Life-cycle asset allocation and unemployment risk[*]

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Abstract

In this paper we extend the traditional life cycle model of saving and portfolio choice to allow for possible long-term unemployment spells to have permanent effects on subsequent labor income prospects. The risk of losing future labor income could imply strong human capital erosion for the investor at any age, dampening the incentive to invest in risky stocks. The resulting optimal portfolio share invested in stocks may be relatively flat in age, more in line with the available evidence and contrary to the predictions of traditional life-cycle models.

Keywords: Life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D91, E21, G11

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1 Introduction

In this paper, we contribute to the life-cycle theory of consumption and portfolio choice by investigating the role of permanent consequences of unemployment risk in shaping optimal investment in risky assets. The analysis is motivated by the recent rise in the average duration of unemployment spells in developed economies. For example, in the US the share of unemployed workers who are jobless for more than one year doubled over the recent Great recession episode, reaching 24% of total unemployment in 2014. Krueger, Cramer and Cho (2014) show that the re-employability of long-term unemployed progressively declines over time and that they are more likely to exit labor force. Recent evidence on the relationship between job openings and unemployment in the US shows that more openings do not lead to more employment among those who are jobless for more than six months, a pattern holding across all ages, industries and education levels (Ghayad and Dickens 2012). Moreover, emprical studies find that unemployment leads to large and persistent earnings losses that are increasing in unemployment duration due to skill deterioration, although the magnitude of this effect varies over time and across industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993; Davis and vonWacther 2011). On the whole, these findings suggest that long-term unemployment may become a trap often not supported by supplementary income provisions, given that unemployment benefits for those who are out of work longer than six months decline rapidly with unemployment duration.

In this paper, we extend a simple life-cycle model in order to account for the possibility of entering long-term unemployment and its possible permanent consequences on the worker's human capital. Related literature on life-cycle portfolio choices shows that households should reduce investments in risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The reason is that human capital can provide a hedge against shocks to stock returns, making financial risk bearing more attractive. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is relatively large relative to accumulated financial wealth. Then, human capital typically decreases relative to financial wealth over the life cycle, leading to a gradual reduction in stock investment till retirement. This model implication is embodied in the popular financial advice of a stock exposure gradually decreasing with age. Should this prescription be still valid when unemployment risk and its long-run consequences are properly taken into account?

Extensions of the standard life-cycle model assigning an explicit role to unemployment risk leave both the observed low stock market participation and age pattern of stock holding during working life largely unexplained (Cocco, Gomes and Maenhout 2005; Bremus and Kuzin, 2014). Some versions of the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labor income: in these models unemployment risk affects income only during the unemployment spell with no consequences on subsequent earnings ability. Bremus and Kuzin (2014), in a richer life-cycle setup, model unemployment persistence using a three-state Markov chain that allows for both short-term and long-term unemployment. Given that there is income reduction during unemployment but no permanent consequences on subsequent earnings ability, the stock holding is still counterfactually decreasing in age till retirement although, on average, lower than what obtained without unemployment risk.

Our paper contributes to the life-cycle literature by investigating how unemployment and its permanent consequences in terms of human capital erosion can shape optimal asset allocation. As in Bremus and Kuzin (2014) we model exogenous working life careers as a three-state Markov chain driving the transitions between the employment, short-term and long-term unemployment, calibrated to broadly match observed US labor market features. The empirical literature on labor market outcomes shows that unemployment leads to skill losses and human capital depreciation (Neal, 1995; Keane and Wolpin, 1997; Edin and Gustavsson, 2008) implying sizable permanent earning losses (Arulampalam et al., 2000; Arulampalam, 2001; Schmieder, von Wachter and Bender, 2013). Consequently, we allow for explicit human capital erosion during unemployment: when unemployed, individuals receive benefits but simultaneously experience a cut (proportional to unemployment duration) in the permanent component of labor income which captures diminished future income prospects.

Our results show that the risk of permanently losing labor income severely reduces the level of human capital at any age and considerably lowers the optimal portfolio share invested in stocks with respect to the case of no unemployment risk. Optimal stock investment is no longer decreasing with age but remains remarkably flat over the whole working life. These findings are broadly consistent with the joint empirical evidence about investment decisions and average unemployment duration across education groups.[1]

[1] In the US, stock investment appears to be positively correlated with the level of education which is inversely related to the average probability of being unemployed. In particular, according to Current Population Survey data from the US Bureau of Labor Statistics, the average unemployment rate among college graduates was 2% in 2014, while it was 6% and 9% for high

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 3 and discuss results in Section 4. Section 5 concludes the paper.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t. The investor starts working at age t_0 and retires with certainty at age t_0+K. Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=0}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j})\,b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right]$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs in case of death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter. Following Cocco, Gomes and Maenhout (2005), we do not model labour supply decisions, whereby ignoring the insurance property of flexible work effort allowing investors to compensate for bad financial returns with higher labour income, as in Gomes, Kotlikoff and Viceira (2008).

2.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labor supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term (u_1) and long-term unemployment (u_2). Unemployment may be short-term and last only one year or it may become long term and last two years. Individual labor market dynamics

school and less than high school educated workers respectively.

are driven by to the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix}$$

where $\pi_{ij} = \text{Prob}\,(s_{t+1} = j|s_t = i)$ with $i, j = e, u_1, u_2$. If the worker is employed at t $(s_t = e)$, she continues the employment spell at $t+1$ $(s_{t+1} = e)$ with probability π_{ee}, otherwise she enters short-term unemployment $(s_{t+1} = u_1)$ with probability π_{eu_1}. Since to become long-term unemployed she must first experience short-term unemployment, we set the probability for the employed to enter long-term unemployment at zero, $\pi_{eu_2} = 0)$. If the worker is short-term unemployed at t $(s_t = u_1)$, then she exits unemployment $(s_{t+1} = e)$ with probability π_{u_1e} or she becomes long-term unemployed $(s_{t+1} = u_2)$ with probability $\pi_{u_1u_2}$; consequently we set $\pi_{u_1u_1} = 0$. Finally, if she's long-term unemployed at t $(s_t = u_2)$, since long-term unemployment lasts only two years, she is re-employed $(s_{t+1} = e)$ with certainty, thus $\pi_{u_2e} = 1$ and $\pi_{u_2u_1} = \pi_{u_2u_2} = 0$.

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labor income given by the following process:

$$Y_{it} = H_{it}N_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{1}$$

where $H_{it} = (F\,(t, \mathbf{Z}_{it})\,P_{it})$ represents the permanent income component. In particular, $F\,(t, \mathbf{Z}_{it}) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics $(\mathbf{Z}_{it})$ such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$\log P_{it} = \log P_{it-1} + \omega_{it} \tag{2}$$

where ω_{it} is distributed as $N(0, \sigma^2_\omega)$. N_{it} denotes the transitory stochastic component and $\log(N_{it})$ is distributed as $N(0, \sigma^2_\varepsilon)$ and uncorrelated with ω_{it}.

In our set-up, differently from Bremus and Kuzin (2014), labor income received by the employed individual at time t depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labor income H. Since the empirical evidence suggests that the longer the unemployment spell the larger the worker's human capital depreciation

(Schmieder, von Wachter and Bender, 2013), we let human capital erosion increase with unemployment duration. Thus, for the worker who at t is re-employed after one-year unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and the worker who is re-employed at t after a two-year unemployment spell experiences a reduction of the permanent income earned when enetring unemployment, H_{it-2}, by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. Then, the permanent component of labor income H_{it} evolves according to:

$$H_{it} = \begin{cases} H_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-2} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{3}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income $H_{it-1} = F_{it-1}P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available: $\xi_2 = 0$. Thus, income received during unemployment is:

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\ 0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e \end{cases} \qquad t = t_0, ..., t_0 + K \tag{4}$$

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labor income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+K}}\right) P_{it_{0+K}} \qquad t_0 + K < t \leq T \tag{5}$$

where the level of the replacement rate λ is meant to capture at least some of the features of Social Security systems. Other, less restrictive, modelling strategies are possible. For example, Campbell, Cocco, Gomes and Maenhout (2001) model a system of mandatory saving for retirement as a given fraction of the (stochastic) labour income that the investor must save for retirement and invest in the riskless asset, with no possibility of consuming it or borrowing against it;[2] at retirement, the value of the wealth so accumulated is transformed into a riskless annuity until death.

[2] Koijen, Nijman and Werker (2011) argue that these mechanisms are suboptimal relative to alternative annuity designs, despite their diffusion across pension systems.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{6}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2005) and Koijen, Nijman and Werker (2010).

At the beginning of each period, financial resources available for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, that we call *cash on hand* $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{7}$$

where R_{it}^P is the portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s) R^f \tag{8}$$

with α_{it}^s and $(1 - \alpha_{it}^s)$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then two financial assets are $B_{it} \geq 0$, $S_{it} \geq 0$, respectively for the riskless asset and stocks, are non negative in each period. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

2.3 Solving the life-cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right. \right. \right.$$
$$\left. \left. \left. + \left(1-p_{t_0+j}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{9}$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right) \left(\alpha_{it}^s R_t^s + \left(1-\alpha_{it}^s\right) R^f \right) + Y_{it+1} \tag{10}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t \left[p_t V_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right. \right.$$
$$\left. \left. + \left(1-p_t\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{11}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state $s_{it}(= e,\ u_1,\ u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit, as:

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} \right.$$
$$+ \beta \left[p_t \sum_{s_{it+1}=e,\,\mathrm{u}_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \widetilde{E_t V}_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right.$$
$$\left. \left. + \left(1-p_t\right) b \sum_{s_{it+1}=e,\,\mathrm{u}_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{12}$$

where $\widetilde{E_t V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and υ_{t+1}. The history dependence that we introduce in our set-up by making unemployment to affect subsequent labor income prospects prevents us to rely on the standard normalization of the problem with respect to the level of P_{it}. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible labor market state (dropping the term involving the bequest motive) as:

$$V(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V(X_{it+1}, P_{it+1}, e) & \text{with prob. } \pi_{e,e} \\ \text{with } P_{it+1} = P_{it} e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V(X_{it+1}, P_{it+1}, u_1) & \text{with prob. } \pi_{e,u_1} \\ \text{with } P_{it+1} = (1 - \Psi_1) P_{it} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_1 F_{it} P_{it} \end{cases} \end{cases}$$

$$V(x_{it}, P_{it}, u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V(X_{it+1}, P_{it+1}, e) & \text{with prob. } \pi_{u_1,e} \\ \text{with } P_{it+1} = (1 - \Psi_1) P_{it-1}\, e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-1} (1 - \Psi_1) P_{it-1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V(x_{it+1}, P_{it+1}, u_2) & \text{with prob. } \pi_{u_1,u_2} \\ \text{with } P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} \end{cases} \end{cases}$$

$$V(X_{it}, P_{it}, u_2) = u(X_{it}) + \beta p_t \begin{cases} V(X_{it+1}, P_{it+1}, e) & \text{with prob. } \pi_{u_2,e} \\ \text{with } P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-2} (1 - \Psi_2)(1 - \Psi_1) P_{it-2} e^{\omega_{it+1}} e^{\varepsilon_{it+1}} \end{cases}$$

This problem has no closed form solution: hence the optimal values for consumption and portfolio shares depending on the values of each state variable at each point in time are obtained by means of numerical techniques. To this aim, we apply the standard backward induction procedure starting form the last possible period of life T. The optimal consumption and portfolio share policy rules are obtained for each possible value of the continuous state variables (X_{it} and

P_{it}) using the standard grid search method.[3] Going backwards, for every period $t = T-1, T-2, ..., t_0$, the Bellman equation (12) is used to obtain the optimal rules for consumption and portfolio shares.

3 Calibration

Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. For reference, we solve the model also abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005); this benchmark scenario is referred to as *Calibration 1.*

The investor begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health Statistics.* As regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008), capturing an intermediate degree of risk aversion, though Cocco, Gomes and Maenhout (2005) and Bremus and Kuzin (2014) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with expected stock premium μ^s fixed at 0.04 . The standard deviation of the returns innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$).

The labor income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). After retirement, income is a constant proportion λ of the final (permanent) labour income, with $\lambda = 0.65$. In the benchmark case, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are $\sigma^2_\omega = 0.0106$ and $\sigma^2_\varepsilon = 0.0738$. In solving the model with unemployment risk,

[3] The problem is solved over a grid of values covering the space of the state variables and the controls in order to ensure that the obtained solution is a global optimum.

the value of all parameters assumed above is maintained. The chosen transition probabilities between the three labor market states broadly reflect the transition rates between employment and the two unemployment states observed on average among US workers:[4]

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.95 & 0 & 0.05 \\ 1 & 0 & 0 \end{pmatrix}$$

The assumed transition matrix yields rather conservative values for the unconditional probabilities of being short-run unemployed (3.84%) and long-run unemployed (0.16%). We set the unemployment benefit replacement rate ξ_1 at the average level observed for the US. In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks,[5] we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0$ for the long-term unemployed.

A well established empirical literature on job displacement shows that job losses affect earnings far beyond the unemploment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement, may range from 30% (Couch and Placzek, 2010) to 40% (Jacobson, Lalond and Sullivan,1993). These earnings losses are shown to be persistent in a range of about 25% (Jacobson, Lalond and Sullivan, 1993) and 15% (Couch and Placzek, 2010) of the pre-displacement earnings. Moreover, these estimates abstract from the effect of the duration of unemployment following job losses, while Cooper (2013) finds, instead, that earning losses are larger the longer the unemployment duration. Thus, to study long-run effects of unemployment on optimal asset allocation, we consider a baseline calibration of the model with unemployment risk (*Calibration 2*), in which the permanent proportion of human capital erosion Ψ following unemployment is equal to 20% and 40% in case of short- term and long-term unemployment spells, respectively. The empirical evidence discussed above refers to workers who are soon re-employed. However, various studies indicate that workers who have been out of the labor force for a long time are less likely to find a job (Krueger, Cramer and Cho, 2014), which

[4] The transition matrix reports the annual transition probabilities obtained by annualizing U.S. average quarterly transitions.

[5] No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.

implies a more substantial human capital erosion. Since the aim of the present paper is to investigate the role of human capital erosion following long-term unemployment experience and given the documented severe shrinkage in employability due to job career interruptions, we allow for a more extreme values of Ψ_2, up to 80% (*Calibration 3*).

4 Results

4.1 Optimal policies

In Figure 1 optimal stock shares are shown fom the benchmark model without unemployment risk and the standard life-cycle result obtains. In particular, the figure plots the optimal stock share as a function of cash on hand for a medium level of the permanent labor income component. For significantly positive levels of the permanent component, labor income acts as an implicit asset and affects the optimal portfolio composition depending on investor's age and wealth. Under the considered standard calibration, labor income, though uncertain, is treated like a risk-free asset. At age 20, the sizable implicit holding of the risk free asset (through human capital) makes it optimal for the less wealthy investors to tilt their portfolio towards the risky financial assets. Indeed, for a wide range of levels of wealth, optimal stock investment is 100%. The optimal stock holding decreases in financial wealth to counterbalance the relatively lower implicit investment in (less risky) human capital.

Figures 2 and 3 display policy functions obtained from our model extended to account for unemployment risk. In this paper, the focus is on the consequences of unemployment in terms of labor income prospects after experiencing job loss. Job losses imply a cut in income during the unemployment spell when the individual receive only a relatively small benefit. We model the most common unemployment insurance scheme in the US, where the average unemployment benefit is 30% of the last wage during the first year of unemployment and zero afterwards. Our results are derived assuming replacement rates $\xi_1 = 0.3$ for short-term and $\xi_2 = 0$ for long-term unemployed.

In addition, our model accounts for the fact that unemployment may have severe effects in terms of individual skill erosion and thus on labor income prospects at re-employment. In the present set-up unemployment induces human capital erosion implying proportional cuts in individual's permanent labor income, with the reduction following a two-year unemployment spell (Ψ_2) being higher than that occurring in case of one-year unemployment (Ψ_1). In Figure 2, we assume *Calibration 2* which implies a moderate human capital depreciation due to job loss, with $\Psi_1 = 0.2$ and $\Psi_2 = 0.4$. Given that the permanent effects of job loss are relatively moderate, optimal asset allocation is not remarkably different from the benchmark case that ignores unemployment risk.

The recent Great Recession spurred attention on the long -term unemployed and on their subsequent labor market prospects. In particular, Krueger, Cramer and Cho (2014) document that the re-employability of long-term unemployed progressively declines over time as well as that they are more likely to exit labor force. In *Calibration* 3 we account for such an extreme consequence of long term unemployment assuming a strong human capital depreciation following a two year unemployment spell. In particular, Ψ_1 is kept at 0.2 and Ψ_2 is increased up to 0.9, implying a 90% erosion of the individual permanent labor income component after the second unemployment year. In Figure 3 the resulting policy functions are shifted abruptly leftwards. The optimal stock share is still decreasing in financial wealth but 100% stock investment is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the benchmark case of no unemployment risk.

4.2 Life Cycle Profiles

On the basis of optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 4 shows the average optimal stock shares plotted against age derived when unemployment risk is ignored (dotted line) and for the case in which it is accounted for (dashed and solid lines).

In case of no unemployment risk, the well known result on the age profile of optimal stock portfolio shares obtains. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and declines with age till retirement.

In case of unemployment risk, with moderate human capital erosion (*Calibration* 2) the optimal share of stocks in the portfolio still declines with age, though being lower at all ages, with a 100% optimal stock share only for very young investors. However, when long-term unemployment implies large skill erosion, as in *Calibration 3*, the optimal stock investment is almost flat and reduced at any age. As seen in the previous section, the risk of permanently losing a substantial portion of future labor income prospects reduces the level of human capital at all ages and increses its riskiness inducing a relatively lower optimal stock investment conditional on financial wealth at all ages. Moreover, the large amount of background risk increases precautionary savings and thus wealth accumulation over time implying less need to tilt the asset allocation towards stocks at any age. Consequently, the age profiles remains remarkably flat overall the working life and during retirement[6]. These results are at odds with previous studies that model unemployment risk. Among them, Bremus and Kuzin (2014) study the effects of unemployment persistence on asset allocation. In their model, the unemployment length affects income only during job loss. Thus, their results on optimal stock investing derived under a standard calibration, namely with moderate risk aversion equal to 5, are indistinguishable from those obtained in a standard life cycle model that disregards unemployment risk at all. Our results show instead that allowing for possible long run consequences of unemployment may significantly dampen the optimal incentive to invest in stocks, even under standard calibrations.

5 Conclusions

Under standard calibrations of the life-cycle model with moderate risk aversion, adding unemployment risk as the possibility of transitory disastrous labor income realizations due to job loss has no significant effect on optimal asset allocation decisions. In this paper we model long-term unemployment risk and its possible permanent consequences in term of human capital depreciation. We show that an even small probability of experiencing high human capital erosion due to unemployment is able to generate optimal conditional stock shares more in line with those observed in the data. In particular, the risk of permanently losing a large fraction of permanent income in case of long-term unemployment implies that investors face a large uncertainty concerning future incomes and social security pension levels which lowers the value of human capital endowment and increases its riskiness, making investors less willing to take on equity market risk at all ages.

[6] The relatively low investment in stocks during retirement is due to the presence of positive bequest motive, common to all parametrizations considered in the paper.

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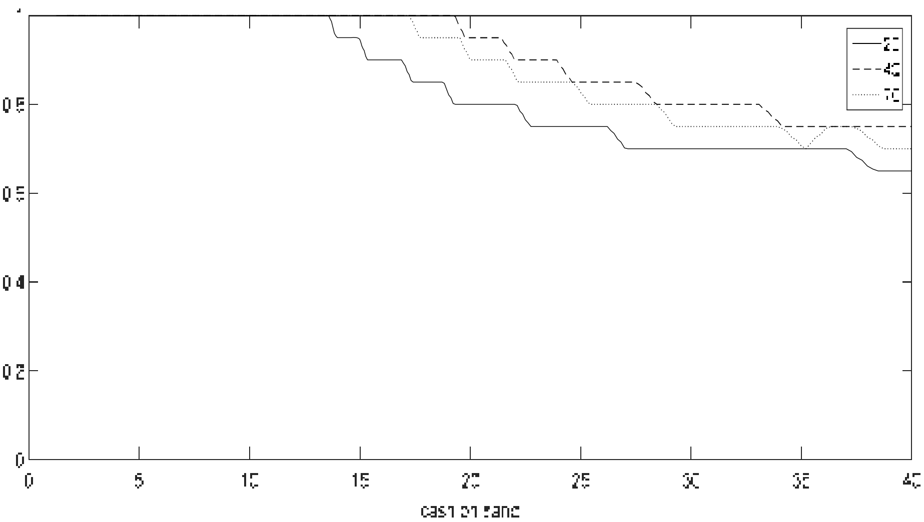
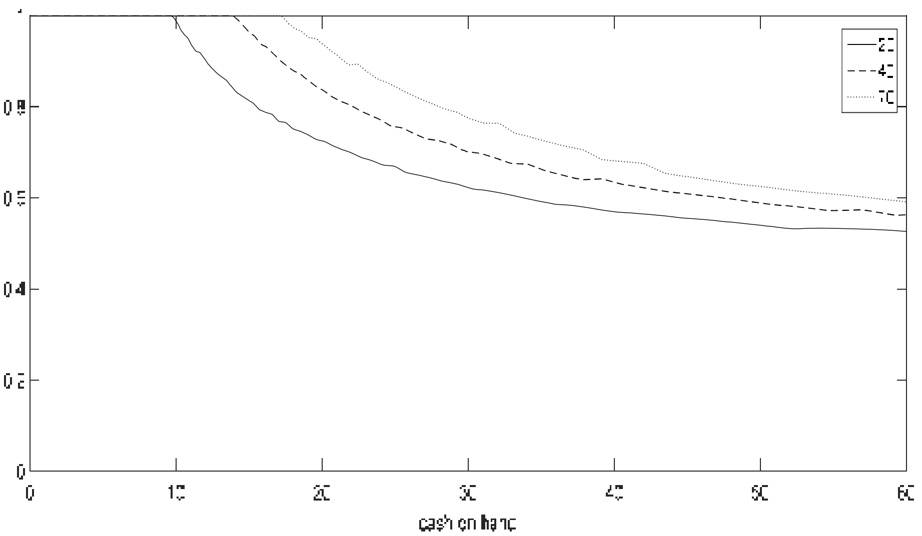
Figure 1 Policy functions: baseline case without unemployment risk



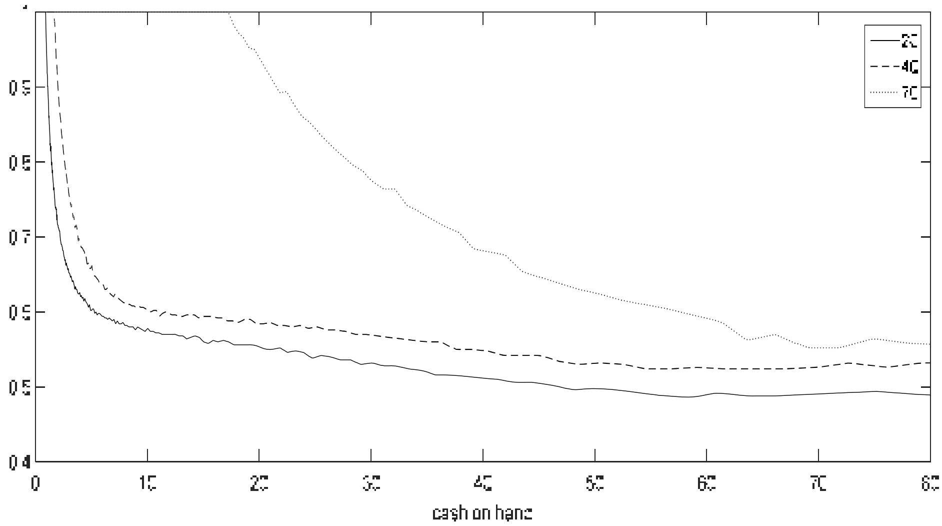
The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in the baseline case, without unemployment risk. The policies are plotted for selected ages 20, 40, and 70.

Figure 2 Policy function: with unemployment risk, low human capital depreciation



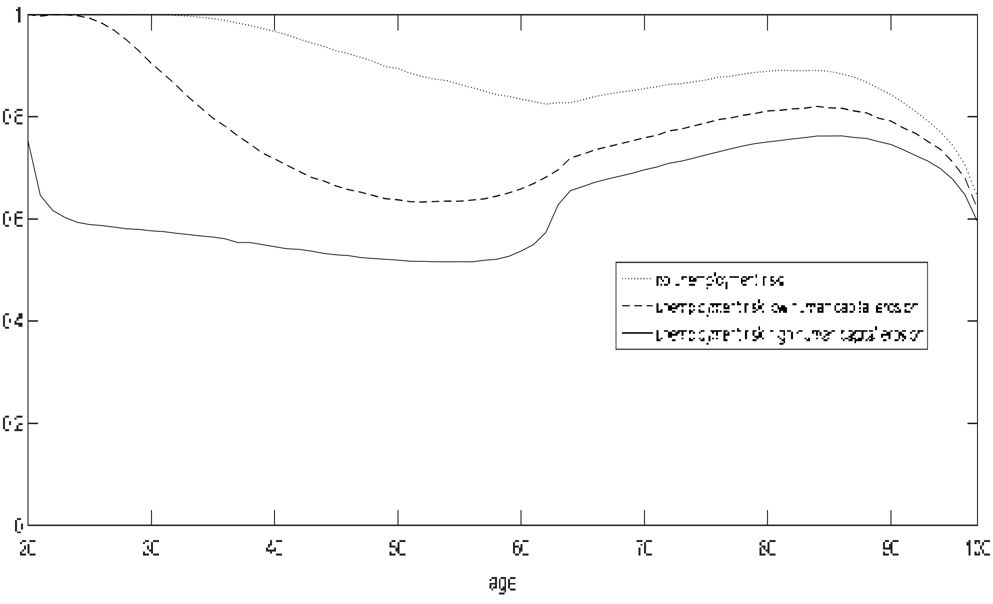
The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in case of unemployment risk. The parameters governing the human capital erosion during short-term and long-term unemployment spells are equal to 0.2 (Ψ_1) and 0.4 (Ψ_2), respectively. The policies are plotted for selected ages: 20, 40, and 70.

Figure 3 Policy functions: with unemployment risk, high human capital depreciation



The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in case of unemployment risk. The parameters governing the human capital erosion during short-term and long-term unemployment spells are equal to 0.2 (Ψ_1) and 0.9 (Ψ_2), respectively. The policies are plotted for selected ages: 20, 40, and 70.

Figure 4 Life cycle profiles



The figure displays the mean simulated stock profiles for individuals of age 20 to 100. Risk aversion ρ=5, social security replacement ratio λ=0.65.Three cases are considered. *Calibration 1* the benchmark model without unemployment risk. *Calibration 2* with unemployment risk and moderate human capital erosion: (Ψ_1=0.2) and (Ψ_2=0.4). *Calibration 3* with unemployment risk and strong human capital erosion: (Ψ_1=0.2) and (Ψ_2=0.9).