Replication of Inter-generational Risk Sharing in Financial Market

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Abstract

Inter-generational risk sharing is often seen as one of the strengths of the Dutch pension system. The ability to absorb financial and actuarial shocks through the funding ratio allows for smoothing of returns over generations. Nevertheless, this implicitly means that generations subsidize each other, which has its disadvantages, especially in the light of incomplete contracts and situations of hard regulation constraints. In this paper, we highlight the advantages of inter-generational risk sharing, as a main characteristic of certain pension plans, and investigate if and how much of this can be replicated by individual participation in the markets. Using a stylized model based on different pension plans such as "hard" defined benefit, "soft" defined benefit, collective defined contribution and "pure" defined contribution (individual investing), this study identifies the effect of one of the most important demographic shock being the increase of life-expectancy (*i.e.* upward shock). We investigate the impact on the share of the possible replication of fund returns by individually investing in the market. Moreover, the effect of this shock is provided separately for both, fund and individual participation, meanwhile a discussion on the heterogeneity of the absorption by different plans is presented.

Keywords: pension plans, individual investment, inter-generation risk-sharing, longevity.

JEL Classification: H55, J26.

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1 Introduction

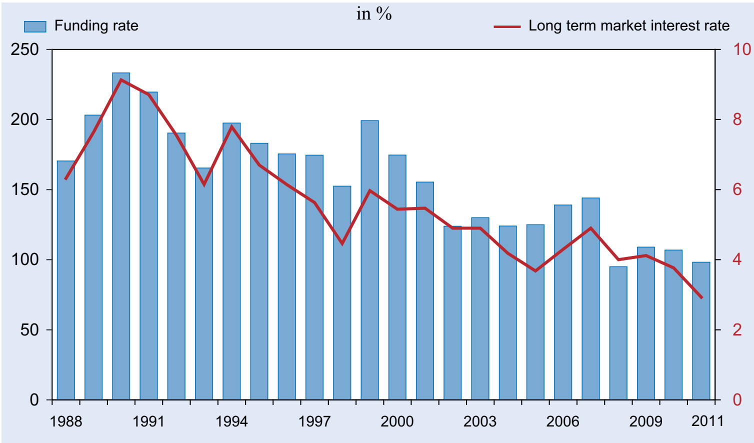
Diamond (1977), Gordon and Varian (1988), Ball and Mankiw (2007) and Gollier (2008) among others, theoretically showed that the inability of the current generations to share their risk with those who are not yet born make markets inefficient. Therefore, the absence of any inter-generational sharing of individual risks, implies that workers face high uncertainty on their future pension income. The inability of the markets to efficiently allocate risk across generations has been used to argue in favor of more public interventions such as introducing sophisticated pension schemes and an appropriate use of financial instruments. Cui et al. (2008) showed that in the collective pension contract, although the pension system participation is a *ex-ante* considered a zero-sum game, there exists welfare enhancing features related to the inter-generational risk sharing not only in the government pay-as-you-go ($PAYG$) but also in the funded plans. Nevertheless, fairness in the pension plan is the key challenge in the mandatory participation.

The original defined benefit (DB) schemes completed the market for the employees by offering life-long stable real cash flows in retirement. Netherlands is one of the countries which no longer provides a "hard" guaranteed pension benefits based on DB plans. In fact, it has become a defined contribution (DC) system that uses a DB accounting framework. This is particularly interesting since most of the other countries have opted for having the redistribution in the first pillar and clear ownership rights as well as *ex-ante* fair risk sharing in the second pillar. More precisely, the Dutch pension systemconsists of a residence based universal first pillar, quasi-mandatory funded second pillar (mandatory except for some specific industries) and the voluntary third pillar. The sustainability issues has not managed to avoided despite the continuous pension reforms the country went through. The recent years' challenge consists on the attempts of improving the matching process between assets and liabilities of the pension plans in the second pillar. The classical asset liability management (ALM) shows that the more risk you take, the higher the expected return is provided and more volatile the funding ratio is.

Academic studies point out the enlarging welfare potential of the Dutch pension funds which is attributed to the inter-generational risk sharing which allows pension funds to take more risk in asset allocation and provides smooth consumption by stabilizing the contribution rates and pension payouts. The ability to absorb financial and actuarial shocks through the funding ratio allows for smoothing of asset returns over generations.

The funding ratio of the Dutch pension fund reached its peak at the end of the 1990's followed by a sharp drop in pension funding during the "dotcom" crisis. The Dutch government imposed supplementary funding requirements in 2002 in order to reduce the risk absorption. The funding ratio slowly recovered from the low levels in 2003 but felt dramatically during the financial crisis (2008) attending the lowest level for a high number of pension funds.

Figure 1: Funding Ratio and Interest Rate Evolution in The Netherlands



Source: Statistics Nederlands; De Nederlandsche Bank; Provided by: Broeders and Ponds (2012), page 66.

As a consequence, the level of trust in these *CDC* pension plans has decreased and the social support for the inter-generational risk sharing is not as strong as it used to be (see figure 1). Current regulations allow that pension fund can cut the benefits and pension-in-payment to restore its solvency level, in the case of under-funding. It is worth to note that the participation in a specific pension fund is still mandatory for the employee. Currently, there is a debate in the Netherlands on a new pension deal which is even more *DC* like. The pension age will be linked to the systemic longevity and there will be roof on the contribution level. Associated to an increase in the strict constraints by the regulatory entity, this research study should be viewed with in the perspective of the proposed changes to the Dutch pension system. It consists in measuring the effect of the hard constraints implemented by the regulator and determining the impact of continuous life-expectancy increase in the current fragile pension sustainability.

Focusing on studying the employer-based supplementary schemes (Pillar II), one can ask himself what would happen to the support of inter-generational risk sharing model when some of the actuarial variables do not follow a random walk, but have a trend? There have been several demographic changes over the last 80 years. In 1932, the average life-expectancy in the Netherlands was 64 years, while today it is 19 years on average after 65. Fertility has decreased and not only do the average women give birth to fewer children but they give birth to their first child later in life. More young people today focus on getting a higher education which leads to a reduced number of years in working life. Furthermore, there is a long-term trend to earlier retirement in many countries while evidence shows that this does not induce a parallel decline in unemployment rates. Given these bio-metrical and societal developments, one can postulate that what we are facing is not just random walk shocks but there exist societal and demographic trends. Therefore, one may think on how will all these developments affect the fairness of current designed pension contracts with respect to inter-generational risk sharing (*resp.* transfers)?

Given that the "hard" promise is no longer part of the Dutch second pillar while the latter still remains a privately mandatory managed pillar, it is important to investigate and measure its uniqueness in providing inter-generational risk sharing. Hence, one could ask: How much of the remaining inter-generational risk sharing in the CDC can be solved by the markets? In other words, we are interested to study what exactly happens in the CDC pension schemes in terms of remaining inter-generational risk sharing. How much is this risk sharing unique and how much can it be replicated by the markets? How can one make the pension deal fair for the young generations and still retain some inter-generational risk sharing? If nowadays, the inter-generational risk sharing is no more considered as the strength of the Dutch pension system, could it be considered as a necessary condition for the youth participation or there are still incentives to do so?

The current CDC pension plans could be described as a "black box" in which redistribution takes place, but it is not really clear what happens in it. Which are the risks that are actually shared and to what extent can they be replicated by the markets? Therefore, providing answers to these questions would lead us to better understand the real value of the proposed pension contracts. In this paper, we investigate the common arguments in the economic literature and in the Dutch debate regarding the inter-generational risk sharing by analyzing a stylized pension contract. Moreover, we aim to determine the amplitude of the policy security constraints. Diverse stylized pension contracts are constructed based on different pension plan such as the "hard" DB plan, the conditional "soft" DB plan, the CDC plan and the individual plan ("pure" DC). Despite the differences in the methodology and the research question this study is closely related to Siegmann (2011).

The paper is organized as follows: next section describes the data. Section 3 presents the methodology used to measure the effect of shocks on inter-generation risk sharing. The main results during the "normal" (baseline) and "shocked" periods are discussed in section 4.

2 Data Description

This is an empirical study based on a simulated stylized contract. Therefore, we consider a pension fund modeled for a number of future years using pension contract specification deal, statistical data and generated scenarios. We try to minimize the number of assumptions and be as much close to the reality as possible. Thus, some of the information used is based on real data and the rest are simulated variables which serve as *proxy* for the corresponding real ones. The population real data and the financial market simulated variables are described in details in the following sections 2.1 and 2.2 respectively. The pension plan characteristics are presented in section 3.1.

2.1 Population characteristics

We focus our study in an open fund approach, where the fund is an infinitely lived instrument with repeated loops of 70 years, corresponding to each cohort. Individuals' lifetime consists on starting to participate to the pension fund at age 25, contributing for 40 years, starting to get benefits at age of 65 and losing the retirement benefits at maximum at age of 95 years. Therefore, since an individual can participate in the fund at most 70 years, at any period in time there are 70 co-existing generations. The population distribution is based on the Dutch population and mortality *per* cohort structure of year 2012. Data are provided by the *CBS* (Centraal Bureau voor de Statistiek). Its structure *per* cohort is a hump shape function (see figure 3) and there exists uncertainty in the agents' life duration. The surviving probability is presented in figure 2. The population is updated each year. The existing cohorts are multiplied to the corresponding surviving probabilities one year later and the new entries are updated to the actual birth rate growth considered constant every 25 years (lag=25).

The population evolution is a key issue for the pension fund since it highly affects the balance asset-liability management. First, we construct one year mortality rate ($q_{x,t}$) and deduce the surviving probabilities ($p\left(x,i\right)$) of each cohort using the Dutch population of 2011 and 2012 as follows:

$$q_{x,t+1} = \frac{Pop_{t+1}^{x+1} - Pop_t^x}{Pop_t^x}$$

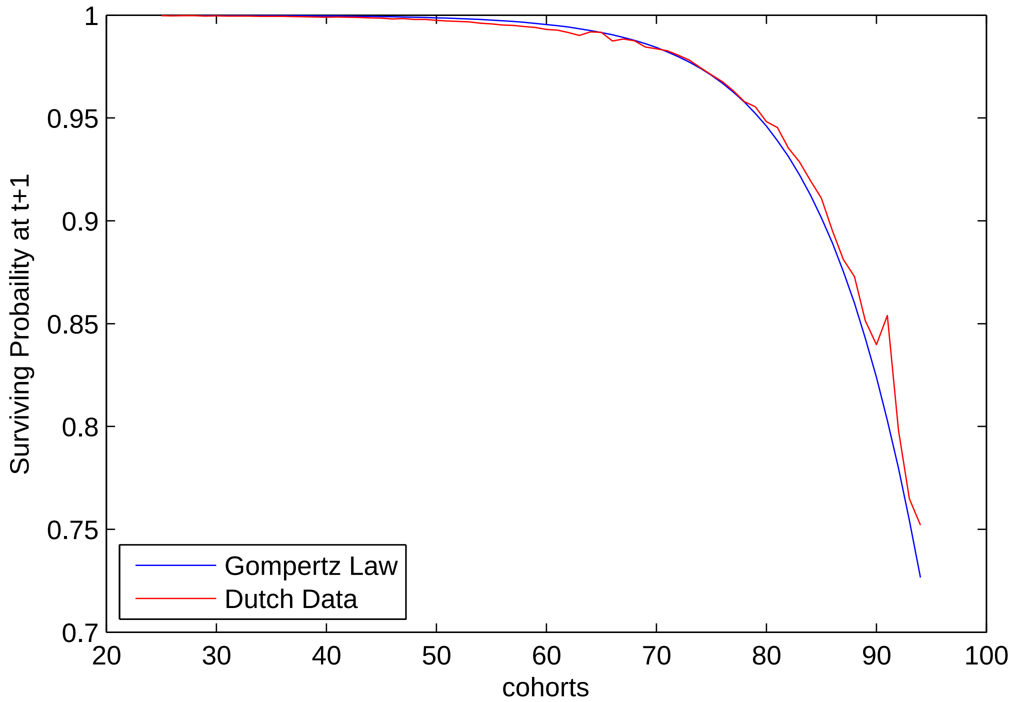
$$p\left(x,i\right) = \prod_{j=0}^{j=i-1} \left(1 - q_{x+j}\right)$$

Secondly, we use the Gompertz (1825) model to *proxy* the population structure. This allows us to have a dynamic population model in which one could reproduce projections of the population considering shocks on different predefined parameters. Gompertz Law[1] states that over a large part of the age range (excluding infancy and youth or very old age) the force of mortality increases with age at a steady exponential rate. Therefore, assuming that the mortality rates increase not only with age but also in time by the same amount every year, the Gompertz Law is written as follows:

$$ln\left(p\left(x,t\right)\right) = \left(1 - e^{t/b}\right) \times e^{(x-m)/b}$$

where, t denotes the survival period, x the current age of the individual, m the modal age at death and b the depression coefficient of the age at death. Parameters ($b = 8, m = 87$) are calibrated based on the initial mortality table.

Figure 2: Conditional Surviving Probability (lag 1)



Source: The conditional surviving probabilities best fit of the real Dutch data (red line) and the Gompertz Law (blue line); estimated parameters: $m = 87$ years, $b = 8$ years; Calculations by the author.

In a lag of one year time, there is no difference in between the real surviving population and the one used as *proxy*. Therefore, we can use the Gompertz law to generate the population survival probability as a function of life-expectancy level in a more dynamic

[1]According to Gompertz the best way to illustrate the law is the following physiological explanation: that a man's power to avoid death is gradually exhausted as his age increases, "congruous with many natural effects, as for instance, the exhaustion of the receiver of an air pump by strokes repeated at equal intervals of time". The actual modern theory tries to explain this law by linking the probability of death to body deterioration over the age ranges.

way in modeling population changes.

We assume that the table of conditional survival probabilities is deterministic and constant in time when the population is assumed to be in "normal" period. The survival probability matrix is constructed as follows:

$$p^x\left(t+i|t\right) = p^x\left(t+i-1|t\right) \times p^x\left(t+i|t+i-1\right)$$

where the $p^x\left(t+1|t\right)$ represents the probability that the representative agent of cohort x would survive at time $t+1$ knowing that he was alive as individual of cohort $x-1$ at time t.

The actual population in the Netherlands is quite favorable for this study since the dependency[2] ratio (DR) is 29.74%. Because of a higher flux of working force than retired people, the support[3] ratio is higher than one. As a *proxy* for the birth rate growth, we use the 25 year historic of population growth already provided by the population historical data of 2012 for the cohorts zero to 25 years old[4]. To avoid the assumptions related to projections, we repeat this 25 year birth rates to provide a historic of 150 years. The birth rate growth is on average 0.6% with a standard deviation of 1.76%.

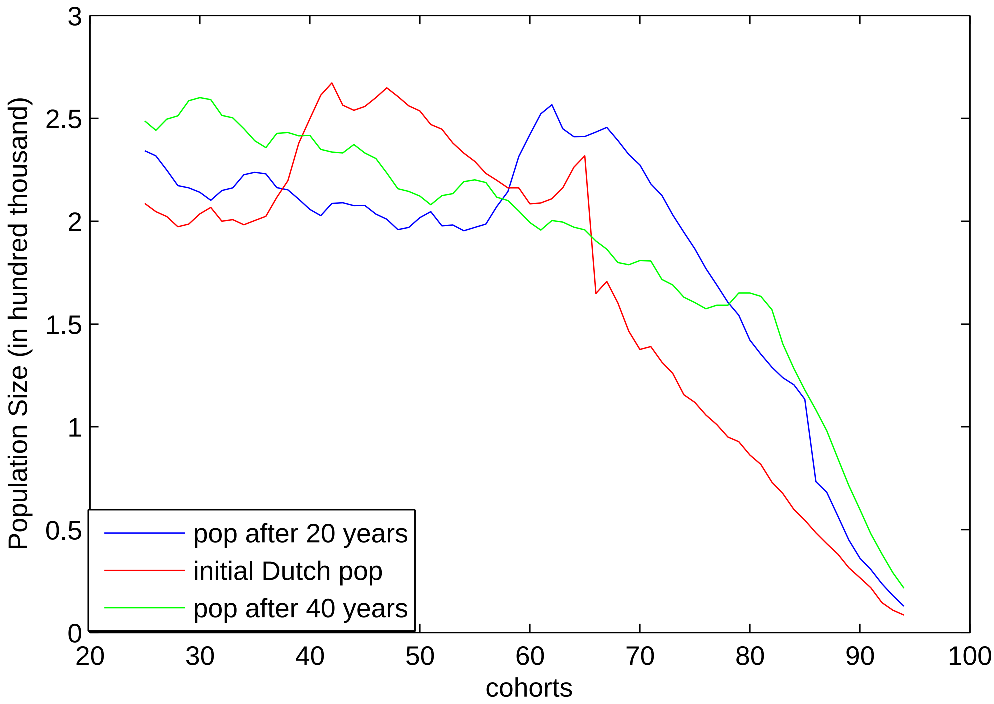
[2]The dependency ratio is considered as:

$$DR = \frac{number\ of\ Old\ people}{number\ of\ Middle\ aged\ people}$$

[3]The support ratio presents the inverse of the dependency ratio.

[4]We assume that mortality for these cohorts during the next 25 years is negligible.

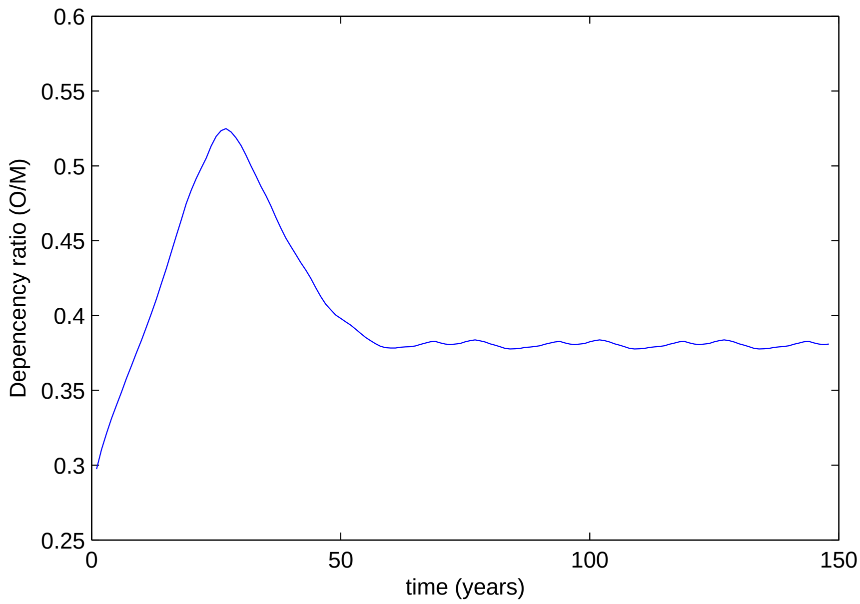
Figure 3: Population Structure *per* Cohort in Time



Source: The number of individuals corresponding to each cohort at the initial time t=1 (in blue), t=20 (in red) and t=40 (in green); Calculations by the author.

Furthermore, the dependency ratio clearly serves as an aging indicator of the population (see figure 4). During the first 26 years it occurs a positive increasing trend ("population aging") followed by a "younging" population during the next 23 years (meaning from year 26 to 48). Nevertheless, even why the population structure reaches stability period with a dependency ratio of around 38%, it still remains older than the starting point of the actual Dutch population in 2012.

Figure 4: Dependency Ratio Dynamics in Time



Source: The dependency ratio in time as the ratio between the old (older than 65 years old) to the young (aged between 25 and 65 years old) total population; Period: "normal"; Calculations by the author.

The descriptive statics of the population dynamics is mentioned on table I. The standard deviation of the birth rate growth for the first 20 years is 1.74% and the standard deviation for the first 40 years is 1.79%. The population structure changes in time nevertheless, the conditional surviving probability stays deterministic. Population structure will be a source of shocking variables in the next sections.

Table I: Descriptive Statistics of Population Evolution

	Dependency ratio		**Mean birth rate growth**	
	After 20 years	After 40 years	During 20 years	During 40 years
Positive stoch. birth rate	49.16%	44.08%	0.58%	0.44%
Stable population	29.74%	29.74%	0%	0%

Source: The dependency ratio and the mean birth rate during 20 and 40 first years; Calculations by the author.

2.2 Financial market characteristics

The asset-liability management (ALM) model is based on Monte Carlo simulations of 1000 possible future economic scenarios for a period of 150 years. Despite the fact that in real life the duration of a specific unchanged pension contracts is shorter than 150 years (pension system is often reformed), in this study we need to use this long historic data because of lost of information caused while constructing different forward looking indicators.

The simulation is based on fixed inflation dynamics. Thus, there is no uncertainty related to price inflation (fixed at 2% *per* year) and wage inflation (fixed at 3% *per* year). The term structure is defined by Vasicek (1977) one-factor[5] model. It is known as one of the earliest non arbitrage models of interest rate based on mean reverting[6] mechanism and its stochastic differential equation is given as follows[7]:

$$dr_t = \kappa_r \left(\mu_r - r_t\right) dt + \sigma_r dW_t;$$

where W_t is a standard Wiener process under the risk neutral framework, σ_r is the standard deviation parameter characterizing the amplitude of the instantaneous random inflow. The speed of adjustment of the interest rate (reversion) towards its long-run normal level is $\kappa_r = 0.05$, the long-term mean is $\mu_r = 0.03$ and the instantaneous volatility is $\sigma_r = 0.05$[8].

However, the term structure does not change its shape in time but becomes flatter. Above 30 years of maturity, the yield curve is considered identical to the corresponding value in 30 years maturity time. Longer the time to maturity, higher the interest rate.

The short-term risk-free instrument is considered as the return on the bank account. It consists on a stochastic risk-free return in time where $R_0^f = 1$ and $\mu(r_f) = 2.81\%$. The expected risk-free return is increasing in time during the first 60 years and is flat around 3.1% after (see figure 5).

[5]The interest rate is derived by only one source of risk being the market risk.

[6]The mean reversion mechanism is explained as follows: when $r_t > \mu_r$, the expected variation of r_t becomes negative and r_t tends to go back to the long-run expected return μ_r all at an adjustment speed of κ_r. The inverse happens when $r_t < \mu_r$. Furthermore, the risk premium in Vasicek model is kept constant.

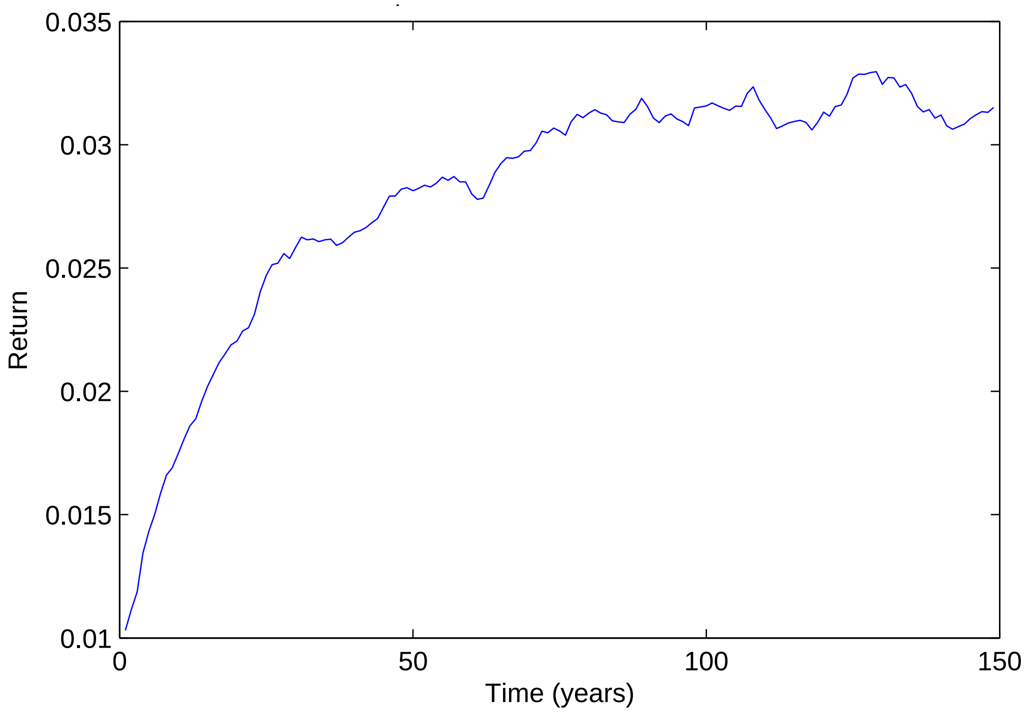
[7]The risk neutral process of short-term interest rate can be written on a general form by using Îto process as follows:

$$dr = m\left(r\right) dt + s\left(r\right) dz$$

where in the case of Vasicek Model, the drift is $m(r) = \kappa\left(\mu - r\right)$ and the instantaneous standard deviation is $s(r) = \sigma$.

[8]Characteristics of the Vasicek 1-factor model are presented in appendix A

Figure 5: Expected Value of Risk-free Return



Source: It is based on the bank account return and corresponds to the short-term risk and the risk-free return; Calculations by the author.

The financial market is composed by two financial equities, a bond and a stock. To keep the model simple, the financial information is exogenous and is not contagious on other parameters such as the demographic structure or the learning process. Bond returns are deduced by the term structure model. We assume that the fund buys the bond of maturity 6 years at the beginning of time t paying its price at maturity 6 years at t, sells it at the end of time t under the price of a bond at maturity 5 years at t, and re-buys bonds at maturity 6 years at the beginning of time $t+1$ at a price of a bond at maturity 6 years at $t+1$.

Therefore, the bond return (r_b) is calculated as follows:

$$r_b = \frac{\left(1 + r_t^{6Y}\right)^6}{\left(1 + r_{t+1}^{5Y}\right)^5} - 1$$

As it concerns the stock simulations, Black and Scholes (1973) model is used to generate equity return scenarios with stochastic short rate.The volatility $\sigma_s = 0.2$, the risk-premium $\lambda = \mu_s - r_f = 0.04$ and no correlation $\rho(r, s) = 0$ are the parameters used for the simulation. Two investing strategies are implemented. On the one hand, relying on the Dutch pension fund characteristics, we consider its investment strategy being a static "constant-mix" (50% on bonds and 50% on stocks) unconditional to the actual fund performance. On the other hand, the individual investment strategy is based on "age-dependent" investment. Thus, based on the simulated scenarios, one could resume the risky market with the following bond, stock and "constant-mix" characteristics (see table II):

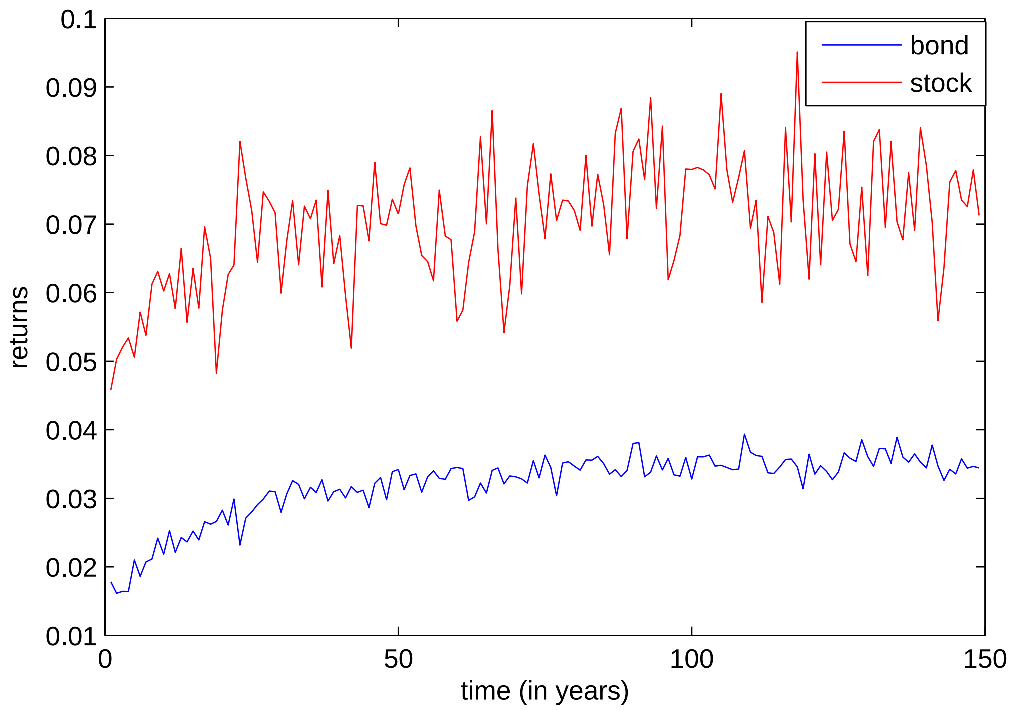
Table II: The Descriptive Statistics of the Financial Market

	Mean	**Volatility**
Bond	3.21%	5.10%
Stock	7.01%	21.72%
constant-mix (50%-50%)	5.11%	11.38%

Source: The statistics are based on risk-return analysis for the stock, the bond and the "constant-mix" portfolio; Calculations by the author.

The bond and stock expected return dynamic evolution in time are represented in figure 6:

Figure 6: Expected Bond and Stock Returns



Source: The expected return for the bond (in blue) and stock (in red) are calculated as an average among all scenarios at given moments in time; Calculations by the author.

3 Methodology

In this section, we will focus on the concrete structure of each pension contract including both collective participation and individual investment. Each representative agent is considered to obtain wage flow w during his 40 years of working life. We normalize the initial remuneration level being $w_{x,t=1} = 1$ the same for all generations $x \in [25 : 64]$. It evolves in time homogeneously for each existing cohort $w_{x,t} = w_{x-1,t-1} \cdot g_t$, for $x \in [25 : 64]$ where g_t is the wage growth between year $t - 1$ and t. In this model, the rate of wage growth is considered constant in time and could be decomposed on the inflation and the real wage growth component. Moreover, because of different reasons such as differences in experience, depreciation of knowledge or age-related trends in physical and mental capabilities a representative worker's productivity could systematically differ over his/her active lifetime period. In this paper we consider homogeneity of wages.

The population data corresponds to the active population since unemployment is supposed to be neglected. The contribution level that active population should pay depends on the type of the pension contract they signed in. Contributions are supposed to be uniform across generations but variable in time and in the state of the nature the system belongs to.

Considering The Netherlands reform of 2003, plans' benefit distribution rule has been transformed from final salary to average salary. Moreover, each contributing year is translated to an accrual rate of $\epsilon = 2\%$ of average wage. Based on the 40 years of total contribution, the representative agent accrues his/her pension rights corresponding to 80% of the average wage indexed to inflation. The agents make it possible to get their accrued benefit paid during 40 past years translated to the real value the year of their retirement. In addition, the pension benefit is considered being contagious to the inflation indexation during the retirement years. Regarding the pension rights, they are considered certainly indexed to inflation while the pension benefit indexation can be full or partial conditional to the pension's performance (funding ratio).

As far as the pension plan is modeled being a Projected Benefit Obligation (PBO[9]), the liabilities are calculated as the present value of claims about the accrued benefit. Therefore, initially assets are calculated as the product of the initial funding ratio with the initial liabilities. As a starting point, we consider the fund having a balance between assets and liabilities ($FR_0 = 1$). The actual fund participants are assumed having been working and fully-contributing for the last 40 years.

[9]The amount of money a company must pay into a pension plan to satisfy all pension entitlements that have been earned by employees up to that date.

As long as, the accrued benefits *per* cohort are the same, the discount elements for each cohort can be summed up to determine the discount element for a given cohort.

$$D^x_{t,s} = \sum_{i=\max(65-x,0)}^{95-x} \frac{p^x(i|t)}{\left(R^{(i)}_{t,s}\right)^i}$$

where $R^{(i)}_{t,s}$ is the yield to maturity i at time t in scenario s, and $p_x(i|t)$ is the surviving probability of cohort x, i years later conditionally to the fact that he/she is alive at time t. The discounting coefficient is a hump shape function of age. The present value of accrued benefit claims is higher for the middle-aged cohorts because they just started collecting benefits or will start doing so. The youngest cohorts are those who did not contribute much in the pension plan yet and they expect to receive the payments quite late in time. The oldest cohorts are the ones who do not have more benefits left to receive and the survival probabilities are quite low, that is why the present value of accrued benefit is low. In line with the existing literature, we calculate liability as target values, as in Cui et al. (2008). In general, the impossibility of forward looking for several variables such as inflation and the fact that pension benefit is indexed on inflation, makes it impossible to deduce the exact future accrued benefits. Therefore, the target liability is the product of the accrued benefit at each age with the corresponding discounted coefficient summed up among all co-existing cohorts.

$$L_{t,s} = \sum_{x=25}^{94} \epsilon \times \min(x-24; 40) \times \bar{w}_{x,t} \times (1+\pi_{t,s})^{\tau}$$

where $\bar{w}_{x,t}$ is the average working age, $\pi_{t,s}$ the inflation (calibrated 2% in this model) and τ the time at which the cohort x was 64 years old.

The assets are calculated as the remuneration of a "constant-mix" investment strategy of: the sum of last year's asset stock plus the actual contribution $C^x_{t,s}$ of all generations younger than 65 minus the actual benefits $B^x_{t,s}$ paid to the retirees till their maximal age 94 years old.

$$A_{t,s} = \left(A_{t-1,s} + \sum_{x=25}^{64} C^x_{t,s} \cdot Pop^x_t - \sum_{x=65}^{94} B^x_{t,s} \cdot Pop^x_t\right) \cdot R^{inv}_t$$

where R^{inv}_t is the gross return on the fund investment strategy at time t.

In consequence to the predefined funding ratio $FR_0 = 1$, assets are initially matched up with the targeted liabilities. At the end of each year, despite the determination of the funding ratio FR_t, variables such as population, surviving probabilities, wage and price inflation level, are updated.

In the following sections 3.1 and 3.2 we will first, describe the specific characteristics of each collective pension plan and secondly, present details of the individual investment.

3.1 Collective Pension Contracts' Characteristics

We will focus on three pension contracts. They differ on the rules of both, the collection of contributions and the distribution of benefits. We can characterize them into two main groups, the pension contracts based on variable contributions related to the fund performance and those consisting on constant ones. The funding ratio level determines the fund performance.

Moreover, in a way to prevent the extreme events of the low or high quantile, we introduce two distinct frameworks by implementing extra constraints that will be considered for each plan. These are considered as the constraints the fund should follow as a result of the predefined regulatory "*Policy Safety Constraints (PSC)*".

- If the $FR < FR_{forbidden}$, the fund is constrained such as, no pension benefit is paid out directly by the plan to its individuals. However, during this year, the pension paid to the individuals is only 80% of the final wage and is issued not by the fund itself but by the insurance company covering the pension fund.
- If the $FR > FFR_{max}$, the fund is constrained such as, no contributions (or to a c_{bound} level) are collected by the working individuals during that year and the pension fund redistributes the excess of funding ratio to the $FR_{surplus}$. The smoothing factor is considered $\gamma = 10\%$ which stays in line with the supervisory recommendation.

$$redistrib_{t,s} = \begin{cases} 1 & if\ FR_{forbidden} < FR_{t-1,s} < FR_{max} \\ 1 + (FR_{t-1,s} - FR_{surplus}) \times \gamma & if\ FR_{t-1,s} \geq FR_{max}. \end{cases}$$

To summarize some of the pension parameters' calibration see table III.

Table III: Fund Parameters

	Forbidden	Min	Floor	Cap	Surplus	Max
Contribution (c_i)	0%	10%	-	-	25%	-
Funding Ratio (FR_i)	0.6	-	1.00	1.30	1.60	1.65

Source: Indicators used for the calibration of the pension system. They are in line with the ones used in Lekniute (2011).

3.1.1 Variable Contribution

The variable contribution on a collective pension contract coincides with the Defined Benefit (DB) pension plans. In this study, we will consider the "pure" defined benefit contract and the conditional indexation one.

Plan DB-"hard" : This contract consists on DB scheme with full pension promise where there is full indexation no-matter the funding performance. It is a traditional average wage contract whose use is more and more limited these days because of its un-sustainability caused by the demographic shocks among others. The contract consists on a full un-conditional pension indexation and adjustable contribution level (see figure 7).

The contribution level is constant for all the active population and at the beginning of each period it is fixed based on the following decision rule:

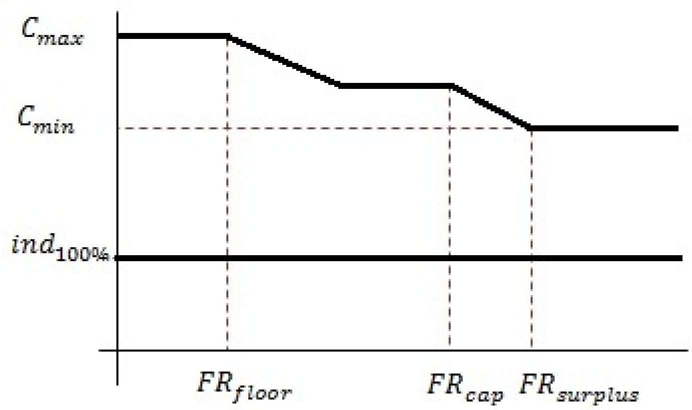
$$c_{t,s} = \begin{cases} c_{max} & if\ FR_{t-1,s} < FR_{floor} \\ c_{max} - \frac{FR_{t-1,s} - FR_{floor}}{FR_{cap} - FR_{floor}} \left(c_{max} - c_{min}\right) & if\ FR_{floor} \leq FR_{t-1,s} \leq \frac{FR_{floor} + FR_{cap}}{2} \\ \frac{c_{min} + c_{max}}{2} & if\ \frac{FR_{floor} + FR_{cap}}{2} < FR_{t-1,s} < FR_{cap} \\ \frac{c_{min} + c_{max}}{2} - \frac{FR_{t-1,s} - FR_{cap}}{FR_{surplus} - FR_{cap}} \left(\frac{c_{max} - c_{min}}{2}\right) & if\ FR_{cap} \leq FR_{t-1,s} \leq FR_{surplus} \\ c_{min} & if\ FR_{t-1,s} > FR_{surplus} \end{cases}$$

The pension benefit indexation is:

$$index_{t,s} = 100\%,\ \forall t \in [1:T]\ \ \forall s \in [1:S]$$

Although the indexation proportion is homogenous among co-existing generations, there is heterogeneity in time for the price inflation and lifetime average wage.

Figure 7: DB-"hard" Pension Contract Characteristics



Source: DB plan, full unconditional indexation and adjusted contribution level on the pension fund's funding ratio.

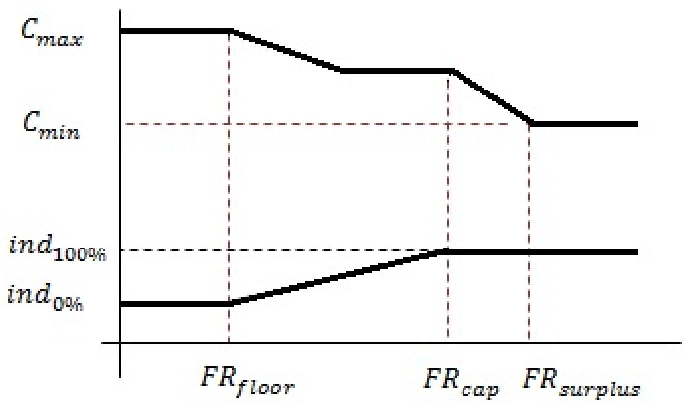
Plan 1: DB**-"soft"** : The attempts to reduce the divergence of the fund performance descending from the DB-"hard" contracts proposed, incited many countries to reform their pension systems. The proposed contract consists on a defined benefit where the pension rights are kept promised but the indexation is contagious to the fund performance. The literature name this a DB-"soft" pension contract. The generosity of the system is still reflected on the funding ratio performance in time.

On the one hand, as it concerns the contributions, the DB-"soft" characteristics (see figure 8) remain the same with the DB-"hard" ones. On the other hand, the benefits are considered partially indexed (ladder indexation) based on the fund performance. The analytical expression of benefit indexation in the conditional price inflation pension plan is determined as follows:

$$index_{t,s} = \begin{cases} 0 & if\ FR_{t-1,s} < FR_{floor} \\ \frac{FR_{t-1,s} - FR_{floor}}{FR_{cap} - FR_{floor}} & if\ FR_{floor} \leq FR_{t-1,s} \leq FR_{cap} \\ 1 & if\ FR_{t-1,s} > FR_{cap} \end{cases}$$

This ladder policy was introduced in the Netherlands in 2005. The policy rule induces that the pension plan can no longer expect to fully index benefits to the price inflation growth. There will be no indexation if the fund's funding ratio is below the FR_{floor} ratio, while there will be full benefit indexation if the fund is doing well and is having a funding ratio above a certain level (FR_{cap}). It is important to emphasize that this pension plan does not consider neither the possibility of cutting pensions, since there is no negative indexation, nor the possibility of fund surplus redistribution.

Figure 8: DB-"soft" Pension Contract Characteristics



Source: DB plan, both adjusted conditional indexation and contribution level on the pension fund's funding ratio.

3.1.2 Fixed contribution

We consider the collective defined contribution (CDC) contract as a plan where contributions are predefined and kept fixed in time. There exist several derivative plans related to the fixed contribution characteristics. In this study we will consider only one type of "hybrid" CDC plan.

Plan CDC : The collective defined contribution contract is a "hybrid" contract standing in between the DB and the DC one. The CDC inherits from the DB pension plan the pension benefit distribution. Despite the fact that CDC is a defined contribution plan, one part of the pension benefit is promised but remains contagious to the fund performance. Moreover, the benefit indexation does not contain pension cuts or surplus re-distributions and is represented by the same ladder equation as in DB-"soft" pension plan. The contribution level is such that the present value of all contributions equalizes the present value of accrued benefits.

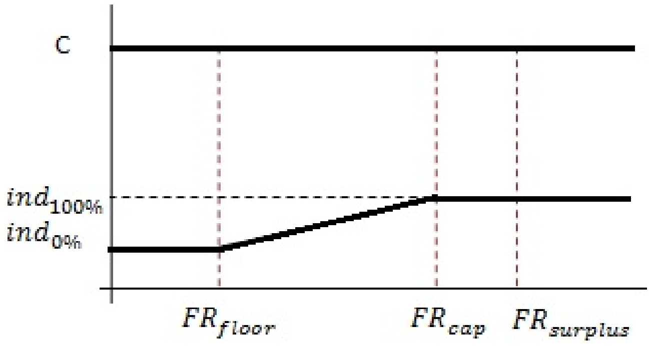
$$c_{t=1,s} \times \sum_{x=25}^{64} w_{t=1}^{x} \times Pop_{t=1}^{x} = \epsilon \times \sum_{x=25}^{64} D_{t=1,s}^{x}$$

Thus, the contribution level c is considered being fixed in time:

$$c_{t,s} = \epsilon \times \sum_{x=25}^{64} \frac{\left(w_{t=1}^{x} \times Pop_{t=1}^{x} \times \sum_{i=65-x}^{95-x} \frac{p^{x}(i|t)}{\left(R_{t,s}^{(i)}\right)^{i}} \right)}{\sum_{x=25}^{64} w_{t=1}^{x} \times Pop_{t=1}^{x}}$$

To associate one fixed contribution level to this fixed plan, we calculate the expected contribution level at time $t = 1$. The contribution and the benefit characteristics are given in figure 9.

Figure 9: CDC Pension Contract Characteristics



Source: "hybrid" CDC plan, adjusted conditional indexation on the pension fund's funding ratio and fixed contribution level.

Each of the three contracts described above, is considered first in a non-constrained framework (no PSC) and secondly under policy security constraints (yes PSC). The introduction of the policy constraints allows for cuts and surplus redistribution (extra element of the existing pension contract characteristics). Finally, results are concluded for the heterogeneity among cohorts. A summary of the plans characteristics is represented in table IV.

Table IV: Summary of Pension Plans' Details

	Contribution	**Benefit 80%**	**Indexation**	**Cuts**	**Surplus**	**PSC cuts**	**MWP**
Plan DB-hard	Variable	Promise	Full	No	No	No & Yes	No & Yes
Plan DB-soft	Variable	Promise	Ladder	No	No	No & Yes	No & Yes
Plan CDC	Fixed	Promise	Ladder	No	No	No & Yes	No & Yes
Plan pure DC	Fix/Var	Unsure	No	No	No	No	No & Yes

Source: Information on the type of contribution, benefit, indexation, cuts (related to pension plan), surplus (related to pension plan), PSC cuts (related to regulatory rules) and MWP. In this paper, we present the results of homogenous wage profile. Results introducing the MWP are available upon request.

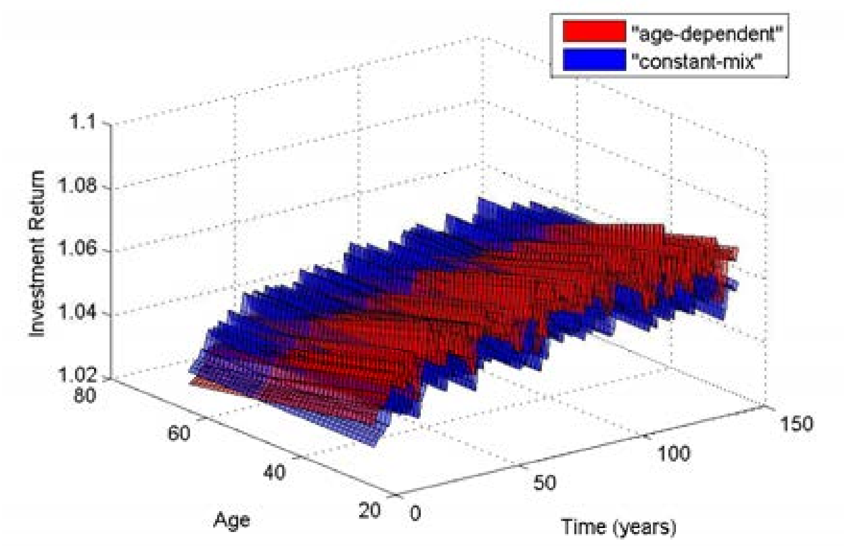
3.2 Individual Investment Characteristics

In our point of view, to make it possible comparing fund participation *versus* individual investment, we consider that the individuals will save the same share of their wage as they would have contributed by participating in the fund. In a first stage, the investment strategy is kept "constant-mix" as the fund does (static constant investment proportions 50% in bonds and 50% in stocks). In a second approach, we consider as individuals follow a life-cycle portfolio investment for their personal accounts, notably an age-dependent investment strategy. There are different ways to reproduce the age-dependent individual investment such as the "100-age" rule, the Malkiel (1996) approach, the Shiller (2005) plan, among others.

Besides the literature accepting the positive correlation between age and bond investment, stands the so called stock market participation puzzle. Young investors typically hold very little stock, progressively increase their risky assets holdings as they age, and decrease their exposure to stock market risk when they approach retirement (see Ameriks and Zeldes (2004) and Campbell (2006)). Unless these empirical studies which contest the theory of more risk-taking at young age compared to older age, arguing that investment position in the market is related to human capital level and the long-run labor risk exposure, in this study we consider the standard rule of thumb 100-age rule. Therefore, contrary to the linear increasing function of the share invested in bonds (25% at 25 years old to 36% at 64 years), the investment part in stocks is a decreasing function with respect to age (75% at 25 years old to 36% at 64 years).

The difference in time between the less-risky constant investment that the fund operates and the age-dependent investment that the individual occurs is given in the following graph 10:

Figure 10: Expected Investment Return "Constant-mix" *versus* "Age-dependent"



Source: Individual investment is considered age-dependent; Fund investment is considered "constant-mix"; Calculations by the author.

The individual investment is considered being a "pure" *DC* plan. The portfolio of each cohort or representative individual evolves in time. At age 64 there is a bucket corresponding to the total retirement income for the individual.

$$Ptf^{x}_{t,s} = C^{x}_{t,s} \times R^{inv}_{t,s}, \quad for \;\; t = 1$$

$$Ptf^{x+1}_{t+1,s} = \left(Ptf^{x}_{t,s} + C^{x+1}_{t+1,s}\right) \times R^{inv}_{t,s}, \quad for \;\; t > 1$$

To redistribute it during the retirement, the 64 year old representative individual buys a level annuity[10], whose expected present value ($EPV^{64}_{t,s}$) is the exactly the total amount of the individual's bucket at age 64.

$$EPV^{64}_{t,s} = \sum_{time=t+1}^{t+R} \frac{Annuity^{65}_{t,s} \cdot p^{64}\left(time - t|t\right)}{\prod_{k=t}^{time}\left(1 + r^{(k-t)}_{t,s}\right)}$$

where $Annuity^{65}_{t,s}$ is the nominal annuity payment corresponding to time t and scenario s, $r^{(k-t)}_{t,s}$ is the nominal spot yield corresponding to scenario s at time t maturity $(k-t)$, T is the max length of pension based on the assumption that no one lives beyond 94 years and $p^{x}(i|t)$ the probability that the annuitant age x at year t will survive at year i.

To put it in a nutshell, the annuity for an individual at age 65 in time t and a given scenario s is calculated as follows :

$$Annuity^{65}_{t,s} = \sum_{time=t+1}^{t+R} \frac{EPV^{64}_{t,s}}{\frac{p^{64}(time-t|t)}{\prod_{k=t}^{time}\left(1+r_{k,s}\right)}} = \sum_{time=t+1}^{t+R} \frac{Ptf^{64}_{t,s}}{\frac{p^{64}(time-t|t)}{\prod_{k=t}^{time}\left(1+r_{k,s}\right)}}$$

In this study, we present the results considering that agents when participating to "pure" defined contribution plan (which corresponds to individual investment) use the "age-dependent" investment strategy.

3.3 Value based pension deals

It is important for the fund manager to measure the value of each existing cohort especially *ex-ante*. Let us consider V the value of contingent claim and $\mathbb{E}^{\mathbb{Q}}_{t}$ the risk neutral expectation under the $\mathbb{Q}$-measure which is calculated as the expectations of outcomes of all future cash flows under risk-neutral scenarios discounting them under risk-free rate (see Cochrane (2001)). The value of the generational account varies *per* cohort. Depending on the individuals' age, there are two profiles called the contributors (agents between 25 and 64 years) and the retirees (agents older than 65). The calculation formula of the

[10]The level annuity is calculated for individuals at age 65 and is kept constant in time for the following years.

value based general account for each subset of generations at time t is given as follows:

$$V_{t,s}^{x}=\begin{cases}\mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=x}^{95}\left(p^x\left(t+(i-x)\,|t\right)B_{t+(i-x),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)\right) & for\ \ x\geq 65\\ \mathbb{E}_t^{\mathbb{Q}}(\sum_{i=x}^{64}\left(-p^x\left(t+(i-x)\,|t\right)C_{t+(i-x),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)+... & \\ ...+\sum_{i=65}^{95}\left(p^x\left(t+(i-x)\,|t\right)B_{(t+(i-x)|t),s}^{i}\prod_{j=t}^{t+(i-x)}\left(R_j^f\right)^{-1}\right)) & for\ \ 25\leq x<65\end{cases}$$

where $R_j^f=1+r_j^f$ and represents the return on investment in short-term risk-free bank account. The scenarios are generated taking in consideration a one year bank account returns, initially normalized at one. The expectations are calculated by taking the mean overall scenarios. $V_{t,s}^{x}$ is the value of the pension deal of an individual representative of cohort x at time t. Since there is no heterogeneity among agents of the same cohort, we consider the value based generation account only for one representative individual.

We analyze the changes between participation in collective pension scheme *versus* individually investing in the market while using the same financial instruments. Hence, we are interested in evaluating the change in the value of each retirement saving option (respectively $V_{t,s}^{x,collec.}$ and $V_{t,s}^{x,indiv.}$). Hence, we measure the difference *per* cohort and study its evolution in time with respect to different exogenous shocks:

$$\Delta V_{t,s}^{x}=V_{t,s}^{x,collec.}-V_{t,s}^{x,indiv.}$$

Higher is the value-based generation account for a given cohort, more expensive this cohort is for the pension fund. Higher this value for the individual investment case, more profitable this cohort is from the swap between their final portfolio at age 65 and the proposed annuity at that time. The value based account method gives important information for the fund itself but not enough in individual's point of view.

3.4 Utility as a measure of a pension contract

It is important to evaluate the contracts not only by using the value-based but also in terms of the utility it provides to each cohort member. Thus, individuals can evaluate not only in terms of expected net benefit (gain/loss) but also measure in terms of utility level. The first reason is that pension system in general is not just to offer individuals benefits during their retirement but it also aims to realize the consumption smoothing of the representative individuals. The second reason why we use a utility comparison is because utility is a measure of agent's well-being. In this study, we aim to conclude how much of the utility reached by fund participation could be replicated by individual investment in the market. Hence, what is remained unable to be replicated in the market by individual investment is a property of the inter-generation risk sharing. Therefore, the utility is the measure that can capture not only the actual value of the pension contract but also its price and the ability to smooth consumption when switching to retirement. Finally, because of the absence of initial values for different cohorts, information for individuals are missing. Hence we focus our analysis on the 25 year old representative agent.

The constant relative risk aversion ($CRRA$) utility function:

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma}$$

γ is the concavity degree of $u(\cdot)$, inter-temporal smoothing (savings precaution). Here we consider inverse of inter-temporal elasticity of substitution $\gamma = 1.5$, as the aim the representative agent is to smooth consumption. The utility function has a constant degree of risk-aversion. In this paper, we will focus on the lifetime utility of the 25 year old representative agent calculated *ex-ante* to capture the price paid for the contract (pension contribution) and its benefit obtained.

$$U^x_{t,s} = \mathbb{E}^{\mathbb{Q}}_t \left(\sum_{i=x}^{64} \frac{p^x\left(t+(i-x)\,|t\right)}{\left(1+{r^f}_{t,s}\right)^{t+i-25}} U\left(W^{i-24}_{t+(i-25),s} - C^{i-24}_{t+(i-25),s}\right) + \sum_{i=65}^{94} \frac{p^x\left(t+(i-x)\,|t\right)}{\left(1+{r^f}_{t,s}\right)^{t+i-25}} U\left(B^{i-24}_{t+(i-25),s}\right) \right)$$

We calculate the utility under fund participation and individual investment. The replicating coefficient ($coef_{replic}$) expresses the share of the fund participation utility replicated by the individual investment. The effects of different shocks and their consequences on this variable are evaluated. Moreover, we emphasize the decomposition of the shock effect on the replicating coefficient. We separately identify the demographic effect on the fund participation and on the individual investment separately.

4 Results

We run the three distinct contracts[11] (DB-"hard", DB-"soft" and CDC) whose characteristics are given in section 3. We construct two different frameworks:

- first step: policy security constraints are omitted (no PSC).
- second step: policy security constraints are introduced (yes PSC).

The first part of this section consists in presenting the results on "normal"[12] period which consist on the fund performance characteristics, the generation account values and the lifetime utility representation of a 25 year old representative individual. In the second part of this section, we present the results obtained under an exogenous life-expectancy

[11]We use Matlab to program the pension system and to manipulate the results.

[12]We define "normal" framework the one developed using the described simulations which serve as a good *proxy* of actual Dutch framework.

positive shock and compare them to the "normal" framework one.

Results of provided for each of the distinct periods ("normal" and "life-expectancy shock") are three-fold. On a first step, we present the fund performance during the 148 years[13]. On the second step, we evaluate the generation account of co-existing cohorts in time. This measure of the value based model allows us to compare at a given moment in time, the generation account level of pension plans and the corresponding individual investments (for the cohorts whose information is available). Finally, on the third step we elaborate the difference between fund participation and individual investment together with the difference among collective plans itself in terms of the level of lifetime utility.

4.1 Results on "normal period"

We consider the "normal period" being the benchmark of this study. Hence, we compare the results related to shocks in "life-expectancy" to the "normal period" ones.

4.1.1 Fund performance

For each plan contract designed, the first part of the table shows the results for the framework where no safety policy constraints (no PSC) are introduced. The second part includes results provided by the introduction of the policy constraints (yes PSC) against the extreme scenarios. In both cases, we consider homogeneity among individuals in terms of labor income. In tables V, VI and VII we exhibit the fund performance in time for each of the corresponding collective pension contracts. Among the represented variables, we highlight some of the distribution quantiles of average funding ratio (FR) during 20 and 40 years respectively, the probability of being underfunded ($P(FR) < 1$), the probability of overstepping either the upper bound limit FR ($P(FR > FR_{max})$) or the lower bound one ($P(FR < FR_{forbidden})$), the probability of having applied pension cuts because of policy regulation ($PSC\ cuts$) and finally the replacement rate (RPR)[14].

In this paper we consider that pension plan characteristics do not change in time and despite the long time duration we study, we decide to focus in presenting the statistics of two particular years such as year 20 and year 40 (see tables V, VI and VII). The 40 years time coincides with the retirement of the first 25 year old agents when we start the model. Because of the long historic and the difficulty in keeping the funding ratio bounded all the time, incites us to focus on some intermediary moment in time. Hence, we chose year 20.

[13]We start with 150 year data but we lose data because of the forward looking calculations.

[14]RPR is the ratio between the pension benefit (indexation included) to the average wage.

Table V: Collective Plan DB-“hard”

Plan DB-hard												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	**P**$(FR < 1)$	**P**$(FR > max)$	**P**$(FR < Forbidden)$	**P**$(index < 0)$	**P**$(index > 1)$	**RPR**	**PSC cuts**
FR_N 40y	0,5234	0,7840	0,9748	1,2304	1,9414	0,5065	0,0793	0,1293	0,0000	0,0000	0,8160	0,0000
FR_N 20y	0,3279	0,6312	0,9018	1,2445	2,7973	0,5637	0,1266	0,2587	0,0000	0,0000	0,8160	0,0000
yes PSC												
FR_N 40y	0,6314	0,8093	0,9820	1,2236	1,8314	0,5024	0,0730	0,0868	0,0000	0,0000	0,6941	0,0783
FR_N 20y	0,5875	0,7673	0,9456	1,2370	2,4185	0,5469	0,1144	0,1443	0,0000	0,0000	0,6306	0,1382

Source: Pension fund statistics in “normal” time, during 20 and 40 years; Type of contract: DB-“hard”; Calculations by the author.

Table VI: Collective Plan DB-“soft”

Plan DB-soft												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	**P**$(FR < 1)$	**P**$(FR > max)$	**P**$(FR < Forbidden)$	**P**$(index < 0)$	**P**$(index > 1)$	**RPR**	**PSC cuts**
FR_N 40y	0,5736	0,8426	1,0313	1,2763	1,9814	0,4542	0,0896	0,0878	0,0000	0,0000	0,8064	0,0000
FR_N 20y	0,4304	0,7605	1,0150	1,3424	2,8962	0,4935	0,1477	0,1755	0,0000	0,0000	0,8058	0,0000
yes PSC												
FR_N 40y	0,6507	0,8499	1,0357	1,2688	1,8751	0,4516	0,0830	0,0639	0,0000	0,0000	0,7224	0,0577
FR_N 20y	0,6083	0,8223	1,0253	1,3086	2,4883	0,4820	0,1332	0,1082	0,0000	0,0000	0,6767	0,1035

Source: Pension fund statistics in “normal” time, during 20 and 40 years; Type of contract: DB-“soft”; Calculations by the author.

Table VII: Collective Plan CDC

Plan CDC												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	**P**$(FR < 1)$	**P**$(FR > max)$	**P**$(FR < Forbidden)$	**P**$(index < 0)$	**P**$(index > 1)$	**RPR**	**PSC cuts**
FR_N 40y	0,5239	0,7879	0,9925	1,2761	2,0722	0,4936	0,0996	0,1269	0,0000	0,0000	0,8060	0,0000
FR_N 20y	0,3343	0,6437	0,9448	1,3710	3,3165	0,5373	0,1617	0,2432	0,0000	0,0000	0,8054	0,0000
yes PSC												
FR_N 40y	0,5004	0,7838	0,9912	1,3090	2,1909	0,4913	0,1140	0,1300	0,0000	0,0000	0,6030	0,1300
FR_N 20y	0,1480	0,6152	0,9766	1,5693	3,8702	0,5127	0,1993	0,2453	0,0000	0,0000	0,5311	0,2453

Source: Pension fund statistics in "normal" time, during 20 and 40 years; Type of contract: CDC; Calculations by the author.

The unconstrained pension DB-"hard" plan (no PSC) is highly influenced by the demographic structure. As mentioned in figure 4, the increase of the dependency ratio shows the population aging phenomenon. This is highly reflected by a low FR level during the first 20 years and its further decrease during the 40 first years especially because of the full pension guarantees promised by this plan (see table V).

Thus, there is a decrease in the FR especially in the scenarios lower than the median. The median results itself show that the funding ratio decreases from 97.48% (during the first 20 years) to 90.18% (during the first 40 years) on average. The higher quantiles (because of the surpluses already stocked) are not much affected by the increase of the dependency ratio. If one calculates the average FR for year the first 60 years, there is a decrease compared to the respective 20 and 40 results. The latter happens because of the increase in the dependency ratio for the population after 60 years compared to the other two moments in time. Figure 11 represents the expected funding ratio dynamics for the no policy constraint framework. As a consequence of the decreasing funding ratio, there is an increase in the probability of the fund to be underfunded and an increase of the funding ratio probability to go beyond the limit bounds (see table V). Because of the guaranteed pension benefit, the replacement rate remains constant.

Focusing on the constraint pension DB-"hard" plan (yes PSC), there is an increase in the prudence conditions. Thus, there is a decrease in the probability of having both, a FR lower than the $forbidden$ value and for having FR higher than the *maximum* one. Moreover, there is an increase in the FR level for the median and lower quantiles and an increase in the higher quantiles as a consequence of the shift from the unconstrained to the constrained plan. The lower quantile of the expected FR exceeds the limit level introduced by the government while the upper *maximum* level is exceeded by the highest quantile. The structure of these pension contracts and the timing of the controlling procedure (1 year lag), induce the probability of PSC cuts being slightly lower than the probability of having a fund ratio lower than the lower limit bound. The contribution cuts induced because of the exceed of the upper limit bound are more difficult to be reduced because of the surplus stock asset value. The safety policies causing benefit cuts, help to increase the fund's buffer and its FR, while the surplus redistribution helps to decrease it. Thus, the probability of being underfunded decreases. For the same reason the probability of being above the maximum accepted level of funding ratio and the probability of being below its lowest limit decreases when policy security constraints are introduced. Finally, the replacement rate is negatively affected by both, the introduction of constraints and time. During the first 20 years, there have been almost 8% of the cases when the policy cuts are applied and this number rises to 14% during the 40 years.

The statistics in time related to plan DB-"soft" go in line with the ones discussed for the DB-"hard" pension plan (see table VI). The decrease of fund sustainability in time is reflected by the decrease in both the funding ratio (FR) and the replacement rate (RPR). Furthermore, time positively impacts the probability of being underfunded, the probability of exceeding of the predefined bounded limits and the share of pension benefit cuts (the latter, in the case when the "policy safety constraints" are inherited).

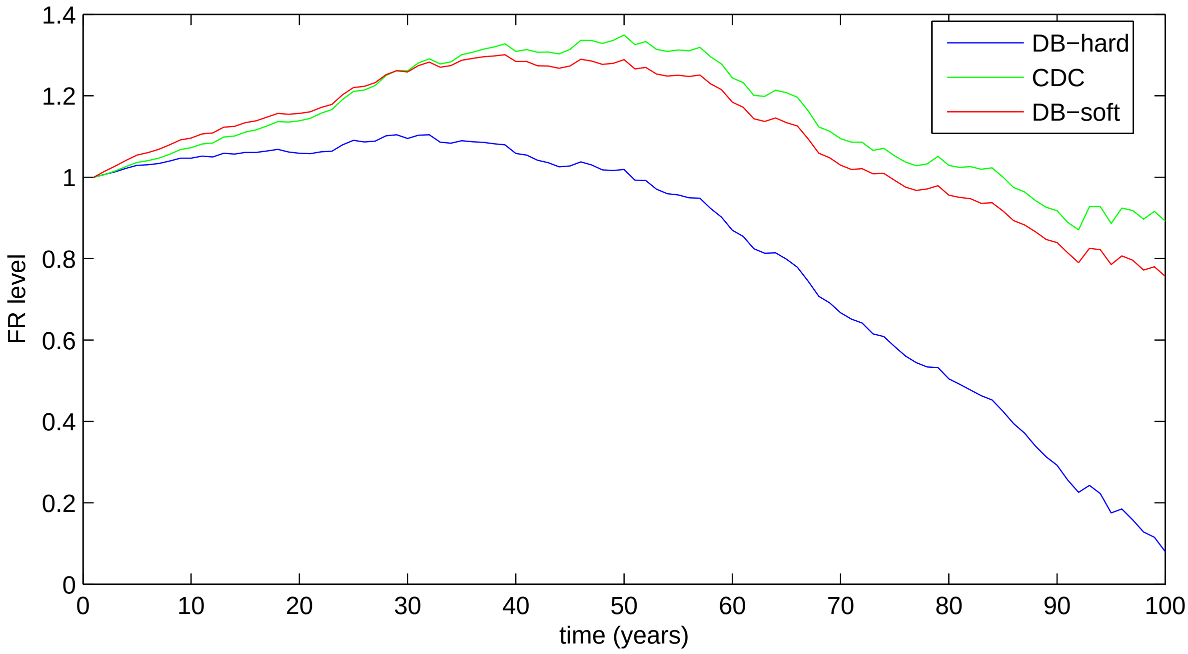
Nevertheless, there is a significant difference in the amplitude of the effects on DB-"soft" plan compared to DB-"hard" one. The flexibility in the pension indexation correlated to the fund performance induces less than full pension benefits distributed in the worst scenarios. Therefore, there is an increase in the average funding ratio and a decrease in both, the probability of being underfunded and the probability of being in extreme scenarios compared to DB-"hard" pension plan results. Because of the pension cutting policy, the probability of the fund to be underfunded slightly decreases. Finally, the replacement rate is no more constant with respect to the fund performance, but it varies. It is evaluated being lower than the respective one in the DB-"hard" plan where "no policy security constraints" are applied.

The CDC plan is defined by fixed contributions ($c_{t,s} = 19.39\%, \ \ \forall t, \ \forall s$) and conditionally indexation benefits (ladder indexation). The contribution level is in between the allowed extreme possible values for the contribution c_{min} and c_{max} and it is calculated based only on the time $t = 1$ available information. Calculating the rate of contribution as an expected value of the contribution levels in time (instead of just one year as we do in this study) could be done only by occurring an *ex-post* calculation because many variables are contagious and/or need predictions. Such variables are the population, the discounting rate, the future wages, and the inflation historic. The difference of the collective CDC compared to the collective DB-"soft" stands on the contribution level which is not contagious to the FR. This allows for shifting the risk to the retired individuals and the workers. A decrease (*resp.* increase) in the FR shifts the risk towards the retired (*resp.* active workers) people. There is a decrease in the RPR compared to the corresponding "sibling" contract with variable contribution (*i.e.* plan DB-"soft"). Nevertheless, there is an increase in the probability of being underfunded and the probability of having a FR outside the allowed limits fixed by the exogenous policy safety rules.

The expected funding ratio dynamics in time for the collective plans is represented in figure 11 and 12 respectively. The unconstrained framework and the generosity in total promising the pension benefit induces a completely unstable and diverging DB-

"hard" pension plan. Moreover, the CDC plan demands constant contributions which are sufficient for the highly aging population during the first 30 years. Its constant level serves in the following years to fill the fund buffer. Contrary to the fund participation, the individual investment keeps the funding ratio constant to one.

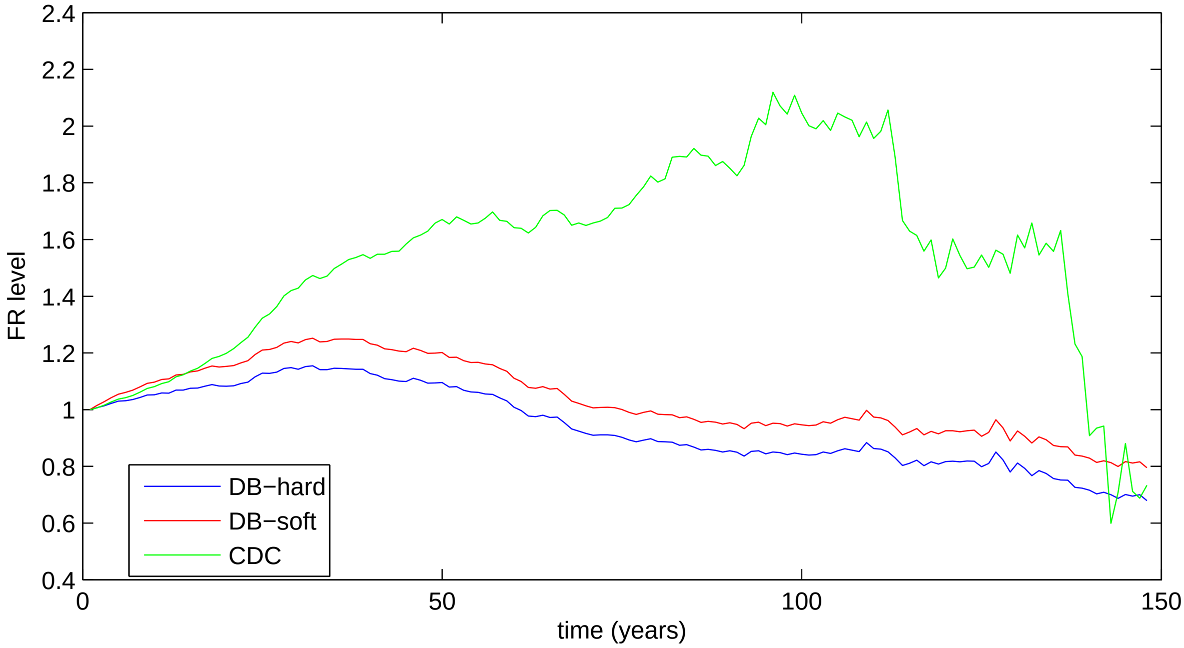
Figure 11: Expected Funding Ratio Dynamics (no PSC)



Source: The funding ratios are initially normalized to one. The DB-"hard" plan diverges; Period: "normal"; Framework: no PSC; Calculations by the author.

The policy safety constraints stabilize the variable contribution plans in terms of convergence and positively affect the CDC plan (see figure 12). The contribution level is high enough to keep the fund over-funded even by overstepping the upper limit borne for about 30 years in a row. It finally gets stabilized after year 110.

Figure 12: Expected Funding Ratio Dynamics (yes PSC)



Source: The funding ratios are initially normalized to one; Period: "normal"; Framework: yes PSC; Calculations by the author.

4.1.2 Generation account results

We use value based model to calculate the generation account (GA) level for co-existing cohorts during both fund participation and individual investment. Because of the homogeneity of individuals in a given cohort, we consider one representative agent for each cohort. To show off the results, we take into consideration three moments in time:

- year 20 (as an intermediate moment);
- year 40 (coincides with the active age for each agent);
- year 70 (maximum number of years the agent is in the system).

Contrary to the open fund whose data allows us to calculate the value of the account of each generation at each moment in time, the individual investment, because of its incompleteness, does not. Its matrix during the first 70 years is a upper triangular one. Thus, during the first 70 years ($t \leq 70$) one can calculate the GA corresponding to only t first co-existing cohorts. Hence, we use the value based model to have a broader vision for all existing cohorts at a given moment in time and focus on the 25 year old representative agent when using the utility measure. The representation of the generation account value for each pension plan in time and the corresponding values of the individual investment are represented in appendix B.

The GA corresponding to individual investment is represented by a hump shape function with respect to the co-existing cohorts (see figure 26 in appendix B). For the collective fund participation (DB-"hard"), the GA corresponds to a decreasing function with respect to the age of the co-existing cohorts (in a no PSC framework). First, this is highly related to the generosity of this collective plan. Secondly, pension benefits are variable when individuals participate to collective pension plans. However, they are considered constant when individual investment is used for pension savings.

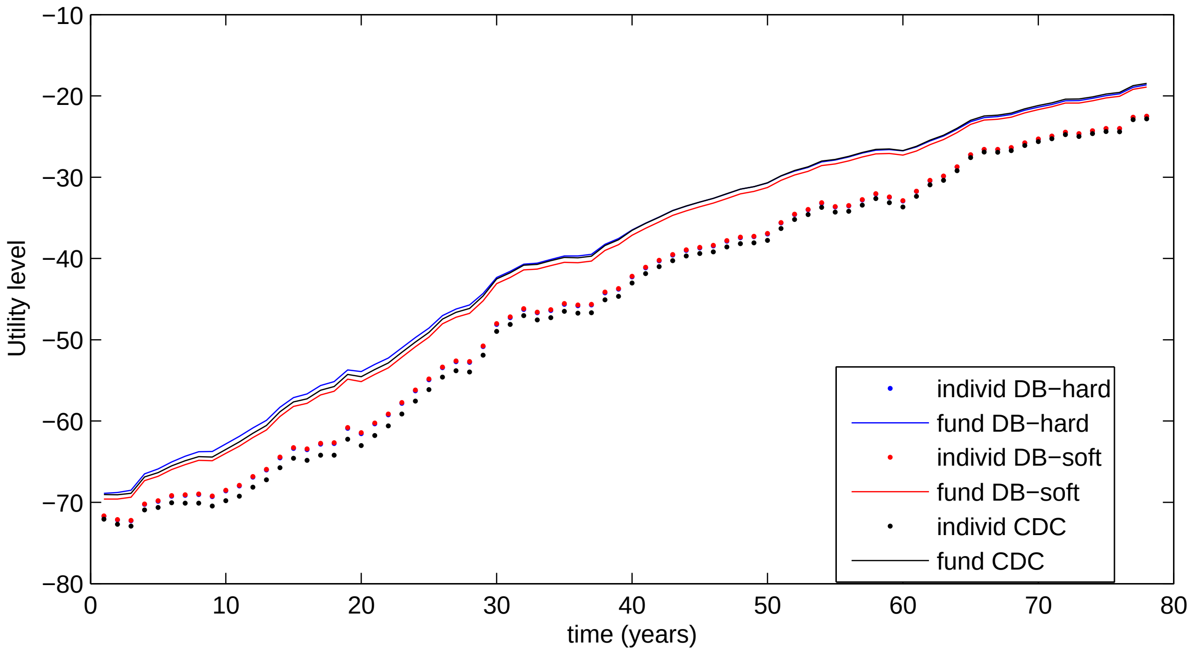
As far as policy rules are implemented, severe cuts may occur. The generation account while participating to collective DB funds is transformed to a hump shape function (see figure 29 and 30 in appendix B). The CDC plan goes in line with the no policy constrained framework because of the trade-off contribution-benefit that this plan offers (see figure 31 in appendix B).

The differences in generation account value, between the fund participation and the corresponding individual investment are positive and increase in time when no policy constraints are implemented. The contrary happens when policy rules are applied. The difference is negative for the DB collective plans. On the one hand, one could conclude that it is better to individually invest and be on a "pure" defined contribution scheme when such policy rules related to fund performance are applied. On the other hand, collective fund participation is more generous when no such safety constraints are taken into consideration. The CDC plan with constraints stays in the middle since the fund participation is optimal for the majority but not all cohorts. Therefore, to take a decision whether fund participation or individual investment is better pension investment choice, we base our argument in utility measure study. Hence, we measure not only the value of the generation account but moreover, we need to determine whether the agent is lifetime better off by individually investing for retirement or by participating to the pension plan while focusing on the individual well-being. Two are the reasons of considering the 25 year old representative agent. First, it is the generation for whom we can have a wider view of data. Secondly, it is the first generation joining the labor market. Hence, in a context where participation is not mandatory they could refuse to participate in the fund if fairness is not ensured. The robustness of these results are discussed as the model is hit by two distinct exogenous shocks.

4.1.3 Utility outcome

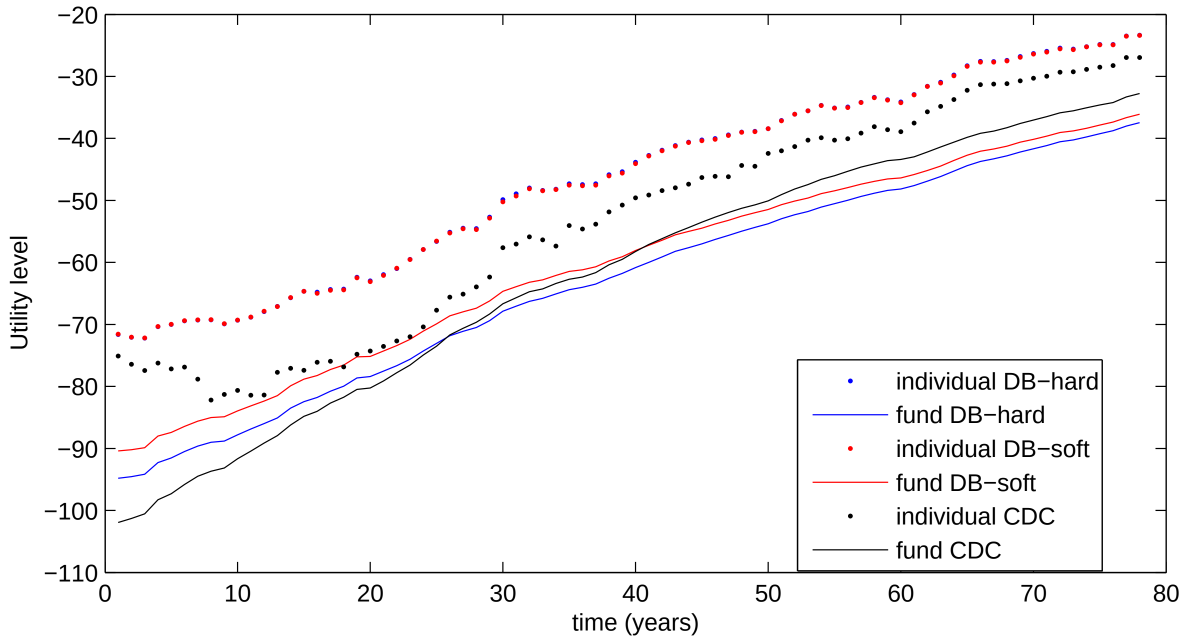
In this study we aim to quantify the proportion of the pension fund performance that could be replicated by individuals in the market. This can be identified by using an individual measure such as the utility. We calculate the lifetime utility of a 25 year old representative agent in time. So far, the price of the signed contract and its future benefits are taken into consideration. Figure 13 shows that under no PSC the fund participation outperforms the individual investment in terms of agent's utility. The inverse happens when and policy rules are applied see figure 14. The differences in the amplitude between each couple individual investment and fund participation depends on the type of the plan.

Figure 13: *Ex-ante* Lifetime Utility (no PSC)



Source: *Ex-ante* Lifetime Utility of a representative 25 year old individual in time; Period: "normal"; Framework: no PSC; Calculations by the author.

Figure 14: *Ex-ante* Lifetime Utility (yes *PSC*)



Source: *Ex-ante* Lifetime Utility of a representative 25 year old individual in time; Period: "normal"; Framework: yes *PSC*; Calculations by the author.

4.2 Results based on "upward demographic shock"

One of the factors determining aging is life-expectancy and together with fertility and migration, they create the three most important factors. An increase in life-expectancy and a decrease in fertility rate are both associated with the aging of the society. Aging is obviously a good development, as it means that people on average live longer. However, there are also worries that aging may negatively impact the economy in general and in particular pension arrangements which are fully funded.

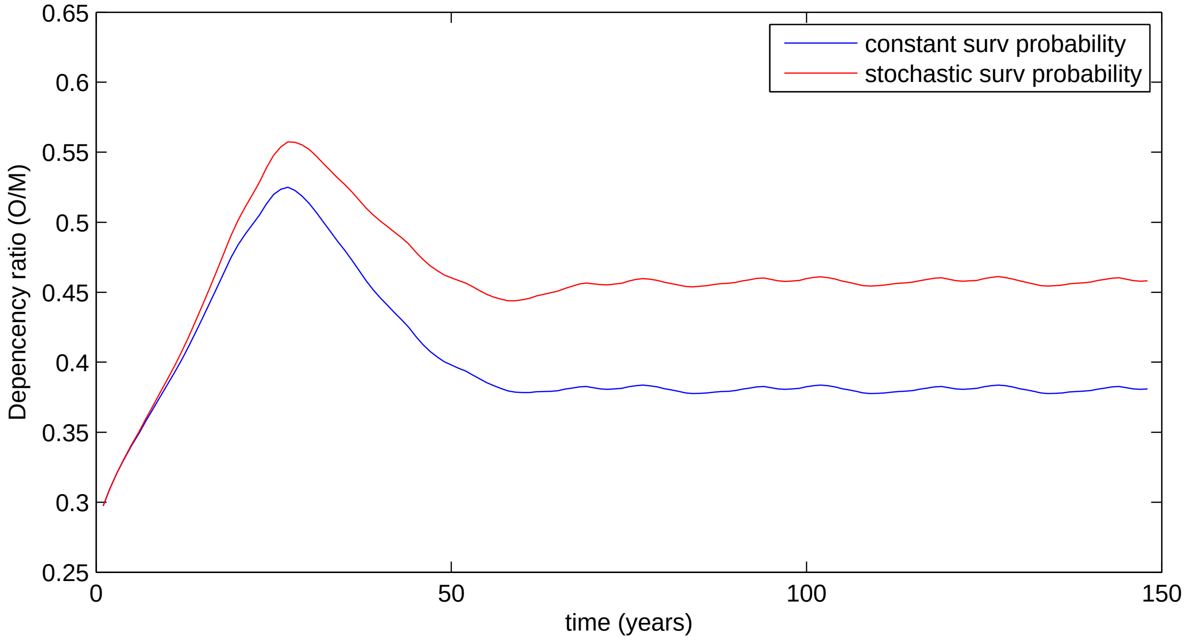
In this section, we conduct an exercise in which we consider an increase in life-expectancy. This phenomenon has been quite often during the last decade. Gompertz law is determined by the parameter modal age at birth that we use to introduce the life-expectancy shock. Fitting the 2011 − 2012 Dutch population to the Gompertz low survival probability, the modal age at death is 87. We introduce a shock in which there is an *increase in the modal age at death* being *one month each year* and when that reaches 92 years, it is kept constant. This upward demographic shock related to the life-expectancy implies an increase in the surviving probability, even why for simplicity in modeling the system, the maximum age an old could become remains constant (94 years old). The table VIII gives the evolution of this variable in time. Compared to the "normal" framework the increase in life-expectancy induces an increase in the dependency ratio reaching its peak of 55.5% after 25 years. Later it stabilizes at about 25 years (see figure 15).

Table VIII: Descriptive Statistics of the Population Evolution (Life-expectancy Shock)

	Modal age at death (in years)	Dependency Ratio (life-expectancy shock)	Dependency Ratio ("normal" framework)
At time t=1	87	29.74%	29.74%
After 20 years	88.67	50.13%	49.16%
After 40 years	90.25	50.09%	44.08%

Source: The descriptive statistics consist on information related to the evolution of population such as the modal age at death and the dependency ratio (in "normal" times and under "life-expectancy shock"); Calculations by the author.

Figure 15: Dependency Ratio Dynamics under Life-Expectancy Shock

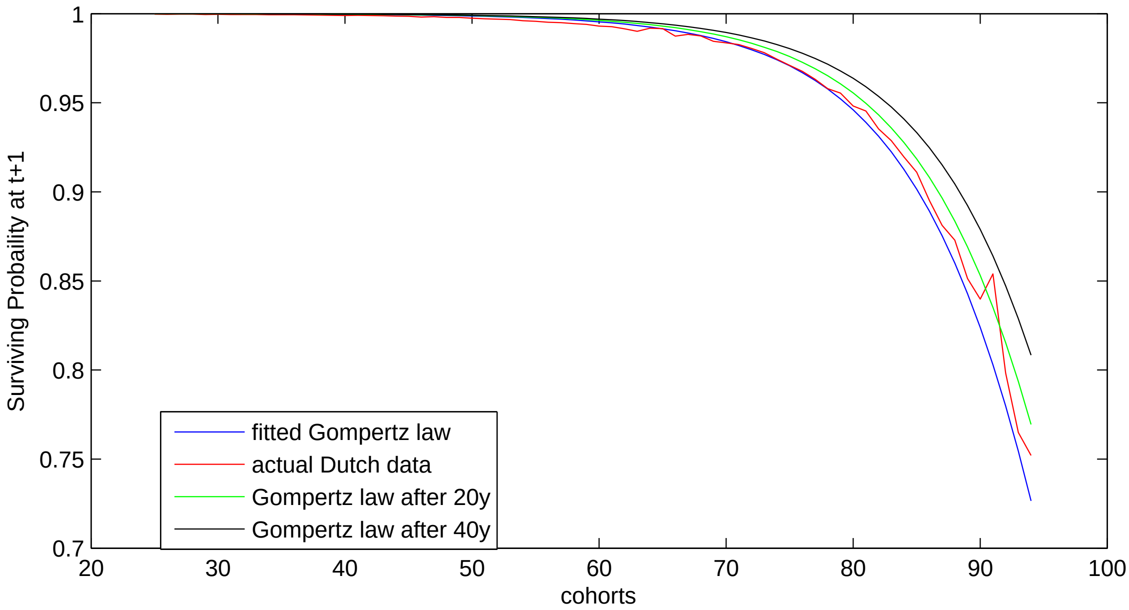


Source: The dependency ratio in time as the ratio between the old (older than 65 years old) to the young (aged between 25 and 65 years old) total population; Period: "life-expectancy shock"; Calculations by the author.

Therefore, the next year surviving probability of each cohort will be no more deterministic and constant in time, but stochastic and time contingent. Since we want to distinguish the effect of the increase in the life-expectancy, in this session we will consider the case where the entries of the 25 year old individuals are the same as the ones in the "normal" period but the conditional surviving probabilities is the only variable changing. Thus, we consider the birth rate growth being the same as the one in section 2.1. If we would like to get only the effect of an increase in life-expectancy, keep the new 25 year old entries constant in time (*i.e.* birth rate growth being zero), the results of the dependency ratio after 20 years would be 52.05% and respectively 55.05% after 40 years.

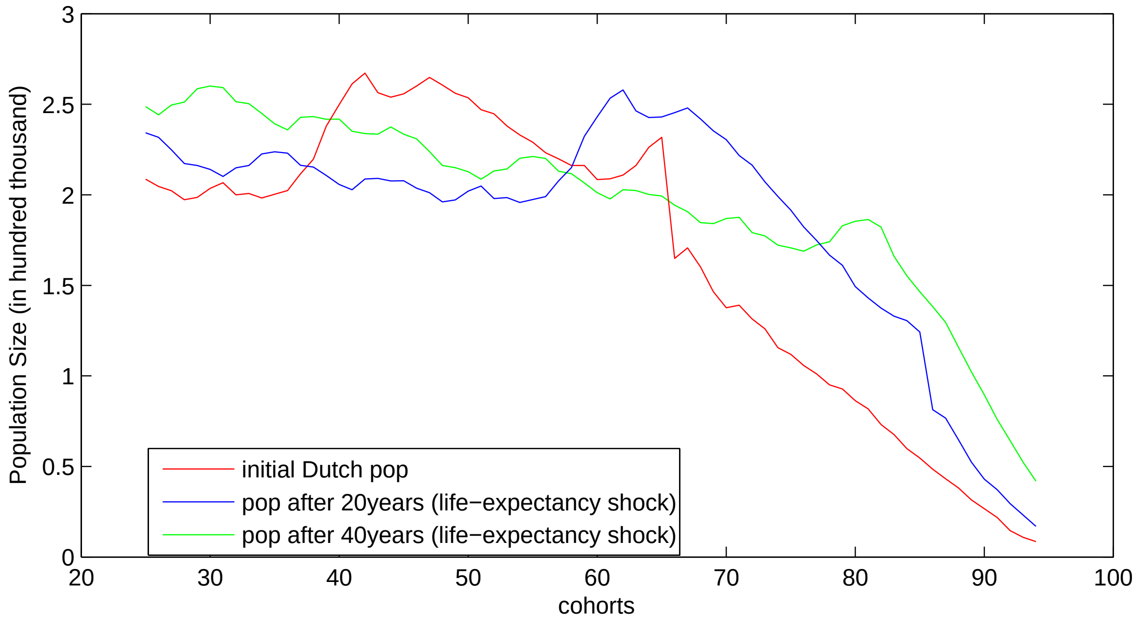
The survival probability dynamics of lag one is presented in figure 16 at time $t = 1$, after 20 years and after 40 years. The corresponding population structure dynamics captured at these 3 moments in time is represented in figure 17.

Figure 16: Surviving Probability (Life-expectancy Shock)



Source: The evolution of the surviving probability as a result of the dynamics of life-expectancy; Period: "life-expectancy shock"; Calculations by the author.

Figure 17: Population Dynamics (Life-expectancy Shock)



Source: The evolution of the population as a result of the dynamics of life-expectancy; Period: "life-expectancy shock"; Calculations by the author.

4.2.1 Fund performance

The aging process caused by an upward demographic shock such as life-expectancy reduces fund sustainability faster than the effect of downward demographic shock. The increase in the surviving probability at old age implies a retirement benefit insured for longer time on average by more individuals. The funding ratio distribution of all plans are lower than the corresponding ones in the "normal" framework. Moreover, the limits are difficult to be maintained under control especially for the first 40 years. The replacement rate decreases and the probability of applying PSC cuts increase.

Table IX: Collective Plans under Positive Life-expectancy Shock

Plan DB-"hard"												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5089	0,7612	0,9471	1,1928	1,8739	0,5302	0,0682	0,1475	0,0000	0,0000	0,8160	0,0000
FR_N 20y	0,3037	0,5805	0,8405	1,1684	2,5688	0,6007	0,1058	0,3003	0,0000	0,0000	0,8160	0,0000
yes PSC												
FR_N 40y	0,6214	0,7874	0,9563	1,1914	1,7806	0,5265	0,0627	0,1008	0,0000	0,0000	0,6685	0,0909
FR_N 20y	0,5658	0,7348	0,8924	1,1657	2,2456	0,5853	0,0960	0,1732	0,0000	0,0000	0,5768	0,1659

Plan DB-"soft"												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5569	0,8195	1,0051	1,2399	1,9092	0,4811	0,0789	0,1036	0,0000	0,0000	0,8058	0,0000
FR_N 20y	0,3997	0,7090	0,9583	1,2564	2,6553	0,5356	0,1258	0,2101	0,0000	0,0000	0,8048	0,0000
yes PSC												
FR_N 40y	0,6344	0,8310	1,0070	1,2347	1,8168	0,4784	0,0728	0,0758	0,0000	0,0000	0,6980	0,0679
FR_N 20y	0,5820	0,7895	0,9774	1,2405	2,3075	0,5240	0,1128	0,1327	0,0000	0,0000	0,6338	0,1268

Plan CDC												
no PSC	**Q2.25%**	**Q25%**	**Q50%**	**Q75%**	**Q97.5%**	$P(FR<1)$	$P(FR>max)$	$P(FR<Forbidden)$	$P(index<0)$	$P(index>1)$	RPR	**PSC cuts**
FR_N 40y	0,5097	0,7641	0,9632	1,2296	1,9957	0,5168	0,0856	0,1443	0,0000	0,0000	0,8054	0,0000
FR_N 20y	0,3149	0,5966	0,8788	1,2684	3,0318	0,5726	0,1351	0,2814	0,0000	0,0000	0,8046	0,0000
yes PSC												
FR_N 40y	0,4878	0,7665	0,9674	1,2738	2,1319	0,5095	0,1021	0,1424	0,0000	0,0000	0,5793	0,1424
FR_N 20y	0,1675	0,5952	0,9294	1,4798	3,6088	0,5347	0,1821	0,2637	0,0000	0,0000	0,5033	0,2637

Source: Pension fund statistics in time, under "life-expectancy shock", during 20 and 40 years; Type of contract: DB-"hard", DB-"soft" and CDC; Framework: no & yes PSC; Calculations by the author.

4.2.2 Generation account results

The positive life-expectancy shock influences at the same time the fund dynamics and the individual performance. The latter comes out in the moment when the pension annuity decision is taken and the swap contract is signed in. Hence, by the rule of thumb, the generation account valuation is contingent to it. The increase in life-expectancy implies an increase in the surviving probability and the conditional surviving probability. As a consequence, the fund liability increases and it is necessary to be balanced by an increase in the asset side. As long as contribution is modeled in these pension contracts being bounded (or fixed), the maximum share of wage that is asked to be contributed by the agents is $c_{max} = 20\%$. Despite, the increase of the liability level, there is a bounded limit for the assets to cover it. Under these circumstances the fund looses his liquidity balance and tends towards lower funding ratios.

Nevertheless, the situation is positively seen by the agents who contribute not much more (sometimes the same c_{max}) and get more during retirement, by increasing the probability of being alive in the system. This is all explained by an increase in the generation account under each pension contract and framework, compared to the "normal framework" results. The concrete generation account values for all co-existing cohorts for each plan under constrained and non-constrained framework are given in appendix C. Therefore, one could give the same conclusions in terms of value based valuation as in "normal" framework since the shape of these functions does not change except their amplitude.

4.2.3 Utility outcome

To better understand the effect of this shock on the agent's point of view and to be able to compare the individual to the fund participation (in the same sense), we base our study on the *ex-ante* lifetime utility. The following figure 18 and 19 (in appendix C) represent the utility of a 25 year old representative agent in time. This lifetime utility depends on the type of the pension plan the representative agent belongs to. It is obvious from the graphic representations that the unconstrained plans, contrary to constrained ones, make higher strong promises such that it is more optimal to be part of a collective plan, for a 25 year old agent, especially under such demographic shock. Equivalently as the results presented in the generation account section, the consequences of the shock are presented by lowering the agent's utility level.

Figure 18: *Ex-ante* Lifetime Utility in Positive Life-expectancy Shock (no PSC)

Source: *Ex-ante* Lifetime Utility of a representative 25 year old individual in time; Period: "life-expectancy shock"; Framework: no PSC; Calculations by the author.

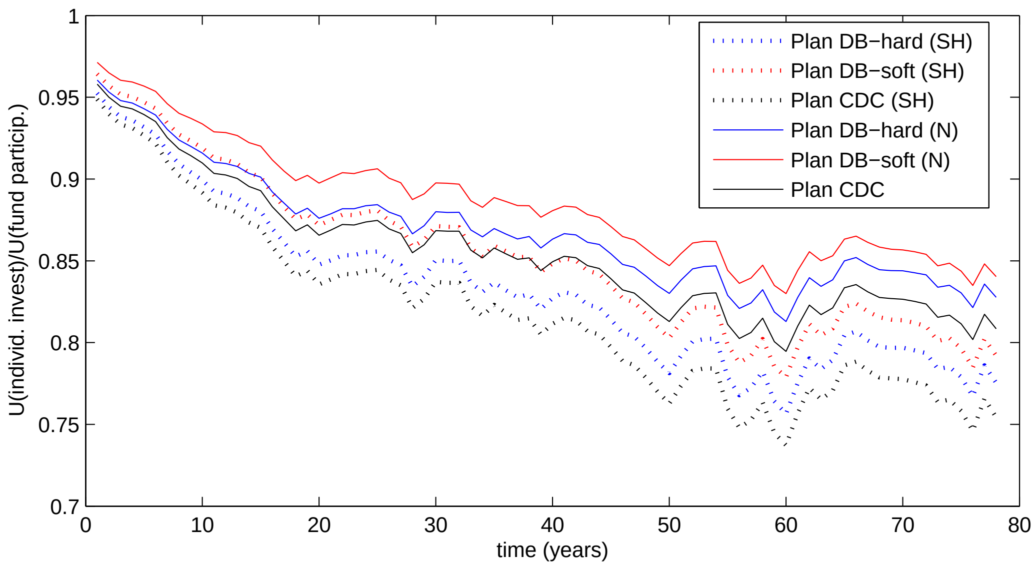
Figure 19: *Ex-ante* Lifetime Utility in Positive Life-expectancy Shock (yes PSC)

Source: *Ex-ante* Lifetime Utility of a representative 25 year old individual in time; Period: "life-expectancy shock"; Framework: yes PSC; Calculations by the author.

Despite the fact that the results stay in line with the "normal framework" while comparing fund participation to individual investment, the amplitude of the shock effect is not the same.

We construct the variable $coef_{replic}$ to capture the proportion of the utility of the fund participation replicated by the representative agent when he/she invests individually. Figure 20 gives these proportions and its dynamics in time during normal and shocked periods. The individual investment replicates around 95% of the fund performance at the starting years of the "normal" period (in terms of utility). It reaches the 85% of the coefficient of replication from year 50 and keeps it stable for the remaining years. The effect of the shock is given on dotted line and represents a decrease in the replication coefficient.

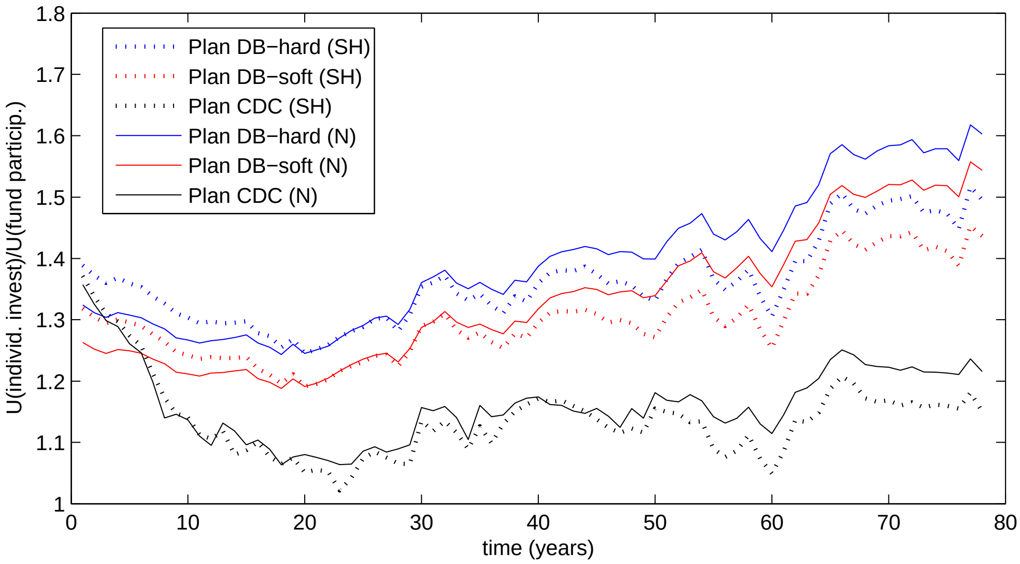
Figure 20: Utility Replication of the Fund Participation by Individual Investment (no PSC)



Source: The dynamics of the $coef_{replic}$ variable representing the proportion of the fund participation replicated by the individual investment in terms of utility; Period: "normal" (N) and "life-expectancy shock" (SH); Framework: no PSC; Calculations by the author.

As soon as the policy security constraints are applied, there is a change in roles. These constraints affect the shape of the corresponding utility for each pension plan as in figure 19. Hence, under the same contribution level, the 25 year old agent would prefer to invest individually rather than to participate in the fund for each of the considered plans. The proportion of the individual investment to the fund participation in terms of utility is given in figure 21. Nevertheless, the shock does not change the optimal choice (with respect to "normal" period) between fund and individual investment.

Figure 21: Utility Replication of the Fund Participation by Individual Investment (yes PSC)



Source: The dynamics of the $coef_{replic}$ variable representing the proportion of the fund participation replicated by the individual investment in terms of utility; Period: "normal" (N) and "life-expectancy shock" (SH); Framework: yes PSC; Calculations by the author.

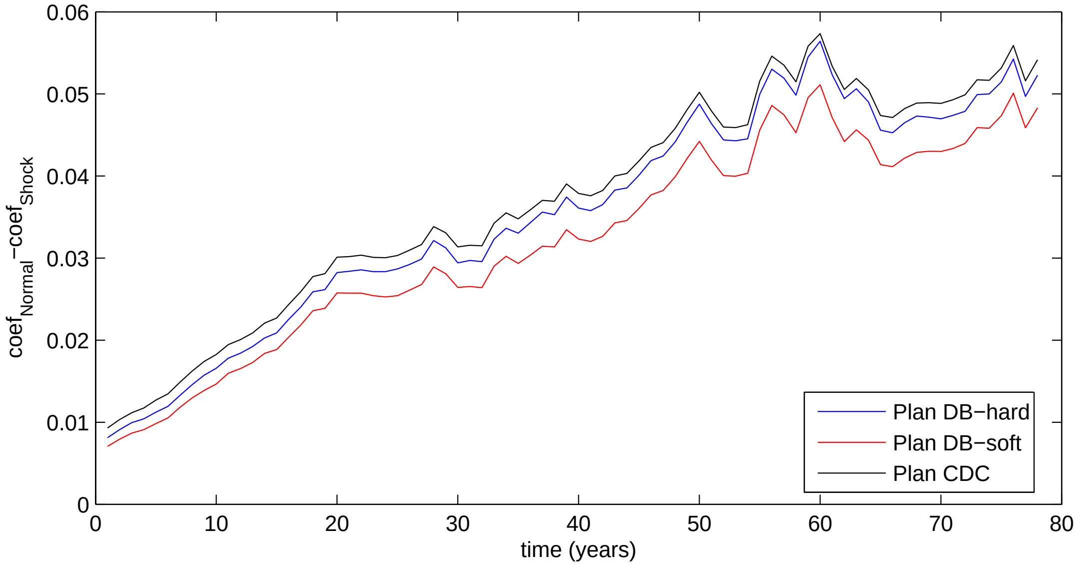
There is a drop in the replication coefficient while the "life-expectancy shock" is taken into consideration. The amplitude of this drop depends on the considered plan.

We define the difference between the normal and the shocked effect. If the individual would have invested using the same investment strategy as the fund, the latter difference would express the characteristic of the pension plan in risk sharing. In this paper we have focused on the case where the individual invests slightly different from the fund. Since the differences are not significant one could interpret the difference as a specific characteristic of the fund. Therefore, figure 22 and figure 23 present the effect of shock (as a difference between the "normal" and the "shocked" variable) in the replication coefficient, respectively with and without policy security constraint implemented. Smaller and flatter it remains the difference (in absolute value) between the "normal" replication coefficient and the one under shock, more one can say that the shock is well amortized.

$$\Delta coef = coef_{replic}^{Normal} - coef_{replic}^{Shock}$$

Therefore, when pension constrained framework is implemented, the CDC contract amortizes this upward demographic shock better among the other pension plans, whose *delta* coefficients diverge. To the contrary, when the no policy constrained rule is applied, the difference between the replicating coefficients in normal and under shock, have a positive trend in time. Nevertheless, it stabilizes 50 years later[15]. The lowest effect of the shock is seen on the "soft" DB plan while the CDC is the plan with the highest consequences of the "life-expectancy shock" framework.

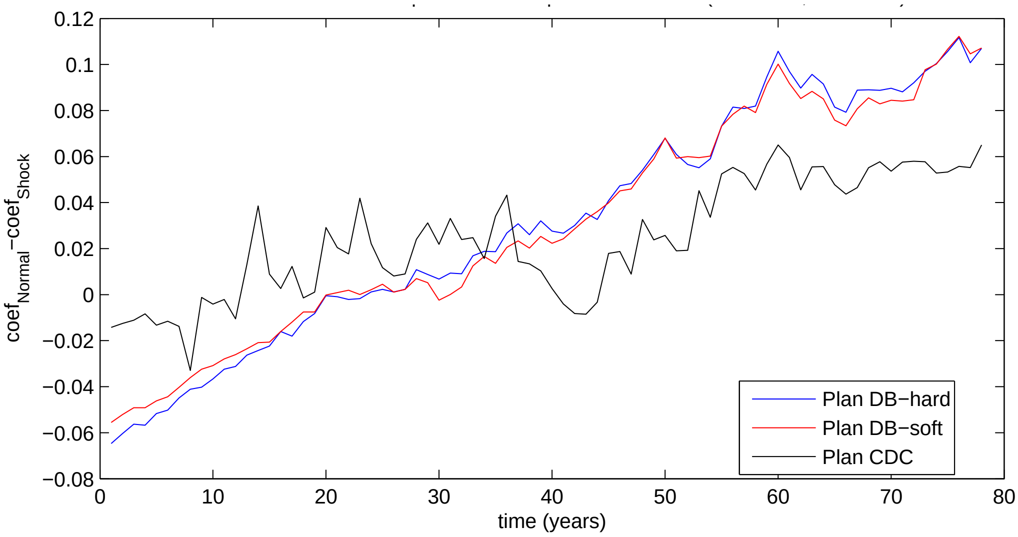
Figure 22: The Shock Effect on the Replicating Coefficient in Different Plans (no PSC)



Source: The replicating coefficient is the ratio of the 25 year old lifetime utility provided by individual investment to the corresponding one provided by fund participation; Period: "life-expectancy shock"; Framework: no PSC; Calculations by the author.

[15]The population structure (Dependency Ratio) stabilizes starting from the years 50.

Figure 23: The Shock Effect on the Replicating Coefficient in Different Plans (yes PSC)



Source: The replicating coefficient is the ratio of the 25 year old lifetime utility provided by individual investment to the corresponding one provided by fund participation; Period: "life-expectancy shock"; Framework: yes PSC; Calculations by the author.

So far, we concluded that this shock does not change the optimal solutions for a 25 year old individual when deciding to participate collectively or to invest individually. The shock affects the replication coefficient differently for different plans. Since the latter is a composition of the utility provided by fund participation and individual investment, last but not least, it is important to decompose the effect of the shock directly in each of these components.

Therefore, figure 24 presents how the shock affects directly the difference between the utility of fund participation (*resp.* individual investment) in "normal" time and the utility of fund participation (*resp.* individual investment) in shocked situation.

$$Units\ of\ utility(fund) = utility(fund)^{Normal} - utility(fund)^{Shock}$$

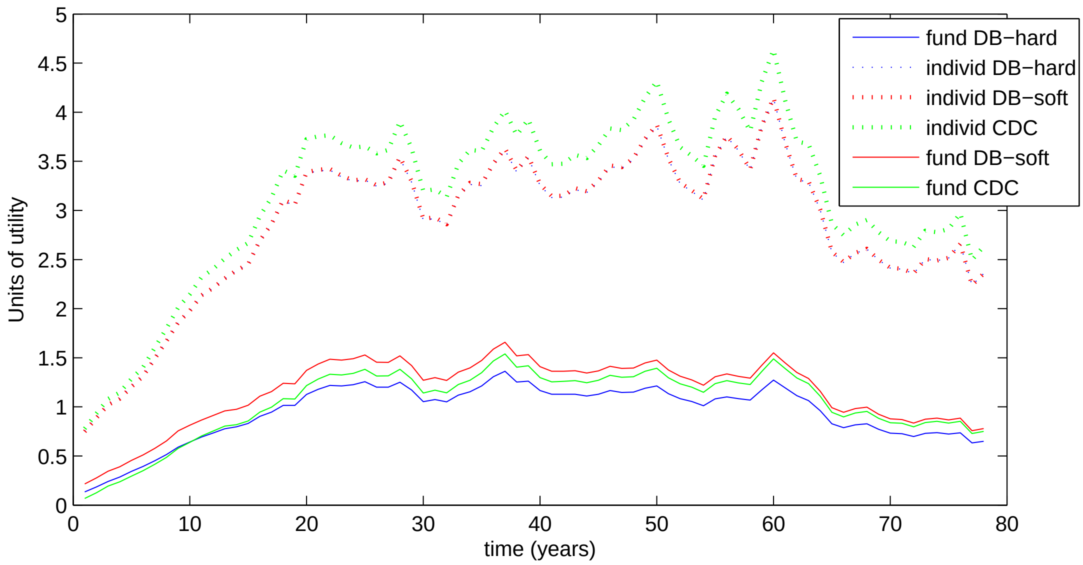
$$Units\ of\ utility(individ) = utility(individ)^{Normal} - utility(individ)^{Shock}$$

Considering the shape of the utility function used, a negative (*resp.* positive) value of $Units\ of\ utility(\cdot)$ induces a positive (*resp.* negative) effect of the shock on the agent's utility.

First, one can conclude that the effect of the shock has a negative impact on the utility regardless its type (*i.e.* fund or individual). On the one hand, based on the no PSC framework, the effect on the fund is smaller than the one by individual investment.

Furthermore, the CDC is the contract that better amortizes during the first 10 years. After 10 years, the DB-“hard” is more profitable for the 25 year old representative individual. The lowest variance (8.5%) and the lowest maximum difference (1.36) is offered by the DB-“hard”. The individual investment under this shock is more risky when it is constructed based on the CDC plan. It presents the highest variance (73%). The DB-“hard” contribution strategy used for the individual investment is more shock amortizing (see table X). Regardless these results, as mentioned in the last paragraph, the replicating coefficient is less affected in the case of DB-“soft” plan (no PSC).

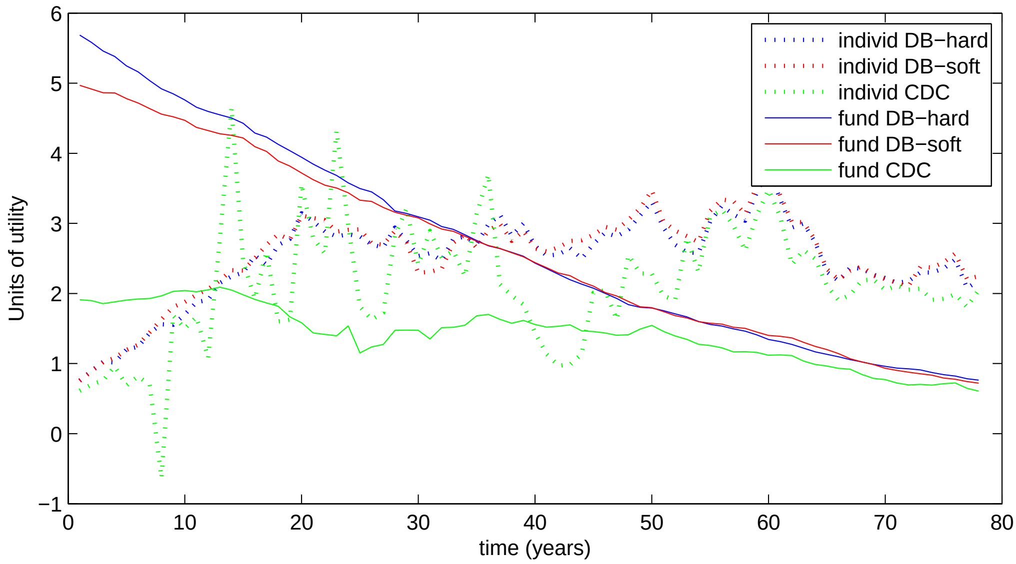
Figure 24: The Effect of Life-expectancy Shock on Fund and Individual Investment (no PSC)



Source: The effect of the shock is considered separately in each pension contract; Period: “life-expectancy shock”; Framework: no PSC; Calculations by the author.

On the other hand, based on the PSC framework, we decompose the effect of the shock on the $coef_{replic}$ into the effect on the fund participation and corresponding individual investment. Figure 25 represents the dynamics in time of these effects. Table X quantifies the descriptive statistics of this shock affecting the utility function.

Figure 25: The Effect of Life-expectancy Shock on Fund and Individual Investment (yes PSC)



Source: The effect of the shock is considered separately in each pension contract; Period: "life-expectancy shock"; Framework: yes PSC; Calculations by the author.

The collective fund amortizing best the shock is the CDC contract. It presents the minimum variance of 17% with almost no trend. It represents the lowest mean and the lowest extreme values, expressed in units of utility. Concerning the corresponding individual plans, the CDC is not the one amortizing the shock the most, this place is substituted by the plan which contributes (*i.e.* invests) the most (the individual plan whose contribution is based on DB-"hard" plan).

Table X: Descriptive Statistics of a Life-expectancy Shock on the Individual Investment and Fund Participation

Fund Participation					
NO PSC NO MWP	**Mean**	**Variance**	**MaxDrawdown**	**Min**	**Max**
Plan DB-hard	0.9466	0.0850	0.5350	0.1336	1.3630
Plan DB-soft	1.1526	0.1181	0.5431	0.2158	1.6583
Plan CDC	1.0473	0.1228	0.5272	0.0677	1.5397
YES PSC NO MWP					
Plan DB-hard	2.7105	2.1823	0.8655	0.7646	5.6864
Plan DB-soft	2.6074	1.7545	0.8551	0.7203	4.9697
Plan CDC	1.4148	0.1669	0.7095	0.6070	2.0897
Individual Investment					
NO PSC NO MWP	**Mean**	**Variance**	**MaxDrawdown**	**Min**	**Max**
Plan DB-hard	2.8556	0.5678	0.4593	0.7522	4.1393
Plan DB-soft	2.8632	0.5753	0.4602	0.7490	4.1476
Plan CDC	3.1616	0.7328	0.4620	0.7782	4.6377
YES PSC NO MWP					
Plan DB-hard	2.4939	0.3709	0.4526	0.7585	3.7937
Plan DB-soft	2.5437	0.3835	0.4526	0.7532	3.8527
Plan CDC	2.1271	0.7692	0.7910	-0.6308	4.6466

Source: The descriptive statistics are calculated in terms of utility level for the 25 year old representative individual; Period: "life-expectancy shock"; Framework: no & yes PSC; Calculations by the author.

5 Conclusions

This paper tries to shed some light on the expression highly used in the recent years: "*the collective defined contribution pension contract is the best choice for inter-generation risk sharing*". The Netherlands was one of the first countries to adopt the CDC contract. It is privately managed and mandatory participation is necessary in this second pillar pension contract. Starting from the regulatory conditions of this plan, being at the same time mandatory and privately managed, one might doubt on the necessity of having this part of the pension savings obligatory. We ask the research question whether it would be possible, to replicate this collective pension performance by individually investing in the market. Moreover, how does this contract react when demographic shocks happen. The regulation constraints negatively affect the individual welfare.

This study is based on stylized pension contract analysis. Using real population data and simulated financial ones, we construct three basic contracts and study them in two main frameworks (when policy safety constraints are implemented, yes PSC; and when

when they are absent in the system, no PSC). One of the successes of this pension contract is its behavior under shock. To test the robustness of the three implemented contracts and to measure how the replication coefficient (the ratio of the utility provided by individually investing to the utility provided by participating to the collective pension plan) and each utility of fund/individual investment react to the exogenous influences, we simulate a demographic shock, being the upward shock (life-expectancy).

The study presents the possibility of individually replicating directly in the market the returns that the collective pension system offers. This replication stays at the level of $80\% - 90\%$ in the case when no policy constraints are implemented and makes it the absolutely best way to future save, when "hard" policy safety constraints are implemented. Therefore, the remained $10\% - 20\%$ is dedicated to the inter-generation specificity of the collective plans. This result, highly raises the question of fairness of this pillar and its mandatory application. The good properties of the CDC plan are shown off when we compare the results among the stylized contracts. The CDC plan appears to be the best choice in several situations except the unconstrained "upward" shocked framework. As it concerns the individual investment, we supposed that the agents save and invest the same amount they would have contributed by participating in the fund. Therefore, agents are supposed to know how much to invest and to have some financial literacy experience.

Although this study remains conducted based on the Dutch economic framework, one could deduce two main policy recommendations. On the one hand, a necessary reform for plans which are still offering "hard" guarantees is needed. Among the collective plans the CDC appears to be the one better amortizing risk and offering sustainability in pension benefits provision. On the other hand, we concluded that it is possible to replicate part of the fund performance by individually investing. What is remained unable to be replicated by the individual investment is characteristic of the pension plan. We modeled the individual investment such that the agent uses a distinct investment strategy compared to the fund itself. This does not allow us to conclude surely (although the results on "constant-mix" investment for individuals are not very distinct) that the remained share unable to be replicate by the individual investment is dedicated to the pension plan characteristics only. Nevertheless, our results propose that if the regulation on pension system is harsh, individual investment is the optimal solution. These arguments are in favor of a voluntary or partially mandatory collective scheme. Nevertheless, we keep in mind that during individual investment the financial literacy knowledge is important.

This study expresses some elements but not everything about the "black box" represented with the name "hybrid" collective defined contribution pension system. The structure of some of the data could be ameliorated and further research ideas could be studied as a future work. So far, we used a Vasicek one factor model to generate the term structure. More sophisticated methods can be used to make a better *proxy* of the market (such as van den Goorbergh et al. (2011)). In this paper, we focus on the pension in the eyes of a 25 year old. The results could be extent to all the cohorts and deduce the effects at any time and cohort. Furthermore, we focus on the effects of demographic shock which is not the only existing shock threatening pension system. The interest rates, or the non mean-reverting stock returns are macro and financial shocks which can hit the pension system especially during crisis and which show post-crisis implications. Finally, it is of high interest to be able to decompose the risk shared into inter and intra generation risk. To do so, heterogeneity in the wages *per* cohort should be introduced.

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Appendices

A : Vasicek 1-factor Model

The solution to this stochastic differential equation is:

$$r(t)=\mu_r+\left(r(\tilde{t})-\mu_r\right)\times e^{-\kappa_r(t-\tilde{t})}+\sigma_r\int_{\tilde{t}}^{t}e^{-\kappa_r(t-\tilde{t})}dW(u),\quad t\geq\tilde{t}$$

We denote $P(t,T)$ the price at time t of a zero-coupon which pays off 1 at time T. In general case, its price can be written as follows:

$$P(t,T)=\mathbb{E}\left[e^{-r(t-T)}\right]$$

According to Vasicek (1977) the price of the zero-coupon can be written as:

$$P(t,T)=A(t,T)\times e^{-B(t,T)r(t)}$$

with $r(t)$ being the short-term interest rate. Moreover, A and B are defined as below:

$$A(t,T)=exp\left(\frac{\left(B\left(t,T\right)-T+t\right)\left(\kappa_r^2\mu_r-\sigma_r^2/2\right)}{\kappa_r^2}-\frac{\sigma_r^2B(t,T)^2}{4\kappa_r}\right)$$

$$B(t,T)=\frac{1-e^{-\kappa_r(T-t)}}{\kappa_r}$$

When $\kappa_r=0$, we have:

$$B(t,T)=T-t$$

$$A(t,T)=\exp\left(\frac{\sigma_r^2\left(T-t\right)^3}{6}\right)$$

The bond prices are log-normally distributed and the dynamics is represented as follows:

$$\frac{dB}{B}=\left(r(t)+\frac{\lambda_r\sigma_r}{\kappa_r}\left(e^{-(T-t)\kappa_r}-1\right)\right)dt+\frac{\sigma_r}{\kappa_r}\left(e^{-(T-t)\kappa_r}-1\right)dW(t)$$

Therefore, the interest rate at time t with maturity T is written as:

$$R(t,T)=-\frac{1}{T-t}ln\left(\mathbb{E}\left(e^{-r(T-t)}\right)\right)=-\frac{1}{T-t}ln\left(A(t,T)\right)+\frac{1}{T-t}ln\left(B(t,T)r(t)\right)$$

On the one hand, one can say that given a set of information at time $\tilde{t}$, the short-term rate is normally distributed $r(t)|F_{\tilde{t}}\sim\mathcal{N}\left(\mathbb{E}_{\tilde{t}}[r(t)];\mathbb{V}_{\tilde{t}}[r(t)]\right)$.

Furthermore, the conditional expectation and variance of the process are respectively represented as:

$$\mathbb{E}_{\tilde{t}}[r(t)]=\mu_r+\left(r\left(t\right)-\mu_r\right)e^{-\kappa_r(\tilde{t}-t)},\quad\tilde{t}\leq t$$

$$\mathbb{V}_{\tilde{t}}[r(t)] = \frac{\sigma_r^2}{2\kappa_r}\left(1 - e^{-2\kappa_r(t-\tilde{t})}\right), \quad \tilde{t} \le t$$

For very large values of t, the expected value and variance of the short-term rate tend to μ_r and $\frac{\sigma_r^2}{2\kappa_r}$ respectively, while the mean reversion reduces the probability of extremely unreasonable large/low interest rates.

On the other hand, one can say that given a set of information at time $\tilde{t}$, the yield to maturity $R(t,T)$ is normally distributed $R(t,T)|F_{\tilde{t}} \sim \mathscr{N}\left(\mu_R\left(\cdot\right); \sigma_R^2\left(\cdot\right)\right)$ with:

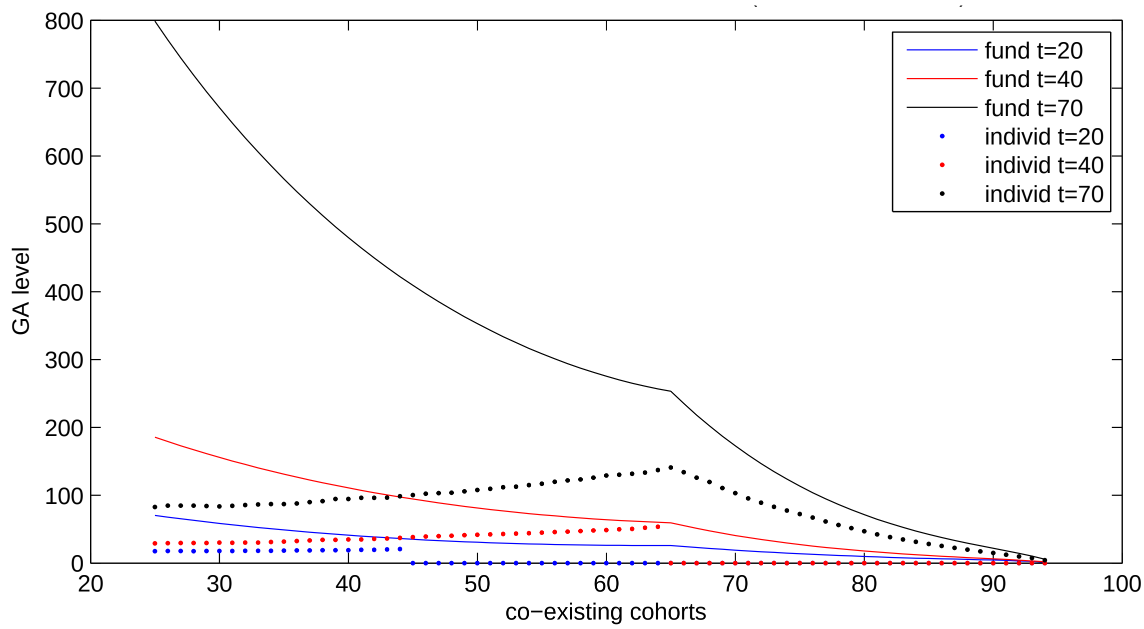
$$\mu_R\left(\cdot\right) = \left(1 - e^{-\kappa_r(t-\tilde{t})}\right)\left(R\left(\tilde{t},\infty\right) + \frac{1-e^{-\kappa_r T}}{\kappa_r T}\left(\mu - R\left(\tilde{t},\infty\right)\right) + \frac{\sigma_r^2\left(1-e^{-\kappa_r T}\right)^2}{4\kappa_r^3 T} + e^{-\kappa_r(t-\tilde{t})}R\left(\tilde{t},T\right)\right) \quad \tilde{t} \le t$$

$$\sigma_R\left(\cdot\right) = \left(\frac{1-e^{-\kappa_r T}}{\kappa_r T}\right)^2\left(1 - e^{-2\kappa_r(t-\tilde{t})}\right)\frac{\sigma_r^2}{2\kappa_r} \quad \tilde{t} \le t$$

where, infinite maturity interest rate $R\left(t,\infty\right) = \mu_r + \frac{\lambda\sigma_r}{\kappa_r} - \frac{\sigma_r^2}{2\kappa_r^2}$ is constant and does not depend on short interest rate. Despite the fact that $R(t,T)$ depends linearly to the short-term interest rate $r(t)$, its shape is independent on the $r(t)$ but depends on t itself. The volatility term structure of the yields is a decreasing function of time to maturity, with limit value zero.

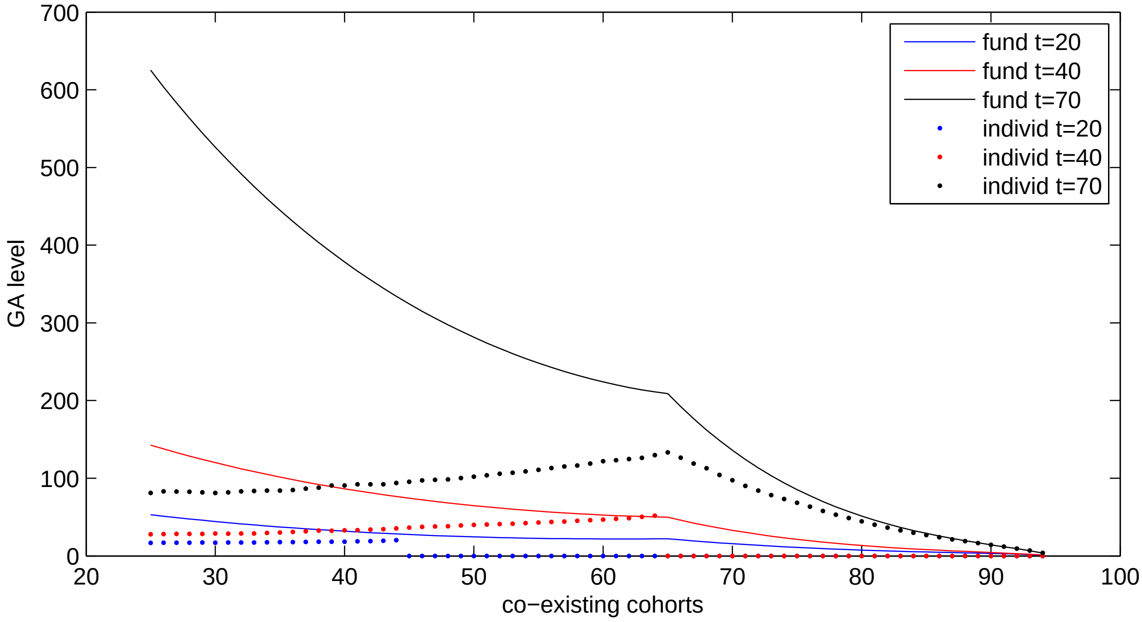
B : GA in "normal" framework

Figure 26: Generation Account Value in Time for DB-"hard" Plan (no PSC)



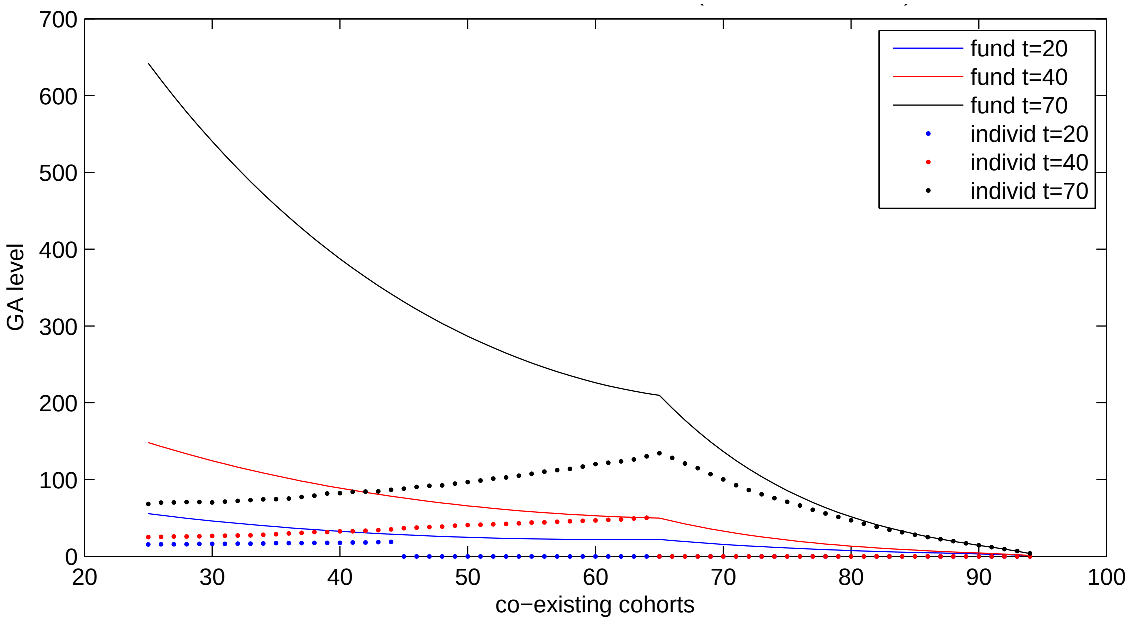
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: "normal"; Plan: DB-"hard"; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 27: Generation Account Value in Time for DB-“soft” Plan (no PSC)



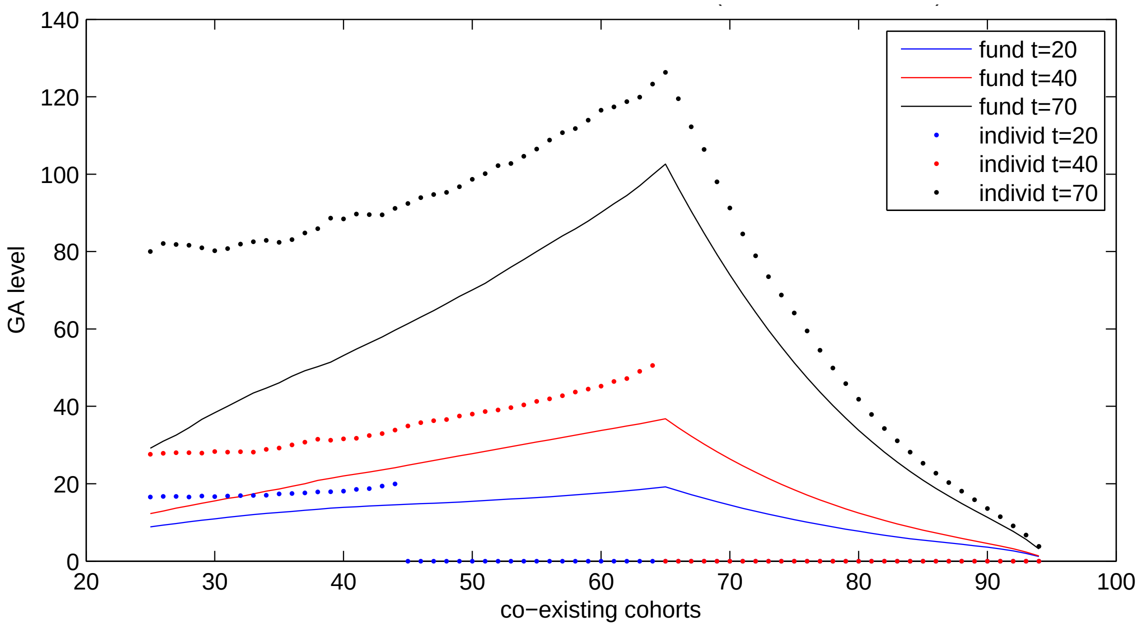
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “normal”; Plan: DB-“soft”; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 28: Generation Account Value in Time for CDC Plan (no PSC)



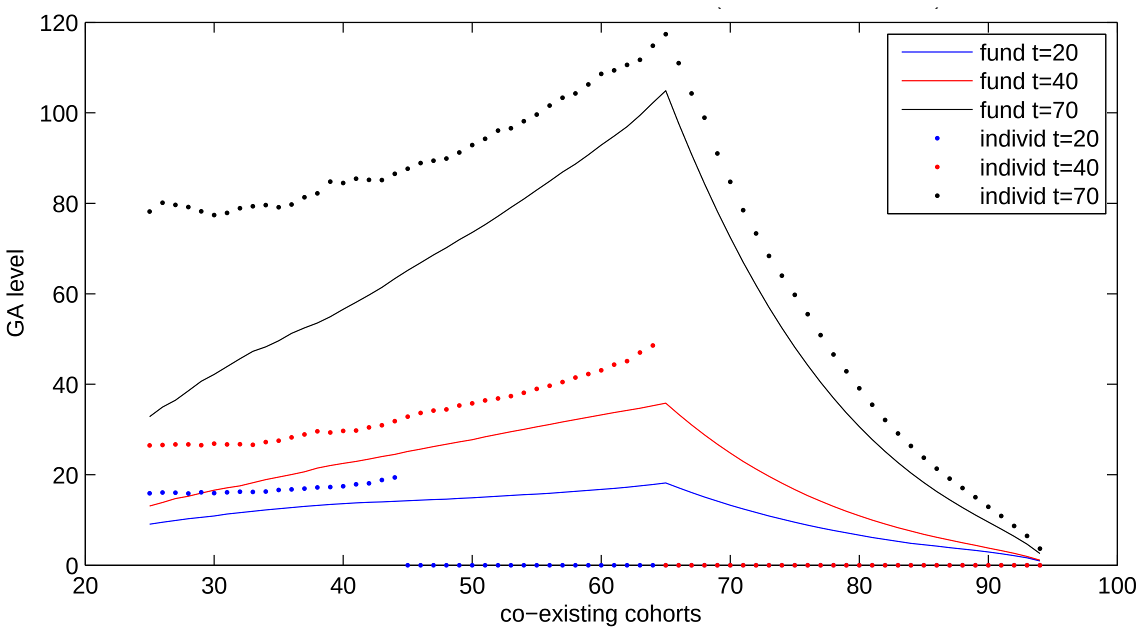
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “normal”; Plan: CDC; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 29: Generation Account Value in Time for DB-“hard” Plan (yes PSC)



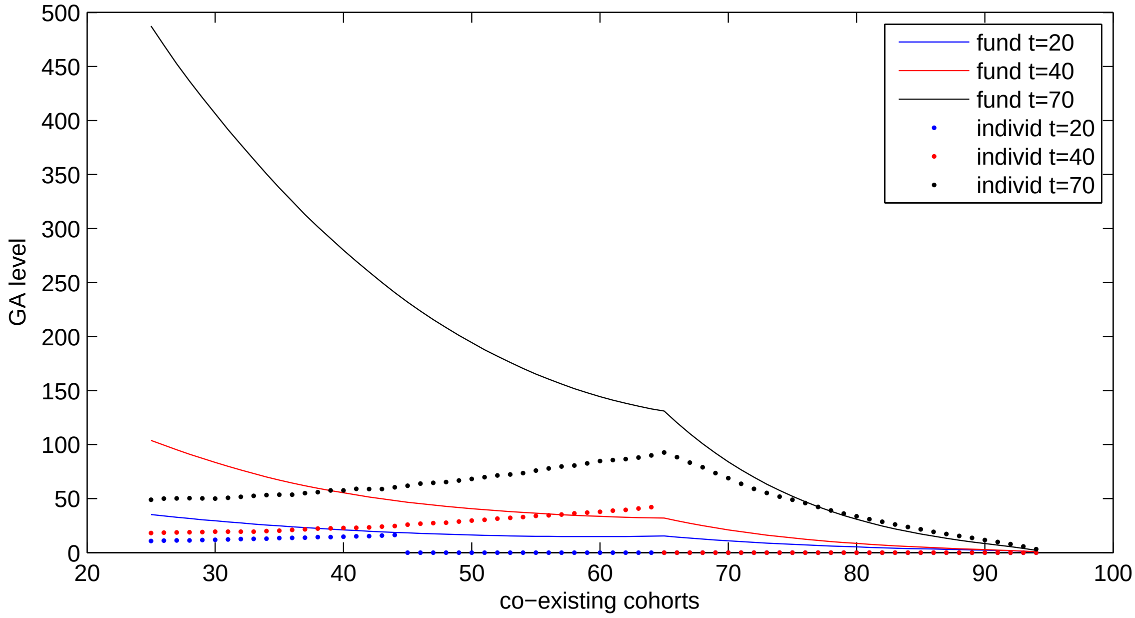
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “normal”; Plan: DB-“hard”; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 30: Generation Account Value in Time for DB-“soft” Plan (yes PSC)



Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “normal”; Plan: DB-“soft”; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.

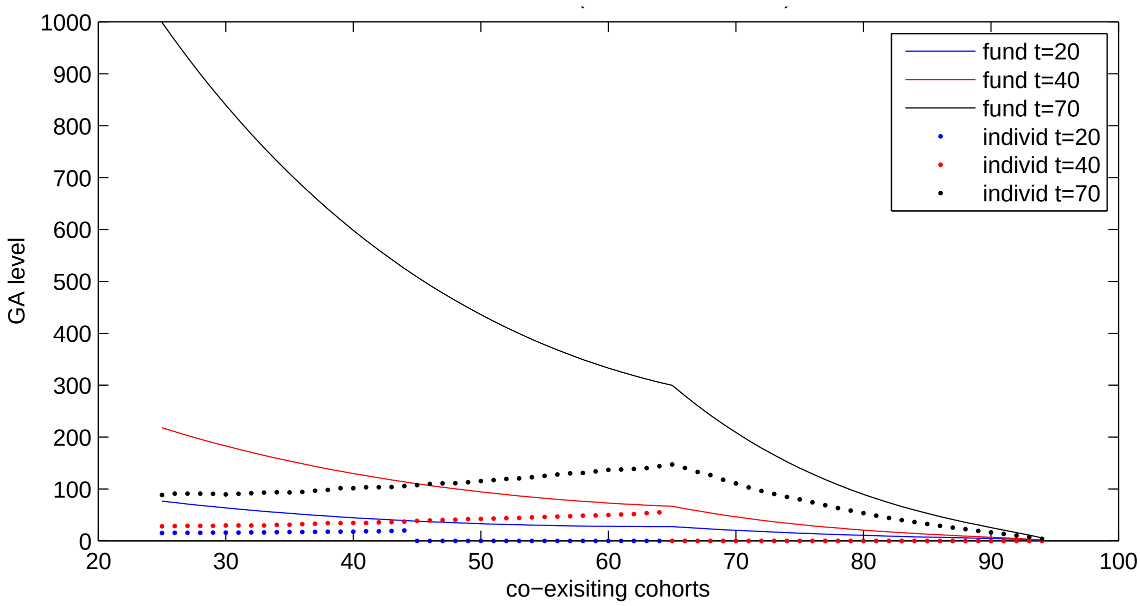
Figure 31: Generation Account Value in Time for CDC Plan (yes PSC)



Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: "normal"; Plan: CDC; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.

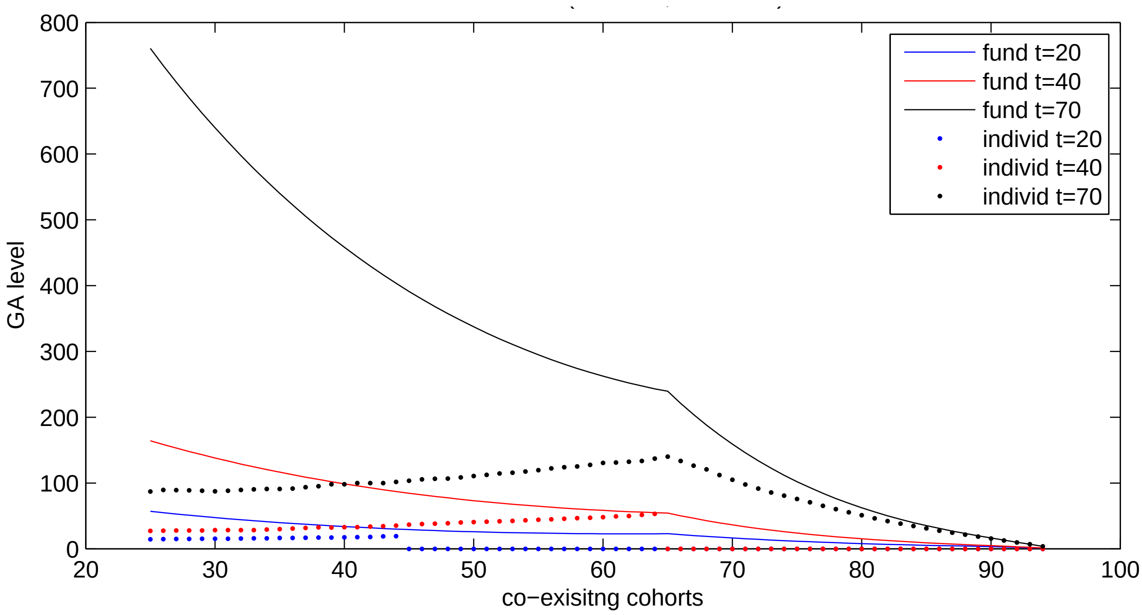
C : GA in “upward demographic shock” framework

Figure 32: Generation Account Value in Time for DB-“hard” Plan (no PSC)



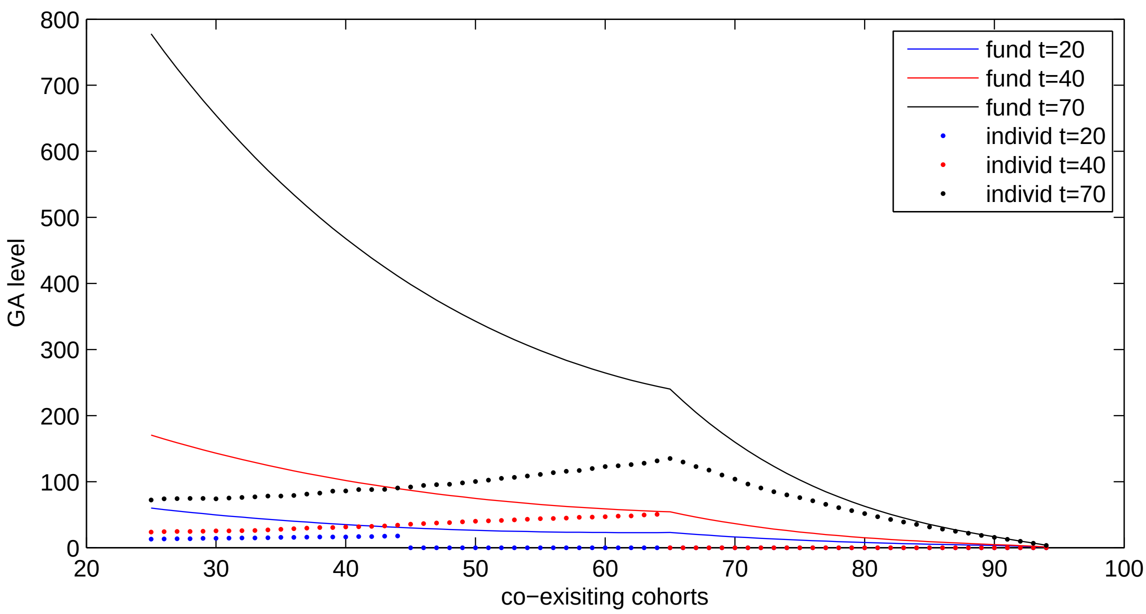
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “life-expectancy shock”; Plan: DB-“hard”; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 33: Generation Account Value in Time for DB-"soft" Plan (no PSC)



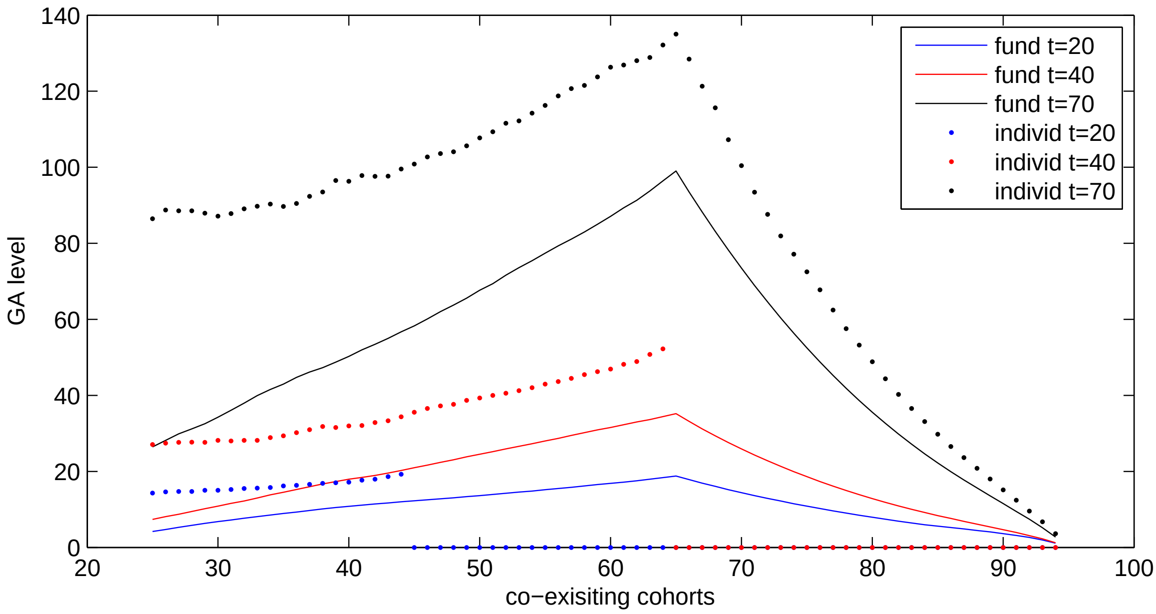
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: "life-expectancy shock"; Plan: DB-"soft"; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 34: Generation Account Value in Time for CDC Plan (no PSC)



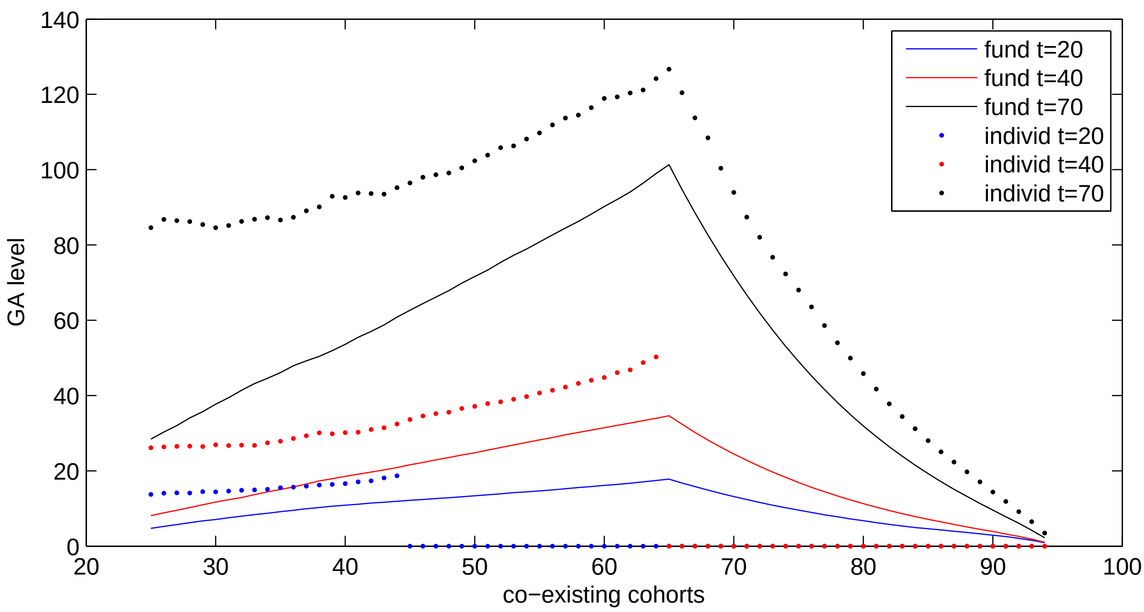
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: "life-expectancy shock"; Plan: CDC; Framework: no PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 35: Generation Account Value in Time for DB-“hard” Plan (yes PSC)



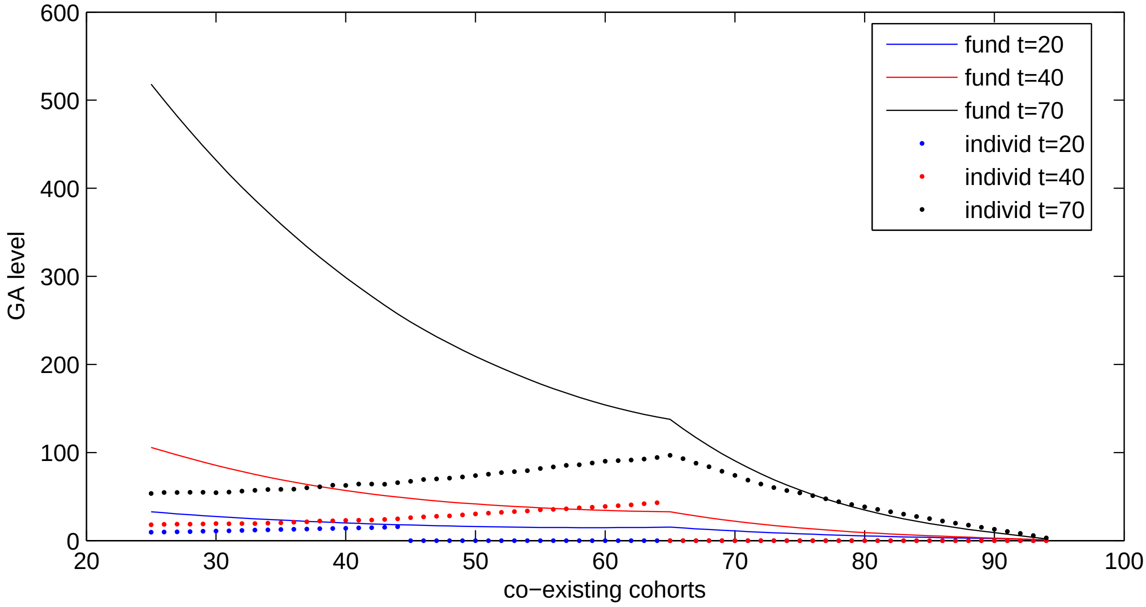
Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “life-expectancy shock”; Plan: DB-“hard”; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 36: Generation Account Value in Time for DB-“soft” Plan (yes PSC)



Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: “life-expectancy shock”; Plan: DB-“soft”; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.

Figure 37: Generation Account Value in Time for CDC Plan (yes PSC)



Source: GA of individual investment and fund participation in time for the co-existing cohorts; Period: "life-expectancy shock"; Plan: CDC; Framework: yes PSC; time: year 20, 40 and 70; Calculations by the author.