.

$$\frac{\dot{C}(t)}{C(t)} = [r(t)-\rho] + \frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} \qquad (A1)$$

As new generations are born without any financial assets ($\bar{a}(t,t)=0$), thus from (8) $\bar{c}(t,t) = (\rho+\beta)\bar{a}^H(t,t)$ and taking into account (16) we get:

$$\frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} = (\rho+\beta)\frac{\eta L(t)\bar{a}^H(t,t) - \beta(A(t)+A^H(t))}{C(t)}$$

Aggregate human wealth is defined as follows:

$$A^H(t) = \left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} + \left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi}$$

where:

$$\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} = \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_t^{\infty} z\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau$$

and $R(t,\tau) = \int_t^{\tau} r(s)ds$

$$\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi} = \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau - \int_t^{\upsilon+\pi} t_W\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_{\upsilon+\pi}^{\infty} z\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau$$

For the simplicity of the analysis let us assume the constant interest rate r (so that equations could be applicable to the steady state).

$$\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi} = \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau - \frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right) + \frac{z}{r(t)+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}$$

We know that age dependent wage can be written as follows:

$$W^N(\upsilon,t) = E(\tau-\upsilon)F_N(k(t),1) = \omega_0 \mathrm{e}^{-\alpha(\tau-\upsilon)}F_N(k(t),1)$$

$$\int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau = \mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)$$

where $\Omega_0(t)$ is defined as follows:

$$\Omega_0(t) = \omega_0\int_t^{\infty}(1-t_L)F_N(k(t),1)\mathrm{e}^{(r+\alpha+\beta)(t-\tau)}d\tau$$

Substituting this definition into the expressions for human wealth of workers and retirees, noted above, we get:

$$\int_{-\infty}^{t-\pi} L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi}d\upsilon =$$

$$= \int_{-\infty}^{t-\pi}\eta \mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)\upsilon + \frac{z}{r+\beta}\right]d\upsilon =$$

$$= L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\mathrm{e}^{-(\alpha+\eta)\pi} - \frac{z}{r+\beta}\mathrm{e}^{-\eta\pi}\right].$$

$$\int_{t-\pi}^{t} L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi}d\upsilon =$$

$$= \int_{t-\pi}^{t}\eta \mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t) - \frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right) + \frac{z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right]d\upsilon =$$

$$= L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\left(1-\mathrm{e}^{-(\alpha+\eta)\pi}\right) - \frac{t_W}{r+\beta}(1-\mathrm{e}^{-\eta\beta}) + \frac{\eta(t_W+z)}{r+\beta}e^{-\beta\pi}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right]$$

Thus aggregate human wealth is:

$$A^H(t) = L(t)\left[\frac{\lambda\Omega_0(t)}{\alpha+\eta} + \eta\mathrm{e}^{-\beta\pi}\frac{t_W+z}{r+\beta}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right]$$

where it was used that:

$$t_W(1-\mathrm{e}^{-\eta\pi}) = z\mathrm{e}^{-\eta\pi} + d^{pens}(t) \Rightarrow t_W + z = \frac{z+d^{pens}(t)}{1-\mathrm{e}^{-\eta\pi}}$$

From the expression for working-age households, taking into account that $t_W = d + (t_W + z)\mathrm{e}^{-\eta\pi}$:

$$\bar{a}^H(t,t) = \Omega_0(t) + \left(\frac{t_W + z}{r(t)+\beta}\right)\left(\mathrm{e}^{-(r(t)+\beta)\pi} - \mathrm{e}^{-\eta\pi}\right) - \frac{t_W}{r(t)+\beta} =$$

$$= \Omega_0(t) + \mathrm{e}^{-\beta\pi}\left(\frac{t_W + z}{r(t)+\beta}\right)\left(\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}\right) - \frac{d(t)}{r(t)+\beta}$$

After substituting this expression in the equation for A^H and eliminating $\Omega_0(t)$ we get:

$$\eta L(t)\bar{a}^H(t,t) = (\alpha+\eta)A^H(t) - \eta\gamma L(t)$$

where

$$\gamma = \frac{d(t)}{r+\beta} + (r+\alpha+\beta)\left(\frac{\mathrm{e}^{-\beta\pi}}{1-\mathrm{e}^{-\eta\pi}}\right)\left(\frac{z+d(t)}{r+\beta}\right)\left(\frac{\mathrm{e}^{-r\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r}\right)$$

Taking into account the expression for $\bar{a}^H(t,t)$ and taking into account (16) we get:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \alpha + n^L - (\rho+\beta)\frac{\eta\gamma L(t) + (\alpha+\eta)A(t)}{C(t)}$$

2. Derivation of aggregate non-interest income

Aggregate non-interest income is defined as follows:

$$WI(t) = \int_{-\infty}^{t} L(\upsilon,t)WI(\upsilon,t)d\upsilon,$$

where $WI(\upsilon,t)$ is defined as follows:

$$WI(\upsilon,\tau) = \begin{cases} (1-t_L)W^N(\upsilon,\tau) - t_W & \text{for } \tau-\upsilon \le \pi, \\ (1-t_L)W^N(\upsilon,\tau) + z & \text{for } \tau-\upsilon > \pi. \end{cases}$$

Thus $WI(t)$ is split into parts, non-interest income of the retirees and non-interest income of the young:

$$WI(t) = \int_{-\infty}^{t-\pi} L(\upsilon,t)\left[(1-t_L)WI(\upsilon,t) + z\right]d\upsilon + \int_{t-\pi}^{t} L(\upsilon,t)\left[(1-t_L)WI(\upsilon,t) - t_W\right]d\upsilon$$

After applying the expression for $W^N(\upsilon,t) = E(t-\upsilon)F_N(k_N(t,1)) = \omega_0\mathrm{e}^{-\alpha(t-\upsilon)}F_N(k_N(t,1))$, which comes from taking into account the definition of the efficiency index $E(\tau-\upsilon)$ and fact that the wage of particular worker born at period υ is equal to the marginal product of labor, adjusted for his or her productivity.

$$WI(t) = \int_{-\infty}^{t-\pi} L(\upsilon,t)\left[(1-t_L)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} F_N(k_N(t,1)) + z\right] d\upsilon +$$
$$+ \int_{t-\pi}^{t} L(\upsilon,t)\left[(1-t_L)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} F_N(k_N(t,1)) - t_W\right] d\upsilon$$

Rearranging we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\left[\int_{-\infty}^{t-\pi} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon + \int_{t-\pi}^{t} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon\right] +$$
$$+ z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon =$$
$$= (1-t_L)F_N(k_N(t,1)) \int_{-\infty}^{t} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon +$$
$$+ z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon$$

Noting from (5) that $\bar{n}(\upsilon,t) = E(t-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(t-\upsilon)}$ and applying the notion of $N(t)$ from (24) we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) + z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon$$

Applying (14) we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) + z\int_{-\infty}^{t-\pi} \eta \mathrm{e}^{\eta\upsilon-\beta t} d\upsilon - t_W \int_{t-\pi}^{t} \eta \mathrm{e}^{\eta\upsilon-\beta t} d\upsilon =$$
$$= (1-t_L)F_N(k(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) - (1-\mathrm{e}^{-\eta\pi})t_W \mathrm{e}^{n_L t} + z\mathrm{e}^{-\eta\pi}\mathrm{e}^{n_L t}$$

Taking into account the fact that the pension system is run on a non balanced manner, thus using (34) the equation of the aggregate income can be simplified as follows:

$$WI(t) = (1-t_L)\frac{\eta\omega_0}{\alpha+\eta}F_N(k_N(t,1))L(t) - D(t)$$

3. Social welfare

$$SW = \int_{t}^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)ln\bar{c}(\upsilon,\tau) + \kappa ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau +$$

$$+ \int_{t}^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)ln\bar{c}(\upsilon,\tau) + \kappa ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau$$

Taking into account the Eurler equation:

$$\bar{c}(\upsilon,\tau) = \bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\tau-t)}$$

$$\bar{c}(\upsilon,\upsilon) = \bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\upsilon-t)}$$

$$(\rho+\beta)[\bar{a}(\upsilon,\upsilon) + \bar{a}^H(\upsilon,\upsilon)] = (\rho+\beta)[\bar{a}(\upsilon,t) + \bar{a}^H(\upsilon,t)]\mathrm{e}^{-(r(t)-\rho)(\upsilon-t)}$$

Applying that $\bar{a}(\upsilon,\upsilon) = 0$ and simplifying we get:

$$\bar{c}(\upsilon,\tau) = (\rho+\beta)(\bar{a}(\upsilon,\tau) + \bar{a}^H(\upsilon,\tau)) = (\rho+\beta)\bar{a}^H(\upsilon,\upsilon)\mathrm{e}^{-(r-\rho)(\upsilon-t)}$$

$$\bar{a}^H_{old}(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon}\mathrm{e}^{\alpha(\upsilon-t)}\right) + \frac{z}{r+\beta}$$

$$\bar{a}^H_{young}(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon}\mathrm{e}^{\alpha(\upsilon-t)}\right) - \frac{tw}{r+\beta} + \frac{tw+z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}$$

$$\begin{aligned} SW(t) = \frac{\mathrm{e}^{n_L t}}{n_L-\rho-\beta}\Big[((1-\kappa)\ln((\rho+\beta)a^H_{old})+ \\ +(1-\kappa)(r-\rho)\frac{\pi\eta+1}{\eta} + \kappa\ln g)\mathrm{e}^{\eta\pi}- \\ -(1-\kappa)\ln((\rho+\beta)a^H_{young})(1-\mathrm{e}^{-\eta\pi})- \\ -(1-\kappa)(r-\rho)(1-\mathrm{e}^{\eta\pi}-\eta\pi\mathrm{e}^{-\eta\pi})\eta^{-1} - \kappa\ln g(1-\mathrm{e}^{-\eta\pi})\Big] \end{aligned}$$

4. Numeric results

Different retirement age

Table 1. Key model variables and policy instruments for $\pi = 55$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	34%	24%	18%	14%	10%	9%	7%	6%	4%	4%
t_W	0,205	0,162	0,131	0,108	0,081	0,074	0,059	0,051	0,035	0,035
z	0,181	0,202	0,218	0,231	0,242	0,247	0,254	0,257	0,263	0,264
$\bar{W}$	0,906	1,012	1,089	1,155	1,210	1,235	1,270	1,287	1,317	1,322
W^N	1,809	1,795	1,771	1,757	1,745	1,718	1,714	1,691	1,691	1,662
$y * n$	1,206	1,346	1,449	1,537	1,611	1,644	1,690	1,712	0,467	1,760
g	0,301	0,337	0,362	0,384	0,403	0,411	0,423	0,428	0,438	0,440
k	8,875	9,759	10,228	10,676	11,037	10,917	11,166	11,011	11,273	10,934
a^G	0,137	0,073	0,144	0,309	0,009	0,538	0,419	0,732	0,053	1,144
c	0,660	0,717	0,754	0,779	0,794	0,801	0,804	0,805	0,802	0,800
c_{share}	54,8%	53,3%	52,1%	50,7%	49,3%	48,8%	47,6%	47,0%	47,0%	45,5%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,53%	1,60%	1,72%	1,80%	1,87%	2,02%	2,05%	2,18%	2,18%	2,36%
d_{share}	-1,47%	-1,58%	-1,52%	-1,41%	-1,66%	-1,32%	-1,44%	-1,31%	-6,18%	-1,27%
SW	0,133	-4,138	-8,421	-12,114	-15,738	-20,200	-23,849	-29,695	-35,148	-45,814
a_o^H	33,457	33,383	32,492	32,092	31,760	30,376	30,326	29,149	29,327	27,816
a_y^H	22,563	23,266	23,056	23,058	23,266	22,179	22,372	21,516	21,947	20,659
SW_o	11,903	9,526	7,511	6,325	5,507	4,547	4,306	3,677	3,747	3,128
SW_y	-11,770	-13,664	-15,933	-18,438	-21,245	-24,748	-28,154	-33,372	-38,895	-48,942

Table 2. Key model variables and policy instruments for $\pi = 60$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	29%	21%	15%	11%	9%	7%	5%	4%	3%	2%
t_W	0,173	0,139	0,107	0,084	0,071	0,057	0,042	0,034	0,026	0,018
z	0,179	0,198	0,215	0,228	0,237	0,244	0,250	0,256	0,258	0,263
$\bar{W}$	0,894	0,992	1,074	1,139	1,184	1,218	1,248	1,278	1,291	1,314
W^N	1,821	1,796	1,783	1,769	1,742	1,730	1,718	1,713	1,691	1,686
$y * n$	1,214	1,347	1,459	1,547	1,608	1,655	1,695	1,735	1,753	1,785
g	0,303	0,337	0,365	0,387	0,402	0,414	0,424	0,434	0,438	0,446
k	9,052	9,784	10,437	10,890	10,987	11,140	11,257	11,461	11,273	11,420
a^G	0,006	0,127	0,051	0,030	0,454	0,415	0,099	0,307	0,333	0,202
c	0,661	0,717	0,755	0,779	0,794	0,801	0,804	0,803	0,802	0,797
c_{share}	54,5%	53,2%	51,8%	50,4%	49,4%	48,4%	47,4%	46,3%	45,7%	44,6%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,88%	1,95%	2,02%	2,05%	2,18%	2,21%
d_{share}	-1,66%	-1,52%	-1,62%	-1,64%	-1,35%	-1,42%	-1,62%	-1,54%	-1,55%	-1,61%
SW	-0,304	-5,345	-9,115	-12,820	-17,428	-20,980	-24,994	-29,691	-36,941	-45,272
a_{old}^H	34,092	33,335	33,104	32,674	31,436	30,930	30,452	30,353	29,171	29,070
a_{young}^H	23,696	23,622	23,962	23,986	23,104	22,918	22,799	22,785	21,956	21,985
W_{young}	11,946	9,004	7,322	6,061	4,811	4,223	3,772	3,577	3,033	3,033
W_{young}	-12,251	-14,349	-16,437	-18,881	-22,239	-25,203	-28,766	-33,267	-39,974	-48,305

Table 3. Key model variables and policy instruments for $\pi = 65$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	26%	18%	13%	10%	7%	5%	4%	3%	2%	1%
t_W	0,152	0,118	0,091	0,074	0,054	0,040	0,033	0,025	0,017	0,009
z	0,175	0,196	0,211	0,223	0,233	0,240	0,246	0,251	0,255	0,258
$\bar{W}$	0,8762	0,9792	1,05315	1,11705	1,1628	1,2024	1,2316	1,2528	1,2741	1,2921
W^N	1,821	1,809	1,783	1,769	1,745	1,741	1,730	1,713	1,702	1,690
$y * n$	1,214	1,356	1,459	1,547	1,611	1,666	1,706	1,735	1,765	1,790
g	0,303	0,339	0,365	0,387	0,403	0,416	0,427	0,434	0,441	0,447
k	9,052	9,983	10,437	10,890	11,037	11,365	11,487	11,461	11,503	11,509
a^G	0,078	0,070	0,122	0,384	0,264	0,075	0,327	0,448	0,419	0,105
c	0,661	0,718	0,755	0,779	0,794	0,800	0,803	0,803	0,800	0,796
c_{share}	54,5%	52,9%	51,8%	50,4%	49,3%	48,0%	47,1%	46,3%	45,3%	44,5%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,53%	1,66%	1,74%	1,87%	1,89%	1,95%	2,05%	2,12%	2,18%
d_{share}	-1,56%	-1,59%	-1,55%	-1,36%	-1,48%	-1,62%	-1,51%	-1,49%	-1,53%	-1,64%
SW	-1,475	-5,893	-10,353	-14,174	-18,483	-21,560	-25,738	-31,103	-37,603	-46,459
a_o^H	33,964	33,980	32,960	32,526	31,454	31,495	31,013	30,202	29,702	29,179
a_y^H	23,989	24,557	24,150	23,983	23,462	23,713	23,381	22,824	22,530	22,238
SW_o	11,410	8,932	6,766	5,501	4,326	3,949	3,481	2,993	2,710	2,501
SW_y	-12,884	-14,825	-17,119	-19,675	-22,809	-25,509	-29,220	-34,096	-40,314	-48,960

Table 4. Key model variables and policy instruments for $\pi = 70$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	23%	16%	11%	6%	4%	3%	2%	1%	1%	1%
t_W	0,133	0,103	0,076	0,046	0,032	0,024	0,016	0,008	0,008	0,008
z	0,173	0,192	0,208	0,230	0,236	0,242	0,246	0,250	0,251	0,251
$\bar{W}$	0,866	0,961	1,040	1,149	1,181	1,209	1,231	1,250	1,255	1,255
W^N	1,833	1,809	1,795	1,757	1,743	1,730	1,715	1,702	1,691	1,673
$y * n$	1,222	1,356	1,469	1,622	1,667	1,706	1,737	1,765	1,772	1,771
g	0,306	0,339	0,367	0,405	0,417	0,427	0,434	0,441	0,443	0,443
k	9,244	9,983	10,646	11,262	11,401	11,487	11,492	11,503	11,394	11,157
a^G	0,073	0,148	0,093	0,475	0,204	0,381	0,410	0,240	0,717	1,130
c	0,662	0,718	0,755	0,794	0,800	0,803	0,803	0,800	0,799	0,799
c_{share}	54,2%	52,9%	51,4%	49,0%	48,0%	47,1%	46,2%	45,3%	45,1%	45,1%
n_L	-0,25%	0,00%	0,25%	0,75%	0,95%	1,15%	1,35%	1,55%	1,65%	1,75%
r	1,41%	1,53%	1,60%	1,80%	1,88%	1,95%	2,04%	2,12%	2,18%	2,29%
d_{share}	-1,57%	-1,50%	-1,58%	-1,36%	-1,55%	-1,49%	-1,50%	-1,59%	-1,45%	-1,32%
SW	-1,838	-7,039	-10,921	-19,114	-22,640	-26,900	-32,164	-38,763	-43,417	-49,396
a_o^H	34,670	33,849	33,583	32,050	31,466	30,870	30,155	29,561	28,965	28,032
a_y^H	24,949	24,761	24,958	24,078	23,860	23,447	22,970	22,609	22,093	21,324
SW_o	11,511	8,409	6,629	4,126	3,493	2,983	2,531	2,204	1,952	1,566
SW_y	-13,349	-15,448	-17,550	-23,240	-26,133	-29,883	-34,695	-40,967	-45,369	-50,962

Table 5. Key model variables and policy instruments for $\pi = 55$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	26%	10%	2%	0%	1%	1%	1%	1%	0%	0%
t_W	0,106	0,053	0,012	0,000	0,007	0,008	0,008	0,008	0,000	0,000
z	0,123	0,159	0,186	0,207	0,224	0,234	0,245	0,251	0,262	0,269
$\bar{W}$	0,409	0,530	0,620	0,689	0,746	0,782	0,818	0,836	0,872	0,895
W^N	1,904	1,881	1,858	1,833	1,821	1,783	1,768	1,745	1,718	1,688
t_L	50%	50%	50%	49%	47%	46%	45%	44%	43%	42%
$y * n$	0,816	1,058	1,238	1,375	1,490	1,560	1,632	1,669	1,741	1,787
g	0,204	0,265	0,310	0,344	0,372	0,390	0,408	0,417	0,435	0,447
k	6,658	8,424	9,614	10,399	11,108	11,161	11,488	11,437	11,561	11,459
a^G	0,022	0,049	0,069	0,107	0,035	0,318	0,549	0,371	0,183	0,453
c	0,462	0,583	0,664	0,719	0,756	0,780	0,794	0,800	0,802	0,796
c_{share}	56,6%	55,1%	53,7%	52,3%	50,8%	50,0%	48,6%	47,9%	46,1%	44,5%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,470%	1,660%	1,737%	1,865%	2,018%	2,199%
d_{share}	-8,3%	-8,3%	-8,2%	-7,6%	-6,3%	-5,4%	-4,7%	-4,1%	-3,6%	-2,9%
SW	23,002	11,687	4,622	-0,826	-5,021	-10,044	-14,091	-18,203	-27,361	-43,012
a^{H_old}	31,801	32,037	31,798	31,704	32,535	31,206	31,171	30,441	29,680	28,623
a^H_{young}	24,713	25,614	25,920	25,731	25,936	24,560	24,326	23,620	23,003	22,003
SW_o	30,162	20,842	15,640	12,157	9,954	7,769	6,671	5,675	4,708	4,104
SW_y	-7,160	-9,155	-11,018	-12,983	-14,976	-17,814	-20,761	-23,878	-32,069	-47,116

Table 6. Key model variables and policy instruments for $\pi = 60$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	21%	6%	0%	0%	0%	1%	1%	1%	2%	2%
t_W	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,017	0,018
z	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,256	0,263
$\bar{W}$	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,854	0,876
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,768	1,757	1,718	1,686
t_L	50%	50%	50%	48%	47%	45%	44%	43%	41%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,740	1,785
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,446
k	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,560	11,420
a^G	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,038	0,202
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,6%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,737%	1,800%	2,018%	2,211%
d_{share}	-8,2%	-8,3%	-8,1%	-7,0%	-6,0%	-4,8%	-4,1%	-3,6%	-2,3%	-1,6%
SW	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-29,208	-45,272
a^{H_o}	31,695	31,905	31,649	32,015	31,629	32,204	31,422	31,402	30,275	29,070
a^H_y	25,103	26,011	26,039	25,944	25,269	25,334	24,492	24,364	23,089	21,985
SW_o	29,672	20,269	15,014	11,376	8,919	7,317	5,886	5,177	3,789	3,033
SW_y	-7,695	-9,766	-11,748	-13,716	-16,034	-18,372	-21,563	-24,367	-32,997	-48,305

Table 7. Key model variables and policy instruments for $\pi = 65$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	16%	3%	0%	0%	0%	1%	1%	0%	1%	1%
t_W	0,063	0,015	0,000	0,000	0,000	0,008	0,008	0,000	0,008	0,009
z	0,118	0,153	0,179	0,199	0,214	0,227	0,236	0,243	0,251	0,258
$\bar{W}$	0,393	0,509	0,596	0,662	0,712	0,756	0,786	0,809	0,838	0,861
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,769	1,757	1,718	1,690
t_L	50%	50%	49%	48%	46%	44%	43%	43%	41%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,633	1,680	1,741	1,790
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,447
k	6,658	8,424	9,614	10,399	10,890	11,384	11,491	11,670	11,561	11,509
a^G	0,017	0,015	0,104	0,580	0,256	0,045	0,136	0,419	0,161	0,105
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,796
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,5%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,736%	1,800%	2,018%	2,184%
d_{share}	-8,3%	-8,3%	-7,5%	-6,4%	-5,4%	-4,3%	-3,6%	-3,5%	-2,3%	-1,6%
SW	20,982	9,335	1,883	-3,802	-8,514	-12,441	-17,031	-20,612	-30,636	-46,459
a^{H_o}	31,595	31,781	32,000	31,867	31,925	32,484	31,695	31,244	30,126	29,179
a^H_y	25,555	26,299	26,316	25,725	25,501	25,550	24,709	24,385	23,130	22,238
SW_o	29,185	19,708	14,276	10,761	8,193	6,607	5,194	4,572	3,191	2,501
SW_y	-8,204	-10,373	-12,393	-14,563	-16,708	-19,049	-22,225	-25,184	-33,827	-48,960

Table 8. Key model variables and policy instruments for $\pi = 70$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	0%	0%	0%	0%	0%	0%	0%	2%	0%	2%
t_W	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,016	0,000	0,017
z	0,117	0,150	0,175	0,195	0,211	0,223	0,231	0,237	0,248	0,253
$\bar{W}$	0,390	0,500	0,585	0,649	0,704	0,742	0,772	0,788	0,828	0,843
W^N	1,927	1,881	1,858	1,833	1,821	1,795	1,770	1,745	1,730	1,686
t_L	53%	53%	49%	47%	45%	44%	43%	41%	41%	39%
$y * n$	0,826	1,058	1,238	1,375	1,490	1,571	1,634	1,669	1,752	1,785
g	0,206	0,265	0,310	0,344	0,372	0,393	0,408	0,417	0,438	0,446
k	6,895	8,424	9,614	10,399	11,108	11,384	11,513	11,437	11,797	11,423
a^G	0,165	1,224	0,506	0,417	0,051	0,245	0,288	0,069	0,110	0,364
c	0,464	0,583	0,664	0,719	0,756	0,779	0,794	0,800	0,801	0,797
c_{share}	56,2%	55,1%	53,7%	52,3%	50,8%	49,6%	48,6%	47,9%	45,7%	44,6%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	0,992%	1,187%	1,294%	1,408%	1,470%	1,599%	1,730%	1,865%	1,951%	2,210%
d_{share}	-10,0%	-8,4%	-7,0%	-5,9%	-5,0%	-4,2%	-3,5%	-2,3%	-2,3%	-0,9%
SW	22,645	8,294	0,554	-5,103	-9,245	-13,703	-18,209	-22,433	-31,152	-48,214
a^{H_o}	31,152	30,133	31,869	32,197	32,982	32,336	31,617	31,180	30,685	29,149
a^H_y	27,020	25,098	26,134	26,007	26,378	25,588	24,814	23,996	23,754	22,043
SW_o	31,262	19,609	13,699	10,066	7,885	6,051	4,682	3,611	2,928	1,800
SW_y	-8,617	-11,315	-13,145	-15,169	-17,130	-19,755	-22,892	-26,044	-34,080	-50,014

Different life expectancy

Table 9. Key model variables and policy instruments for $\beta = 1.43\%$ with an optimal t_L

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,017
z	0,161	0,183	0,203	0,218	0,230	0,240	0,248	0,255	0,262
$\bar{W}$	0,537	0,611	0,676	0,726	0,766	0,800	0,827	0,852	0,874
W^N	1,869	1,866	1,838	1,808	1,784	1,766	1,734	1,713	1,682
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,094	1,244	1,378	1,479	1,561	1,630	1,685	1,735	1,781
y	2,804	2,800	2,756	2,712	2,676	2,648	2,601	2,569	2,522
g	0,274	0,311	0,345	0,370	0,390	0,407	0,421	0,434	0,445
k	8,601	9,751	10,471	10,882	11,178	11,431	11,401	11,455	11,329
a^G	0,093	0,187	0,040	0,352	0,195	0,333	0,049	0,029	0,139
c	0,617	0,682	0,738	0,775	0,799	0,814	0,823	0,824	0,818
c_{share}	56,36%	54,85%	53,56%	52,41%	51,22%	49,95%	48,81%	47,46%	45,91%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,63%	-0,43%	-0,18%	0,07%	0,32%	0,57%	0,87%	1,17%	1,57%
r	1,24%	1,25%	1,39%	1,53%	1,66%	1,75%	1,93%	2,05%	2,24%
dt	-0,089	-0,101	-0,096	-0,089	-0,075	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	7,451	3,316	-1,897	-6,219	-9,910	-13,304	-18,050	-23,224	-33,123
a^{H_o}	29,884	30,566	30,709	30,121	30,095	29,857	29,233	28,723	27,665
a^H_y	24,261	25,101	24,833	24,008	23,604	23,218	22,399	21,859	20,885
SW_o	16,612	13,679	10,106	7,686	5,944	4,820	3,535	2,795	2,010
SW_y	-9,162	-10,363	-12,003	-13,904	-15,854	-18,123	-21,584	-26,019	-35,134

Table 10. Key model variables and policy instruments for $\beta = 1.25\%$ with an optimal t_L

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,018
z	0,162	0,182	0,202	0,218	0,231	0,240	0,249	0,256	0,263
$\bar{W}$	0,540	0,608	0,675	0,726	0,771	0,801	0,831	0,854	0,876
W^N	1,881	1,858	1,833	1,809	1,795	1,768	1,742	1,718	1,686
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,101	1,238	1,375	1,480	1,571	1,632	1,692	1,740	1,785
y	2,822	2,786	2,750	2,713	2,692	2,653	2,612	2,577	2,529
g	0,275	0,310	0,344	0,370	0,393	0,408	0,423	0,435	0,446
k	8,766	9,614	10,399	10,890	11,384	11,488	11,550	11,560	11,420
a^G	0,106	0,203	0,044	0,402	0,238	0,400	0,062	0,038	0,202
c	0,602	0,664	0,719	0,756	0,779	0,794	0,802	0,802	0,797
c_{share}	54,70%	53,65%	52,31%	51,08%	49,63%	48,61%	47,36%	46,11%	44,62%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,89%	2,02%	2,21%
dt	-0,090	-0,100	-0,096	-0,089	-0,076	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	9,109	3,266	-2,340	-7,115	-11,055	-15,676	-21,687	-29,208	-45,272
a^{H_o}	32,163	31,649	32,015	31,629	32,204	31,422	30,986	30,275	29,070
a^H_y	26,200	26,039	25,944	25,269	25,334	24,492	23,800	23,089	21,985
SW_o	19,260	15,014	11,376	8,919	7,317	5,886	4,603	3,789	3,033
SW_y	-10,151	-11,748	-13,716	-16,034	-18,372	-21,563	-26,290	-32,997	-48,305

Table 11. Key model variables and policy instruments for $\beta = 1.43\%$ and $d = 0$

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	49%	36%	27%	21%	16%	13%	10%	8%	6%
t_W	0,254	0,214	0,176	0,146	0,121	0,102	0,082	0,067	0,051
z	0,156	0,176	0,197	0,213	0,225	0,236	0,245	0,252	0,258
$\bar{W}$	0,521	0,588	0,655	0,709	0,750	0,786	0,817	0,839	0,861
W^N	1,8129	1,7962	1,7807	1,7656	1,7460	1,7341	1,7126	1,6885	1,6561
t_L	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,38
$y * n$	1,061	1,197	1,336	1,445	1,528	1,601	1,664	1,710	1,754
y	2,719	2,694	2,671	2,648	2,619	2,601	2,569	2,533	2,484
g	0,265	0,299	0,334	0,361	0,382	0,400	0,416	0,428	0,438
k	7,847	8,692	9,529	10,133	10,479	10,830	10,984	10,972	10,821
a^G	0,166	0,198	0,241	0,287	0,331	0,394	0,470	0,556	0,704
c	0,610	0,675	0,733	0,772	0,798	0,814	0,823	0,825	0,820
c_{share}	57,46%	56,34%	54,87%	53,46%	52,22%	50,84%	49,45%	48,24%	46,79%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,63%	-0,43%	-0,18%	0,07%	0,32%	0,57%	0,87%	1,17%	1,57%
r	1,51%	1,59%	1,67%	1,75%	1,86%	1,93%	2,05%	2,20%	2,40%
SW	2,132	-1,969	-5,761	-8,998	-12,228	-15,195	-19,471	-24,826	-35,007
a^{H_o}	32,171	31,913	31,720	31,394	30,694	30,369	29,494	28,422	26,954
a^H_y	20,610	21,089	21,570	21,796	21,634	21,660	21,250	20,626	19,689
SW_o	11,936	9,207	7,069	5,571	4,347	3,601	2,771	2,077	1,339
SW_y	-9,804	-11,176	-12,830	-14,569	-16,575	-18,796	-22,242	-26,904	-36,346

Table 12. Key model variables and policy instruments for $\beta = 1.25\%$ and $d = 0$

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	49%	36%	27%	21%	16%	13%	10%	8%	6%
t_W	0,255	0,215	0,176	0,146	0,121	0,101	0,082	0,067	0,051
z	0,157	0,176	0,197	0,213	0,225	0,235	0,245	0,252	0,259
$\bar{W}$	0,523	0,588	0,656	0,710	0,749	0,784	0,817	0,840	0,864
W^N	1,821	1,797	1,783	1,769	1,745	1,730	1,713	1,691	1,662
t_L	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380
$y * n$	1,066	1,198	1,337	1,447	1,527	1,597	1,665	1,712	1,760
y	2,731	2,695	2,675	2,653	2,618	2,595	2,570	2,536	2,493
g	0,266	0,299	0,334	0,362	0,382	0,399	0,416	0,428	0,440
k	7,949	8,697	9,568	10,183	10,463	10,751	10,995	11,011	10,934
a^G	0,185	0,217	0,269	0,324	0,373	0,443	0,556	0,684	0,955
c	0,597	0,659	0,716	0,754	0,779	0,794	0,803	0,805	0,800
c_{share}	55,98%	55,03%	53,54%	52,13%	51,02%	49,75%	48,25%	47,03%	45,48%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,87%	1,95%	2,05%	2,18%	2,36%
SW	3,018	-2,125	-6,366	-10,171	-14,304	-18,275	-23,791	-31,447	-47,783
a^{H_o}	34,212	33,441	33,349	33,015	32,007	31,442	30,793	29,719	28,351
a^H_y	22,035	22,183	22,760	22,998	22,616	22,471	22,233	21,608	20,745
SW_o	13,931	10,477	8,238	6,617	5,159	4,276	3,525	2,827	2,178
SW_y	-10,913	-12,601	-14,604	-16,788	-19,463	-22,551	-27,315	-34,274	-49,961

Different labor productivity

Table 13. Key model variables and policy instruments for $\alpha = 1.25\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	32%	21%	6%	0%	0%	0%	1%	1%	1%	1%	2%
t_W	0,109	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,008	0,018
z	0,102	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,255	0,263
$\bar{W}$	0,340	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,849	0,876
W^N	1,904	1,904	1,881	1,858	1,833	1,809	1,795	1,768	1,757	1,730	1,686
t_L	50%	50%	50%	50%	48%	47%	45%	44%	43%	42%	40%
$y * n$	0,692	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,730	1,785
g	0,173	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,432	0,446
k	5,650	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,646	11,420
a^G	0,037	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,237	0,202
c	0,398	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	57,46%	56,65%	55,10%	53,65%	52,31%	51,08%	49,63%	48,61%	47,57%	46,39%	44,62%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,09%	1,09%	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,80%	1,95%	2,21%
d_{share}	-8,23%	-8,24%	-8,32%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-3,57%	-2,90%	-1,61%
SW	28,207	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-26,187	-45,272
a^{H_o}	30,922	31,695	31,905	31,649	32,015	31,629	32,204	31,422	31,402	30,497	29,070
a^H_y	24,122	25,103	26,011	26,039	25,944	25,269	25,334	24,492	24,364	23,480	21,985
SW_o	34,975	29,672	20,269	15,014	11,376	8,919	7,317	5,886	5,177	4,189	3,033
SW_y	-6,768	-7,695	-9,766	-11,748	-13,716	-16,034	-18,372	-21,563	-24,367	-30,376	-48,305

Table 14. Key model variables and policy instruments for $\alpha = 1.2\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	32%	21%	7%	0%	0%	0%	1%	1%	1%	1%	2%
t_W	0,113	0,087	0,038	0,000	0,000	0,000	0,008	0,008	0,008	0,009	0,018
z	0,106	0,125	0,161	0,188	0,209	0,224	0,236	0,245	0,252	0,261	0,268
$\bar{W}$	0,354	0,416	0,538	0,628	0,695	0,747	0,786	0,816	0,839	0,869	0,894
W^N	1,904	1,904	1,881	1,858	1,833	1,809	1,783	1,757	1,745	1,730	1,684
t_L	50%	50%	50%	50%	49%	47%	45%	44%	43%	42%	40%
$y * n$	0,714	0,840	1,085	1,266	1,403	1,507	1,587	1,647	1,694	1,753	1,805
g	0,179	0,210	0,271	0,317	0,351	0,377	0,397	0,412	0,423	0,438	0,451
k	5,826	6,853	8,640	9,832	10,611	11,092	11,350	11,438	11,605	11,803	11,518
a^G	0,008	0,008	0,071	0,143	0,642	0,340	0,163	0,316	0,108	0,169	0,139
c	0,410	0,476	0,598	0,679	0,734	0,770	0,793	0,806	0,812	0,813	0,806
c_{share}	57,46%	56,64%	55,10%	53,65%	52,31%	51,08%	49,96%	48,96%	47,94%	46,39%	44,68%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,09%	1,09%	1,19%	1,29%	1,41%	1,53%	1,66%	1,80%	1,87%	1,95%	2,22%
d_{share}	-8,31%	-8,32%	-8,22%	-8,16%	-7,02%	-6,04%	-4,88%	-4,13%	-3,61%	-2,93%	-1,63%
SW	27,798	21,631	10,261	3,177	-2,381	-7,021	-11,429	-15,985	-19,468	-25,742	-44,686
a^{H_o}	31,477	32,272	32,483	32,210	32,087	32,150	31,970	31,179	31,136	30,939	29,355
a^H_y	24,396	25,421	26,209	26,417	25,831	25,609	25,055	24,213	24,071	23,758	22,140
SW_o	34,441	29,172	19,843	14,647	11,183	8,636	6,737	5,356	4,673	3,982	2,776
SW_y	-6,643	-7,541	-9,583	-11,470	-13,564	-15,657	-18,166	-21,341	-24,141	-29,723	-47,461

Table 15. Key model variables and policy instruments for $\alpha = 1.1\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	34%	23%	8%	0%	0%	0%	1%	1%	0%	1%	2%
t_W	0,129	0,103	0,046	0,000	0,000	0,000	0,008	0,009	0,000	0,009	0,019
z	0,114	0,134	0,171	0,201	0,222	0,237	0,249	0,258	0,264	0,271	0,280
$\bar{W}$	0,381	0,446	0,571	0,671	0,740	0,791	0,830	0,860	0,880	0,905	0,935
W^N	1,881	1,881	1,858	1,857	1,833	1,809	1,783	1,757	1,741	1,718	1,684
t_L	50%	50%	50%	50%	49%	47%	45%	44%	44%	42%	40%
$y * n$	0,753	0,882	1,130	1,327	1,463	1,565	1,642	1,700	1,741	1,790	1,848
g	0,188	0,220	0,282	0,332	0,366	0,391	0,411	0,425	0,435	0,447	0,462
k	5,993	7,021	8,770	10,300	11,062	11,518	11,748	11,807	11,879	11,888	11,797
a^G	0,026	0,050	0,080	0,007	0,521	0,203	0,029	0,191	0,523	0,019	0,013
c	0,436	0,503	0,628	0,712	0,765	0,799	0,821	0,832	0,837	0,837	0,826
c_{share}	57,88%	57,08%	55,59%	53,65%	52,31%	51,08%	49,96%	48,96%	48,05%	46,77%	44,69%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,19%	1,19%	1,29%	1,29%	1,41%	1,53%	1,66%	1,80%	1,89%	2,02%	2,22%
d_{share}	-8,26%	-8,22%	-8,21%	-8,32%	-7,17%	-6,17%	-4,98%	-4,22%	-4,05%	-2,99%	-1,66%
SW	24,037	18,587	8,168	2,990	-2,368	-6,829	-11,074	-15,467	-19,052	-25,522	-43,193
a^{H_o}	31,290	32,089	32,228	33,399	33,230	33,250	33,019	32,163	31,441	31,120	30,169
a_y^H	23,611	24,638	25,550	27,207	26,577	26,320	25,716	24,827	24,302	23,745	22,630
SW_o	30,536	25,958	17,504	13,889	10,523	8,055	6,218	4,883	4,229	3,285	2,345
SW_y	-6,499	-7,371	-9,336	-10,899	-12,891	-14,884	-17,291	-20,349	-23,281	-28,807	-45,538