Labor supply, savings and consumption inequality under different pension benefit arrangements

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Abstract

We analyze the trade-off among insurance, labor supply and savings incentives arising in the design of pay-as-you-go (PAYG) pension benefits. We consider two extreme types of pension benefits: i) a flat benefit (FB) system that pays the same pension regardless of the amount of previous contributions and ii) a notional defined contribution (NDC) system in which benefits are perfectly linked to previous contributions. The FB system promotes lower labor supply, lower consumption inequality and generally crowds out capital less. If the level of pension contributions is low, the FB system brings a higher welfare due to higher consumption insurance. At higher levels of contributions, the NDC system leads to higher welfare due to lower labor supply distortions. General equilibrium effects tilt the welfare result in favor of FB pensions in dynamically efficient economies. The analysis suggests that pension benefit design should depend on the size of the pension system and the size of idiosyncratic risk. The welfare results can explain the contrasting reforms of pension benefits implemented in different countries, as well as the empirical correlations between pension benefit progressivity, on the one hand, and the size of pension systems and idiosyncratic risk, on the other hand.

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1 Introduction

Most of the reforms of pay-as-you-go (PAYG) pension systems enacted in the past 20 years involved changes in the relation between pension contributions and pension benefits. Countries like Germany, Italy, France and Poland switched to a Notional Defined Contribution (NDC) pension system or a points system in which pension benefits are perfectly linked to contributions paid. One of the main aims of this reform was to improve labor supply incentives of agents. In sharp contrast to this type of reform, we have the example of the UK pension system reform that eliminated the earnings related part of the pension system, keeping only the flat benefit (FB) pension.

In this paper, we show that these opposing choices for pension benefits arrangements can be explained from a welfare point of view if we take into consideration that economies differ with respect to the size of idiosyncratic income risk and the level of pension contributions. We also determine the implications of the progressivity of pension benefits for labor supply, savings and consumption inequality. The novelty of our paper resides in showing that the impact of the different macroeconomic and inequality outcomes on the ex-ante utility of agents depends on the the level of pension contribution and the size of the idiosyncratic risk prevailing in the economy.

The choice of the pension benefit arrangement implies a trade-off among consumption insurance, labor supply incentives and savings incentives. We study this trade-off in a general equilibrium overlapping generations model with incomplete markets, endogeneous retirement and a PAYG pension system. In order to build an analytically tractable model, we assume that agents live only two periods. In the first period of their life agents supply labor inelastically, pay the contribution to the PAYG pension system and save in risk free capital. In the second period of their life, agents are hit by idiosyncratic productivity shocks, chose when to retire and receive the pension benefit.

We compare the steady states of two economies that differ only regarding the way pension benefits are linked to life-time contributions: i) an economy with a flat benefit (FB) pension system and ii) an economy with an NDC pension system. Because the contribution to the NDC system is nondistortionary for labour supply, agents work longer than under the FB system. Hence, aggregate labour is higher in the NDC system. However, consumption inequality is also higher in the NDC system, because pension benefits are perfectly linked with the agents' life-time labor income.

We show that the impact of the pension benefit arrangement on consumption inequality and labor supply determines the relative welfare of the two pension systems, if we abstract from general equilibrium effects working through prices and the government budget constraint. The FB system brings a higher welfare than the NDC system at low levels of pension contributions due to the consumption insurance it provides. As the size of the pension contribution increases, the welfare losses from labor supply distortions start to dominate the welfare gains from consumption insurance and the NDC system eventually provides a higher welfare. The threshold contribution up to which the FB system brings a higher welfare increases with the size of the idiosyncratic productivity risk.

The impact of the pension benefit arrangement on savings is not straightforward as it depends on two opposing factors. On the one hand, because people work longer under the NDC pension system, they need to save less for old age consumption. On the other hand, due to the existence of idiosyncratic productivity shocks, agents make precautionary savings. The level of precautionary savings is higher in the NDC pension system because this type of pension system does not offer consumption insurance. We show that the former effect dominates the latter if labor supply is relatively elastic. If labor supply is relatively inelastic, the NDC pension system still crowds out capital accumulation more than the FB pension system as long as the level of the idiosyncratic risk is not very high.

Hence, an important finding of the paper is that the strengthening of work incentives in the NDC pension system can come at the expense of lower savings. This outcome impacts the steady state welfare through general equilibrium effects. In dynamically efficient economies, the fact that the FB pension system crowds out capital formation less than the NDC pension system has a less negative impact on welfare through the effect on prices (wage and interest rate). The general equilibrium effects reinforce the welfare gains from consumption insurance of the FB pension system at low levels of contributions. As the level of contribution increases, compared to the NDC pension system, the FB pension system brings higher welfare losses due to labor supply distortions but lower welfare losses through general equilibrium effects. Which of the two effects dominates is a quantitative question. A calibrated version of the model shows that general equilibrium effects dominate and the FB pension system brings higher welfare at any level of pension contributions.

In dynamically inefficient economies, the fact that the FB system crowds out capital less than the NDC system leads to a smaller positive impact on welfare through prices. A calibrated version of the model shows that, for realistic levels of idiosyncratic productivity risk, the FB pension system still brings a higher welfare than the NDC at low levels of pension contributions. However, as the size of pension contribution increases the NDC pension system becomes better from a welfare point of view.

We confront the findings in our model with cross-country OECD data. We document: i) a negative relationship between the progressivity of pension benefits[1] and the size of the pension system proxied by the replacement rate and ii) a positive relation between disposable income inequality of working age agents and the progressivity of pension benefits. The data agrees with the welfare analysis presented in the paper: less progressive pension systems such as the NDC pension system are implemented in countries where the level of the pension system is high or the size of productivity risk is low.

[1]The FB pension system is very progressive, while the NDC pension system entails no pension benefit progressivity.

The present paper considers both the benefits of PAYG pension systems stemming from better insurance in economies with incomplete markets and the costs of labor supply and savings distortions. Storesletten et al. (2004) show that the US pension system reduces consumption variance with 20% due to the particular, convex shaped link between pension contributions and pension benefits. Imrohoroglu et al. (1995) obtain that, due to its consumption insurance property, a PAYG with a replacement rate of 30% is welfare improving in the US. However, these papers do not consider the labor supply distortions of PAYG pension contributions. Gruber and Wise (1998), Erosa et al. (2012), Wallenius (2013), Alonso-Ortiz (2014), Bagchi (2015) show that pension systems adversely impact on the economy by reducing the labor supply of agents.

Our paper is also closely related to the literature analyzing pension reforms in economies with incomplete markets. Nishiyama and Smetters (2007) analyze the consequences of a 50% privatization of the US pension system. They conclude that this is welfare decreasing because of the lower insurance of idiosyncratic productivity shocks. However, if the privatization is accompanied by an increase in the progressivity of pension benefits and is financed by a consumption tax, then it can produce efficiency gains. Also focusing on the US pension system, Huggett and Parra (2010) determine the pension benefit function that is optimal from an ex-ante point of view. They consider benefit functions that are constant, linear or quadratic with respect to life-time income. The results show that the optimal benefit function entails more progressivity than the one currently in place. Using a model calibrated on German data, Fehr and Habermann (2008) show that, if pension contributions are a constant proportion of income, a FB pension system is optimal. The highest welfare efficiency gains are obtained by setting a 0% pension contribution for income up to 30% of average income and splitting the contributions equally between a FB and a NDC pension system.

We depart from this literature on pension reforms in a number of ways. First, we explicitly analyse the impact of the pension benefit arrangement on labor supply, savings and consumption inequality. Second, we develop a very tractable model that can be used to perform extensive cross country policy analyses. Finally, we compare the ex-ante utility of FB and NDC pension benefit arrangements for different levels of pension contributions and different sizes of the idiosyncratic productivity shock. In this way, we can explain the cross country heterogeneity in pension benefit arrangements.

The setup of our model is related to Harenberg and Ludwig (2015). Unlike them, however, we consider the retirement decision of agents and abstract from aggregate shocks. The most important difference is that we also consider a NDC pension system and study its macroeconomic and welfare implications in comparison with the FB pension system.

The rest of the paper is structured in the following way. Section 2 presents the model, while Section 3 compares the steady states of the economies with a FB and a NDC pension system, respectively. Section 4 illustrates the welfare implications of the two pension benefit arrangements in calibrated versions of the model. We analyze the correlation between pension progressivity, on one hand, and the size of the pension system and the size of the idiosyncratic productivity risk, on the other hand, using cross-country OECD data in section 5. Section 6 concludes the paper. All the proofs are presented in the Appendix.

2 The model

We consider an economy with two overlapping generations: young (y) and old (o). There are N^y young agents and N^o old agents. Population grows at rate $n = \frac{N^o}{N^y} - 1$. Young agents can be thought of as the group of prime-aged workers (25-55 years) who decide how much to consume (c^y) and save (a). They supply work inelastically. Old agents can be considered as the group of old workers (55-85 years old). They will decide how much to consume (c^o) and work (l^o), but they will not save. We assume that agents face no uncertainty regarding the length of their life and that they die with no assets.

Agents have the same productivity z when young. When old, however, they are hit by an idiosyncratic productivity shock z_i distributed according to the c.d.f. $F(z, \sigma^2)$. At this point, we make no assumption regarding $F(z, \sigma^2)$.

2.1 Households

Agents decide how much to consume, save and work maximizing their expected utility.

$$\max_{c_t^y, c_{i,t+1}^o, l_{i,t+1}^o, a_t} \quad u(c_t^y) + \beta E u(c_{t+1}^o, l_{t+1}^o)$$

$$c_t^y + a_t = w_t z(1 - \tau_t)$$

$$c_{i,t+1}^o = a_t(1 + r_{t+1}) + w_{t+1} z_i l_{i,t+1}^o (1 - \tau_{t+1}) + ben_{t+1}$$

$$l_{t+1}^o \in [0, 1)$$

where i represents the type of the agent, defined by the idiosyncratic shock z_i received when old.

We denote by w_t the wage per unit of efficiency and by r_t the interest rate at time t. The contribution to the pension system is τ_t. The pension benefit is represented by ben_t. We assume that the pension benefit can depend on the contributions paid by agents and, hence, on the labour supplied by agents[2].

[2] In means tested pension systems, the pension benefit can also depend on the size of an agent's savings, hence distorting the intertemporal consumption-savings decision. In the present paper we abstract from this possibility.

We consider a form for life-time utility that is consistent with a balanced growth path:

$$u(c^y) + \beta E u(c^o, l^o) = \begin{cases} \frac{(c^y)^{\theta(1-\sigma)}}{1-\sigma} + \beta E \frac{((c^o)^{\theta}(1-l^o)^{1-\theta})^{1-\sigma}}{1-\sigma} & \text{, if } \sigma \neq 1 \\ \theta ln(c^y) + \beta E(\theta ln(c^o) + (1-\theta) ln(1-l^o)) & \text{, if } \sigma = 1 \end{cases} \quad (1)$$

The first order conditions of the household's problem are:

$$\frac{u_l(c^o_{i,t+1}, l^o_{i,t+1})}{u_c(c^o_{i,t+1}, l^o_{i,t+1})} = -\left(w_{t+1} z_i (1 - \tau_{t+1}) + \frac{\partial ben_{t+1}}{\partial l^o_{i,t+1}} \right) \quad (2)$$

$$u_c(c^y_t) = \beta(1 + r_{t+1}) E[u_c(c^o_{t+1}, l^o_{t+1})] \quad (3)$$

If pension benefits are linked to pension contributions, a higher labor supply when old increases pension benefits, i.e. $\frac{\partial ben_{t+1}}{\partial l^o_{i,t+1}} > 0$. Relation (2) shows that such a pension system lowers the intratemporal distortion between consumption and labor supply.

2.2 The pension system

The government runs a PAYG pension system. The contributions to the pension system come from a proportional tax on labor income τ_t. We consider two different pension benefits arrangements:

1. A flat benefit (FB) or Beveridge pension system. In this system, the benefit received from the pension system does not depend on the previous contributions of the agent. We normalize the flat benefit by expressing it as a proportion of the average wage:

$$ben^{FB}_t = b_t w_t z$$

   In this type of pension benefit arrangement, the intratemporal consumption-leisure choice of equation (2) becomes:

$$\frac{u_l(c^o_{i,t+1}, l^o_{i,t+1})}{u_c(c^o_{i,t+1}, l^o_{i,t+1})} = -w_{t+1} z_i (1 - \tau_{t+1}) \quad (4)$$

2. A Notional Defined Contribution (NDC) system. In this type of pension system, the benefits are proportional to the contributions made throughout the lifetime:

$$ben^{NDC}_{i,t} = w_{t-1} z \tau_{t-1}(1 + r^P_t) + w_t z_i l^o_{i,t} \tau_t$$

   where r^P_t is the rate of return of the pension system given by the wage bill growth rate:

$$1 + r^P_t = \frac{w_t L_t}{w_{t-1} L_{t-1}}$$

where L_t is aggregate labor supply in the economy.

The return of the NDC pension system is set in such a way that the budget of the pension system is balanced in the steady state.

In this type of pension benefit arrangement, the consumption-leisure choice reflected by equation (2) becomes:

$$\frac{u_l(c^o_{i,t+1}, l^o_{i,t+1})}{u_c(c^o_{i,t+1}, l^o_{i,t+1})} = -w_{t+1}z_i \tag{5}$$

Hence, the NDC pension system does not impose any distortions on the labor supply of old agents.

2.3 Firms

Firms operate a technology $F(A_t, K_t, L_t)$. Profit maximization yields the following conditions:

$$r_t + \delta = \frac{\partial F(A_t, K_t, L_t)}{\partial K_t}$$
$$w_t = \frac{\partial F(A_t, K_t, L_t)}{\partial L_t}$$

where A_t is the total factor productivity, K_t is the capital stock, δ is the depreciation rate of capital.

2.4 General equilibrium

In the following, we define the stationary competitive equilibrium of the economy described in sections 2.1-2.3.

Definition 1. *A stationary competitive equilibrium of the model is represented by a set of time-invariant allocations $\{c^y, c^o_i, l^o_i, a\}$, prices $\{w, r\}$ and policies $\{\tau, ben_i\}$ such that:*

1. *Agents take optimal decisions given prices;*
2. *Firms take the optimal decision given prices;*
3. *The budget of the pension system balances;*

$$w\tau L = N^o \int_i ben_i dF(z_i)$$

4. *Capital market clears:*

$$K = N^o a$$

5. *Labour market clears:*

$$L = N^y z + N^o \int_i z_i l_i^o dF(z_i)$$

In the next sections, all variables will be expressed in terms of labor per capita $l = \frac{L}{N^o}$ and capital to labor ratio $k = \frac{K}{L}$.

3 Steady state analysis

In this section, we compare the steady states of the economies with a FB pension system and a NDC pension system in terms of macroeconomic variables, consumption inequality and welfare. In order to obtain analytical results, we impose a number of assumptions on the preferences and on the parameters of the model.

Assumption 1. *Capital depreciates fully ($\delta = 1$), the technology is of Cobb-Douglas type $F(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$ and there is no technological progress ($A_t = A_{t+1} = A$). Preferences of agents are represented by the life-time utility described in relation (1) with $\sigma = 1$.*

Proposition 1 presents the solution of the stationary competitive equilibrium under the Assumptions summarized in 1.

Proposition 1. *Under Assumption 1, the level of labor ($l = \frac{L}{N^o}$) and capital to labor ratio ($k = \frac{K}{L}$) in the stationary competitive equilibrium of the economies with a FB pension system and a NDC pension system, respectively, are equal to:*

$$l^{FB} = \frac{(1+n+\theta)(1-\alpha)(1-\tau)z}{1-\alpha\theta-(1-\alpha)\theta\tau} \tag{6}$$

$$k^{FB} = \left(\frac{Az(1-\alpha)(1-\tau)s^{FB}}{l^{FB}}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

$$l^{NDC} = \frac{(1+n+\theta-\tau(1-\theta)(1+n))(1-\alpha)z}{1-\alpha\theta} \tag{8}$$

$$k^{NDC} = \left(\frac{Az(1-\alpha)(1-\tau)s^{NDC}}{l^{NDC}}\right)^{\frac{1}{1-\alpha}} \tag{9}$$

where

$$s^{FB} = \frac{\beta\Gamma^{FB}}{\theta + \beta\Gamma^{FB}} \quad (10)$$

$$\Gamma^{FB} = \frac{\alpha}{1-\alpha} l^{FB} E\left[\frac{1}{\frac{\alpha}{1-\alpha} l^{FB} + \tau l^{FB} + z_i(1-\tau)}\right] \quad (11)$$

$$s^{FB} = \frac{\beta\Gamma^{NDC}}{\theta + \beta\Gamma^{NDC}} \quad (12)$$

$$\Gamma^{NDC} = \frac{\alpha}{1-\alpha} l^{NDC} E\left[\frac{1}{\frac{\alpha}{1-\alpha} l^{NDC} + z_i + z\tau(1+n)}\right] \quad (13)$$

Labor supply is decreasing in the pension contribution rate τ in both types of pension systems. Equations (4) and (5) show that the pension contribution distorts the intratemporal choice between labor supply and consumption in the FB pension system, but not in the NDC pension system[3]. However, it lowers the accumulation of savings under both types of pension systems. The smaller amount of capital leads, in equilibrium, to a lower demand for labor also in the case of the NDC pension system.

The pension contribution impacts the capital to labour ratio through three channels: i) the distortion of the intertemporal savings-consumption decision - reflected by the term $1-\tau$ -, ii) the savings rate s and iii) the labor supply l. An increase in the contribution to the pension system lowers the capital to labour ratio through the distortion of intertemporal choices and through the lower savings rate. However, an increase in the contribution to the pension system also lowers labour supply and this triggers an increase in savings and, hence, in capital.

3.1 Macroeconomic variables and consumption inequality

In the following, we compare labor supply, capital to labor ratio and consumption inequality in the steady state of the economies with a FB pension system and a NDC pension system, respectively. To make the two economies comparable, we consider that the same level of pension contributions τ prevails in both economies. Proposition 2 shows the impact of the type of pension benefit arrangement on the aggregate variables.

Proposition 2. *Under Assumption 1, comparing the labor supply and capital to labor ratio under the FB and NDC pension systems, we obtain the following results:*

1. *Labour supply is higher in the economy with a NDC pension system, i.e.* $l^{NDC} > l^{FB}$*;*

[3]The contribution to the NDC pension system would distort the intratemporal decision of young people, if they supplied labor elastically. However, it would still be the case that the NDC pension system imposes lower labor supply distortions than the FB pension system.

2. *In an economy without idiosyncratic productivity risk, the NDC pension system crowds out capital formation more than the FB pension system, i.e.* $k^{NDC}_{\sigma=0} < k^{FB}_{\sigma=0}$*;*

3. *There exists* $\theta^* \in (0, 1]$ *such that:*

   - *if* $\theta < \theta^*$ *(labor supply is elastic), the capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the FB pension system than under the NDC pension system, i.e.* $\frac{\partial k^{FB}}{\partial \sigma^2} > \frac{\partial k^{NDC}}{\partial \sigma^2}$*. Consequently, the NDC pension system crowds out capital formation more than the FB pension system at any level of idiosyncratic risk, i.e.* $k^{NDC} < k^{FB}$*;*
   - *if* $\theta > \theta^*$ *(labor supply is inelastic), capital to labor ratio increases with respect to the size of the idiosyncratic risk more under the NDC pension system than under the FB pension system, i.e.* $\frac{\partial k^{FB}}{\partial \sigma^2} < \frac{\partial k^{NDC}}{\partial \sigma^2}$*. In this case the FB pension system can crowd out capital formation more than the NDC pension system, i.e.* $k^{NDC} > k^{FB}$*, but only if the size of idiosyncratic risk is high.*

The fact that labor supply is higher under the NDC pension system is intuitive, since this type of pension system does not distort the labor supply-consumption decision.

The capital prevailing under the two pension systems is determined by two opposing effects. On the one hand, because people work longer in the NDC pension system, they can insure the same old-age consumption level by making lower savings. Consequently, in an economy with no idiosyncratic shocks the capital to labor ratio in the NDC pension system is lower than in the FB pension system, i.e. $k^{NDC}_{\sigma=0} < k^{FB}_{\sigma=0}$.

On the other hand, as we show in the Appendix, capital is increasing in the size of the idiosyncratic risk under both types of pension systems. This is because agents make higher precautionary savings. Moreover, capital under the NDC system increases faster with respect to σ^2 if labor supply is sufficiently inelastic ($\theta > \theta^*$). This effect is also intuitive. An increase in the size of the idiosyncratic risk triggers higher precautionary savings, but the effect is more pronounced in an economy with an NDC pension system because this offers lower consumption insurance (Proposition 3). It follows that capital in the NDC pension system economy can be higher than in the FB pension system economy, but only when the size of idiosyncratic risk is high.

If labor supply is elastic ($\theta < \theta^*$), agents adjust their labor supply significantly when the size of the idiosyncratic risk is higher and, hence, increase their savings to a lesser degree. Consequently, when $\theta < \theta^*$ the NDC pension system economy crowds out capital formation more than the FB pension system at any level of idiosyncratic productivity risk.

Next, we investigate the consumption insurance properties of the two pension systems.

Proposition 3. *Under Assumption 1, consumption inequality is lower in the FB pension system economy than in the NDC pension system economy. Both types of pension systems lower consumption inequality compared to an economy with no pension system.*

The FB pension system lowers consumption inequality because it provides a return that decreases with the productivity of agents. This comes from the fact that benefits are flat, but contributions are proportional to income, which is given by an agent's productivity. The NDC pension system, however, does not provide direct consumption insurance, because the pension benefits are perfectly linked to the contributions. Compared to an economy without a pension system, the reduction in consumption inequality in the NDC pension system is brought about by the fact that the crowding out of capital accumulation also reduces labor supply. Hence, all types of agents retire earlier in the economy with a NDC pension system, lowering their exposure to idiosyncratic productivity shocks.

3.2 Welfare comparison

We turn to assessing the impact that the different macroeconomic and consumption inequality outcomes have on the ex-ante utility of agents living under the two types of pension systems. We consider the implication of increasing the contribution to the pension system for the utility that agents have at birth:

$$SWF = U(c^y) + \beta EU(c^o, l^o)$$

The following proposition presents the trade-off among consumption insurance, labor supply and capital accumulation that defines the welfare in the two economies.

Proposition 4. *Under Assumption 1, the change in welfare caused by a marginal increase in the contribution to the pension system is composed of the following three terms:*

1. *the welfare loss from lower consumption due to the higher contribution to the pension system* (ω_1')*;*
2. *the welfare gain from higher consumption due to the higher pension benefit* (ω_1'')*;*
3. *the impact of general equilibrium effects* (ω_2)*.*

	FB	*NDC*
ω_1'	$-\frac{\theta}{1-\tau} - \frac{\beta\theta}{1-\tau}$	$-\frac{\theta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau}$
ω_1''	$\beta\frac{1-\alpha}{\alpha}\Gamma^{FB}$	$\beta\frac{1-\alpha}{\alpha}z(1+n)\frac{\Gamma^{NDC}}{l^{NDC}}$
ω_2	$\frac{\partial ln(k^{FB})}{\partial\tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FB}\right) + \frac{\beta\tau\Gamma^{FB}\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))}$	$\frac{\partial ln(k^{NDC})}{\partial\tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}\right)$

$$\omega_1 = \omega_1' + \omega_1'' \tag{14}$$

$$\frac{\partial SWF}{\partial\tau} = \omega_1 + \omega_2 \tag{15}$$

An increase in the contribution to the pension system produces a reallocation in the income sources for consumption, replacing consumption from wage income with consumption from the pension benefit. Both old and young agents give up part of their consumption from wage income in order to pay the higher contribution to the pension system. The welfare loss they incur from the lower consumption is measured by the term ω_1'. In return for paying a higher pension contribution, old agents receive a higher pension benefit that increases their consumption. The term ω_1'' measures the welfare gain from this increase in consumption. This welfare gain increases with Γ. The proof of Proposition 2 shows that Γ increases with the size of the idiosyncratic risk and, hence, with the consumption inequality in the economy. Intuitively, the higher is consumption inequality in the economy, the more valuable is the insurance provided by the pension system and the higher is the present value of the additional pension benefit agents will receive. We call $\omega_1 = \omega_1' + \omega_1''$ *the consumption reallocation effect*.

The last component of the marginal change in welfare, ω_2, captures the impact of the higher contribution to the pension system working through general equilibrium effects. A higher contribution reduces savings and, hence, crowds out capital formation. Under both types of pension systems, the lower level of capital impacts on welfare through factor prices: wages decrease and the return on capital increases.

Whether a marginal increase in the contribution to the FB or the NDC pension system has a positive or negative effect on welfare through prices depends on the following term:

$$\Phi = \theta\alpha(\beta + 1) - \beta(1 - \alpha)\Gamma \tag{16}$$

On the one hand, a marginal decrease in the capital to labor ratio leads to a higher interest rate on savings and increases consumption when old. The welfare gain of this effect is given by $\beta(1 - \alpha)\Gamma$. On the other hand, a marginal decrease in the capital to labor ratio leads to a lower wage, decreasing consumption in both periods of life. The welfare loss from lower wages is given by $\theta\alpha(\beta + 1)$. The sign of Φ gives the net impact on welfare of a marginal change in capital to labor ratio through prices.

In an economy with no idiosyncratic productivity shocks, if $\Phi_{\sigma=0,\tau=0} > 0$, then the economy is dynamically efficient and capital crowding out has a negative effect on welfare. The opposite is true in a dynamically inefficient economy: $\Phi_{\sigma=0,\tau=0} < 0$ and capital crowding out has a positive effect on welfare.

In the presence of idiosyncratic productivity shocks ($\sigma > 0$), there are calibrations such that $\Phi < 0$ even in dynamically efficient economies. Consequently, we can have a positive welfare effect of the pension system through prices even in dynamically efficient economies. Intuitively, a lower capital to labor ratio increases the interest rate and, hence, consumption when old. The present value of the higher old age consumption increases with the size of idiosyncratic risk. Hence, a higher σ implies an increase in Γ and a more pronounced positive impact of a higher interest rate on welfare.

In the case of the FB pension system, the impact on welfare through general equilibrium effects has an additional term that does not appear in the case of the NDC pension system:

$$\Omega = \beta\tau\Gamma^{FB}\frac{\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))} > 0 \tag{17}$$

Consequently, an increase in the contribution to the FB pension system has one effect on welfare that is positive irrespective of the dynamic efficiency of the economy. This term increases with the level of pension contributions. Hence, it is quantitatively important at high levels of pension contributions.

The intuition for Ω resides in the fact that a decrease in the capital to labor ratio leads to an increase in aggregate labor supply (equation (18)): a decrease in the level of savings requires an increase in labor supply in order to insure the same old age consumption. In the FB system a higher aggregate labor supply increases pension benefits and, hence, consumption of all old agents through the balanced budget condition of the PAYG pension system. In contrast, in the NDC pension system, the pension benefit of each agent depends on her own labor supply and not on the aggregate labor supply[4].

We compare the sizes of ω_1 and ω_2 between the two types of pension systems. We start by analyzing the two terms in an economy with no uncertainty, i.e. $\sigma = 0$.

Proposition 5. *Under Assumption 1, in an economy with no idiosyncratic risk ($\sigma = 0$) a marginal increase in the contribution to the pension system has:*

1. *a higher (lower) positive (negative) impact on welfare through the consumption reallocation effect in the case of NDC system than in the case of FB system, i.e. $\omega_1^{NDC}|_{\sigma=0} \geq \omega_1^{FB}|_{\sigma=0}$. However, the introduction of idiosyncratic productivity risk increases (decreases) the welfare gains (losses) from the consumption reallocation effect of the FB system more, i.e. $\frac{\partial(\omega_1^{FB}-\omega_1^{NDC})}{\partial\sigma^2} > 0$;*

2. *a lower negative impact on welfare through general equilibrium effects in the case of the FB pension system ($\omega_2^{FB}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$), if the economy is dynamically efficient in the absence of idiosyncratic shocks ($\Phi_{\sigma=0,\tau=0} > 0$);*

3. *a higher positive impact on welfare through general equilibrium effects in the case of the NDC pension system, ($\omega_2^{NDC} > \omega_2^{FB}$), if if the economy is dynamically inefficient in the absence of idiosyncratic shocks ($\Phi_{\sigma=0,\tau=0} < 0$) and the level of pension contribution τ is low.*

Let us analyze the consequences of Proposition 5 in an economy that is dynamically efficient in the absence of idiosyncratic shocks, i.e. $\Phi|_{\sigma=0,\tau=0} > 0$. In the absence of idiosyncratic shocks, at the marginal introduction of the pension system, the impact on welfare

[4]Note than in the steady state the return of the NDC pension system is $r^P = (1+n)$, independent of aggregate labor supply.

through the consumption reallocation effect is the same in the case of the two pension systems ($\omega_1^{FB}|_{\sigma=0,\tau=0} = \omega_1^{NDC}|_{\sigma=0,\tau=0}$). For $\tau > 0$, the NDC pension system brings a higher (lower) welfare gain (loss) through consumption reallocation because it distorts labor supply less.

When we introduce idiosyncratic productivity risk, the consumption reallocation effect increases in the FB economy more than in the NDC economy because of the insurance provided by the FB pension system. Hence, at the marginal introduction of the pension system, the welfare gain (loss) from consumption reallocation will be higher (lower) in the FB pension system, i.e. $\omega_1^{FB}|_{\tau=0} > \omega_1^{NDC}|_{\tau=0}$ if $\sigma > 0$. As τ increases, the welfare gains from better consumption insurance under the FB pension system diminish while welfare losses from labor supply distortions increase. Hence, $\omega_1^{NDC} > \omega_1^{FB}$ for a high enough level of τ.

In conclusion, if we abstract from general equilibrium effects, we obtain that the FB pension system brings a higher welfare at low levels of pension contributions due to consumption insurance, while the NDC pension system brings a higher welfare at high levels of pension contributions due to lower labor supply distortions.

In the absence of idiosyncratic shocks, general equilibrium effects favor the FB pension system at all levels of pension contributions ($\omega_2^{FB} > \omega_2^{NDC}$). Two effects determine this result. First, the FB pension system crowds out capital formation less than the NDC pension system and this has a less negative impact on welfare through prices. Second, the impact of the FB system on welfare through general equilibrium effects contains an additional term Ω identified in equation (17). This term is positive and increases with the level of pension contributions.

At low levels of pension contributions, both the general equilibrium effects and the consumption reallocation effect favor the FB system. As the level of pension contributions increases, the two effects act in opposite directions: compared to the NDC system, the FB system has a less (more) positive (negative) impact on welfare through the consumption reallocation effect, but a less negative impact through general equilibrium effects. Which type of pension system brings a higher welfare at high level of pension contributions is, hence, a quantitative question. We explore it using a calibrated version of the model in Section 4.

An additional observation is necessary. Proposition 5 states that, in the absence of idiosyncratic shocks, the FB pension system has a less negative impact on welfare through general equilibrium effects. In the presence of idiosyncratic productivity shocks, this does not always hold. As Proposition 2 shows, the FB pension system can crowd out capital formation more than the NDC pension system if labor supply is relatively inelastic and the size of idiosyncratic risk is high. In this case, at low levels of pension contributions, the FB pension system can have a more negative impact on welfare through general equilibrium effects. However, when idiosyncratic risk is high, the welfare benefits of the FB system coming from insurance are also very high. Overall, the FB system can still bring a higher welfare.

In the case of an economy that is dynamically inefficient in the absence of idiosyncratic shocks, i.e. $\Phi|_{\sigma=0,\tau=0} < 0$, the general equilibrium effects favor the NDC pension system, but only at low levels of pension contributions. At high level of pension contributions, the term Ω becomes quantitatively important and the FB system brings a higher welfare through general equilibrium effects. However, at low levels of pension contributions the FB pension system brings higher welfare gains due to consumption insurance. Which of the two effects dominates is again a quantitative question that we investigate in Section 4.

4 Numerical examples

In the previous section we were able to establish that the FB pension system brings a higher welfare at low levels of pension contributions in dynamically efficient economies. In all other cases, the consumption reallocation effect and the general equilibrium effects act in opposing directions. Hence, we were not able to determine which of the two pension systems dominates from a welfare point of view. To this end, we present two numerical examples in this section.

We consider that the idiosyncratic productivity shock has a lognormal distribution with $z = 1, \sigma = 1$. We calibrate $\beta = 0.7, \theta = 0.35, n = 0$. Assuming that a period of the model represents 30 years, the size of the idiosyncratic productivity risk and the time preference at yearly frequency are $\sigma_y^2 \approx 0.016, \beta_y \approx 0.99$. For α we take two values: i) $\alpha = 0.3$ - in this case the economy is dynamically efficient in the absence of idiosyncratic productivity shocks ($\Phi_{\sigma=0,\tau=0} > 0$) and ii) $\alpha = 0.2$ - in this case the economy is dynamically inefficient in the absence of idiosyncratic productivity shocks ($\Phi_{\sigma=0,\tau=0} < 0$).

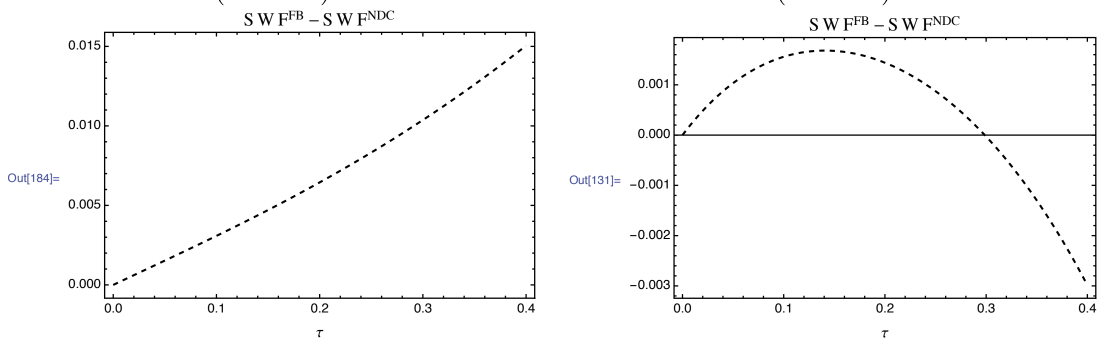
4.1 Welfare analysis

Figure 1 presents the difference in ex-ante utility between the FB and the NDC pension economies at different levels of pension contribution for $\alpha = 0.3$ and $\alpha = 0.2$, respectively. Figure 2 presents the decomposition of the change in welfare caused by a marginal increase in the contribution to the pension system presented in Proposition 4.

Figure 1: Welfare comparison

(a) Dynamically efficient economy ($\alpha = 0.3$)

(b) Dynamically inefficient economy ($\alpha = 0.2$)



In the dynamically efficient economy ($\alpha = 0.3$), the FB pension system brings a higher welfare than the NDC pension system irrespective of the level of the pension contribution ($SWF^{FB} - SWF^{NDC} > 0$).

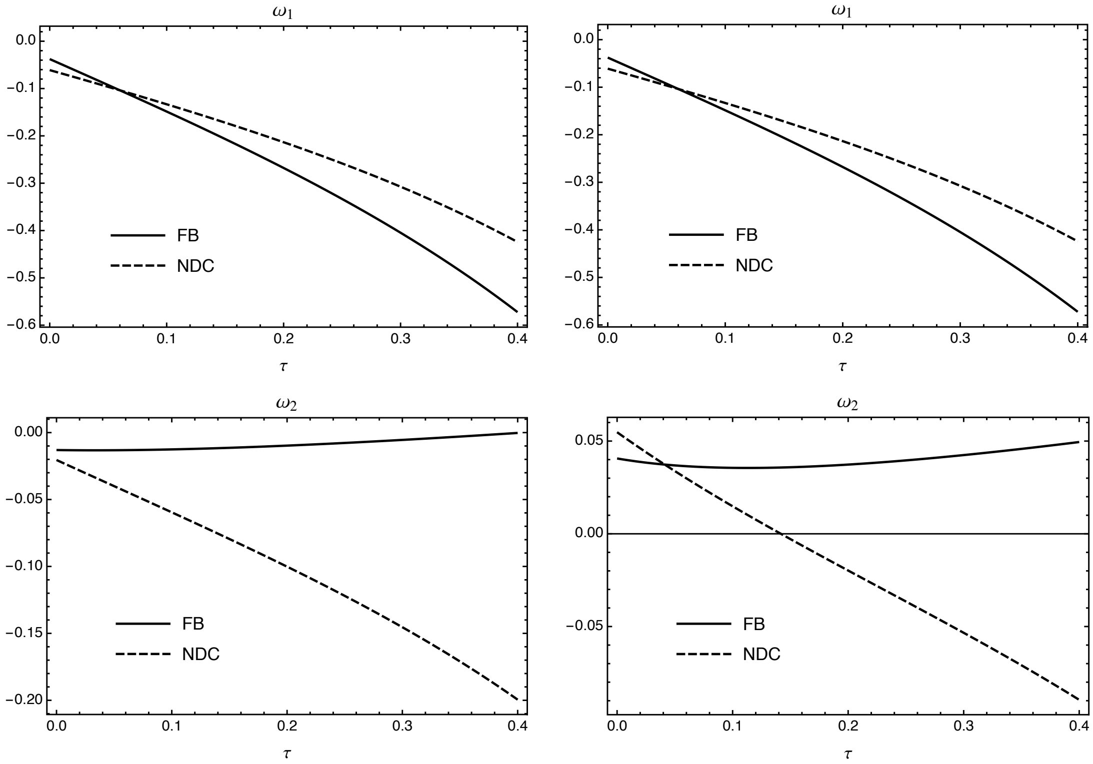
On the one hand, as figure 2 shows, the FB pension system has a lower negative impact on welfare through consumption reallocation ($\omega_1^{FB} > \omega_1^{NDC}$) when the level of contribution is small. This is due to the consumption insurance that the FB pension provides. As the level of the pension contribution increases, the welfare loss from labor supply distortions start to dominate the welfare gains from the consumption insurance offered by the FB pension system. For high levels of pension contributions, the marginal welfare losses from consumption reallocation become higher in the FB system ($\omega_1^{FB} < \omega_1^{NDC}$).

On the other hand, the NDC pension system has a more negative impact on welfare through general equilibrium effects at any level of pension contribution ($\omega_2^{NDC} < \omega_2^{FB}$). This is due to the fact that the economy is dynamically efficient and the NDC pension system crowds out capital formation more than the FB pension system. Moreover, as the level of pension contribution τ increases, the negative impact on welfare through general equilibrium effects amplifies under the NDC pension system, but dampens under the FB pension system. This happens because of two effects: i) the difference in capital to labor ratio between the two pension system increases with the level of pension contribution (see proof of Proposition (2)) and ii) the impact of the FB pension system on welfare through general equilibrium effects contains an additional term Ω (relation (17)) that is always positive and increases with the level of pension contribution.

Overall, the general equilibrium effects dominate and the welfare is higher in the FB pension system at any level of pension contributions.

Figure 2: The components of marginal welfare change

(a) Dynamically efficient economy ($\alpha = 0.3$) (b) Dynamically inefficient economy ($\alpha = 0.2$)



In the dynamically inefficient economy ($\alpha = 0.2$, figure 2), the two effects of the pension systems on welfare are qualitatively the same as in the case of the dynamically efficient economy with one exception: at low levels of pension contributions, the NDC pension system brings higher welfare gains from general equilibrium effects. This is in line with the results of Proposition 5.

Unlike in the case of the dynamically efficient economy, the consumption reallocation effect dominates the general equilibrium effects. As a result, the FB pension system brings a higher welfare at low values of pension contributions, while the NDC pension system brings a higher welfare at high levels of pension contributions (figure 1). Experimenting with various calibrations, we found that the general equilibrium effects can dominate at low levels of pension contributions even when $\alpha = 0.2$, but only when the size of the idiosyncratic risk is unrealistically low ($\sigma < 0.4$). In this case, the NDC pension system brings a higher welfare at any level of pension contribution.

4.2 Comparison with a partial equilibrium model

In this section we analyze the role played by general equilibrium effects in determining the relative welfare of FB and NDC pension systems. We consider a version of the model in which the interest rate and the wage do not change when we vary the contribution to the pension system. We label this a partial equilibrium analysis and we consider it relevant for small open economies.

Figure 3: General equilibrium, dynamically efficient economy ($\alpha = 0.3$)

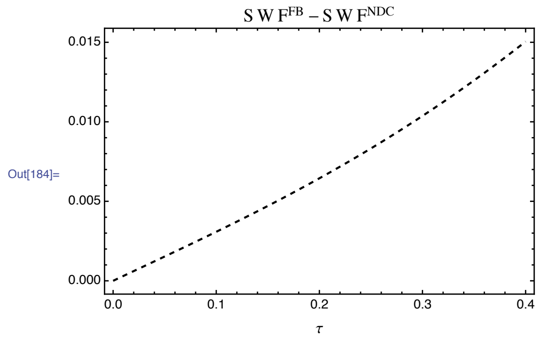
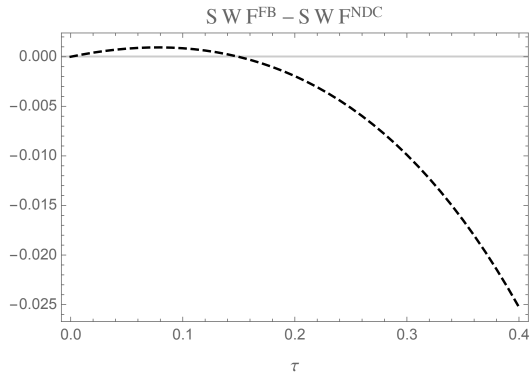


Figure 4: Partial equilibrium, dynamically efficient economy ($\alpha = 0.3$)



Figures 3 and 4 present the difference between the welfare of the FB pension economy and the NDC pension economy in a general equilibrium and in a partial equilibrium setting, respectively, for the case of the dynamically efficient economy. In the general equilibrium setting, the FB pension system brings a higher welfare than the NDC pension system at any level of pension contribution. In partial equilibrium, the difference in welfare is given by the difference in ω_1 between the FB and the NDC pension economies. The welfare is higher in the FB pension system for low levels of pension contributions. As the level of pension contribution increases, the welfare of the FB pension system becomes lower due to the labor supply distortions.

We conclude that general equilibrium effects play a decisive role in the welfare comparison between the two types of pension systems in dynamically efficient economies and they may tilt the balance in favour of the FB pension system even at high levels of pension contributions.

5 A cross-country analysis of pension benefit arrangements

In this section we analyze the relationship between the arrangement of pension benefits, on the one hand, and the size of the pension system and idiosyncratic risk, on the other hand, using cross-country data from OECD for the following countries: Sweden, France, Germany, Netherlands, Austria, Belgium, Finland, Italy, Spain, UK, US, Canada, Australia, New Zealand, Japan.

As a proxy for the size of the idiosyncratic productivity risk, we consider the Gini coefficient of disposable income (income after taxes and transfers) for the population aged 18-64 (working age). We consider the average of data recorded for 1974-2012. The pension progressivity index is taken from OECD Pensions at a Glance (2005). The size of the pension system is proxied by the gross replacement rate of the agent with an average life-time income provided by OECD in Pensions at a Glance (2005).

Figure 5 presents the relationship between the progressivity of pension benefits and the size of the pension system. We find a strong negative relation between the two variables. This correlation is in accordance with our welfare analysis conducted abstracting from general equilibrium effects. In section 3.2 we found that a more progressive pension system (in our case the FB pension system) brings a higher welfare than a less progressive pension system (the NDC pension system) at low levels of pension contributions due to the insurance provided. At higher levels of pension contributions, the less progressive pension system brings a higher welfare because of lower labor supply distortions.

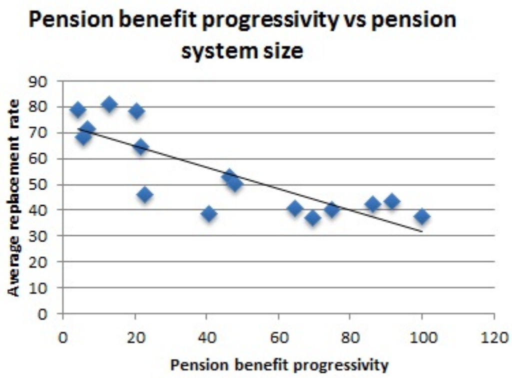


Figure 5

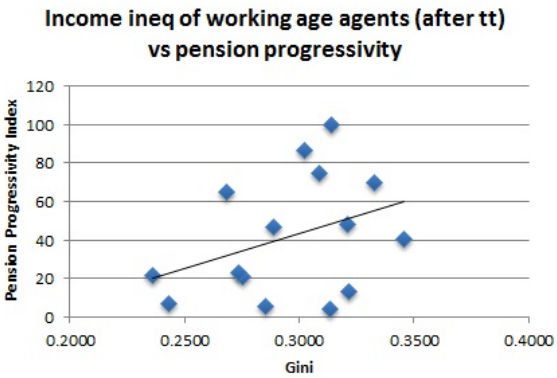


Figure 6

Figure 6 presents the relation between the disposable income inequality of working age agents and the progressivity of pension benefits. The relationship between the two variables is positive, indicating that countries with a higher income Gini have more progressive pension benefit arrangements. This correlation can also we explained from a welfare perspective since more progressive pension systems offer better consumption insurance.

6 Conclusion

In this paper we analyzed the implications for macroeconomic variables, consumption inequality and welfare of two extreme types of pension benefits arrangements, the FB and NDC pension systems. We found that the FB pension system leads to lower labor supply, but also lower consumption inequality. As long as the level of idiosyncratic risk is not very high, the FB pension system also crowds out capital formation less than the NDC pension system.

Comparing the welfare of the two pension systems, if we abstract from general equilibrium effects, the FB pension system brings a higher welfare at low levels of pension contributions. As the level of pension contribution becomes higher, the NDC pension system brings a higher welfare due to the lower labor supply distortions. This result can explain why countries with a high size of the PAYG pension system like Germany, Italy, France and Poland switched to a perfect link between pension contributions and pension benefits. Countries with a small size of the pension system like the UK prefer a FB pension system because of the consumption insurance it provides.

When we also consider the impact of general equilibrium effects on the welfare of the pension systems, the results depend on the dynamic efficiency of the economy. In a dynamically efficient economy, the FB pension system still brings a higher welfare for low levels of pension contributions. At higher level of pension contributions, the general equilibrium effects have a less adverse impact on welfare in the case of the FB pension system but it is not clear whether they dominate the higher welfare losses from labor supply distortions. A calibrated version of the model indicates that general equilibrium effects dominate. However, it may prove useful to verify whether this result holds also in a more detailed model. For example, it would be interesting to analyze how other tax and transfer programs ran by the government interact with the PAYG pension system. Progressive income taxes also offer some degree of insurance, thus reducing the importance of the FB pension system.

The tractability of our model makes it an extremely useful tool to analyse other issues related to pension arrangements. One straightforward extension of the present paper is to determine what (non)linear combination of the FB or NDC pension system would be better from a welfare perspective at different levels of idiosyncratic risk and of pension contributions.

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Appendix

Proof of Proposition 1

With the preferences considered in Assumption 1, using (2) and the budget constraints of the agents and government, we find closed form solutions for consumption and labor as a function of savings and prices:

FB	NDC
$c^y = wz(1-\tau) - a$	$c^y = wz(1-\tau) - a$
$c_i^o = \theta(a(1+r) + wz_i(1-\tau) + w\tau l)$	$c_i^o = \theta(a(1+r) + wz_i + wz\tau(1+n))$
$l_i^o = \theta - (1-\theta)\frac{a(1+r)+w\tau l}{wz_i(1-\tau)}$	$l_i^o = \theta - (1-\theta)\frac{a(1+r)+wz\tau(1+n)}{wz_i}$

In the above relations we replace the formula for the gross rate of return from the firm's problem:

$$1 + r = \frac{\alpha}{1-\alpha}\frac{l}{a}w$$

and obtain:

FB	NDC
$c^y = wz(1-\tau) - a$	$c^y = wz(1-\tau) - a$
$c_i^o = \theta w(\frac{\alpha}{1-\alpha}l + z_i(1-\tau) + \tau l)$	$c_i^o = \theta w(\frac{\alpha}{1-\alpha}l + z_i + z\tau(1+n))$
$l_i^o = \theta - (1-\theta)\frac{\frac{\alpha}{1-\alpha}l+\tau}{z_i(1-\tau)}$	$l_i^o = \theta - (1-\theta)\frac{\frac{\alpha}{1-\alpha}l+z\tau(1+n)}{z_i}$

The expressions for consumption are substituted in (3).

We solve for the level of assets:

$$a = \frac{\beta\Gamma}{\beta\Gamma+\theta}z(1-\tau)(1-\alpha)\left(\frac{a}{l}\right)^{\alpha} \Rightarrow l = sz(1-\tau)(1-\alpha)k^{\alpha-1} \tag{18}$$

where $\Gamma^{FB} = \frac{\alpha}{1-\alpha}lE_{z_i}\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z_i(1-\tau)+\tau l}\right]$, $\Gamma^{NDC} = \frac{\alpha}{1-\alpha}lE_{z_i}\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z_i+z\tau(1+n)}\right]$ and $s = \frac{\beta\Gamma}{\beta\Gamma+\theta}$.

We aggregate individual labor supply in order to obtain a closed form expression for it:

$$L = N^y z + N^o\int_i z_i l_i dF(z_i) \Rightarrow$$

$$l^{FB} = \frac{(1+n+\theta)(1-\alpha)(1-\tau)z}{1-\alpha\theta-\theta\tau(1-\alpha)} \tag{19}$$

$$l^{NDC} = \frac{(1+n+\theta-(1-\theta)\tau(1+n))(1-\alpha)z}{1-\alpha\theta} \tag{20}$$

We obtain the capital to labor ratio by substituting (20) in (18).

Proof of Proposition 2

Using (6) and (8), we compare labor supply under the two pension systems:

$$l^{NDC} - l^{FB} = \frac{(1-\alpha)(1-\alpha(1-\tau)-\tau)\tau\theta(1-\theta)}{(1-\alpha\theta)(1-\alpha\theta(1-\tau)-\tau\theta)} > 0$$

Using (7) and (9), we show that capital accumulation is lower in the NDC pension system in absence of idiosyncratic productivity shocks ($\sigma = 0$) at all levels of pension contributions:

$$sgn\left(k^{NDC}_{\sigma=0} - k^{FB}_{\sigma=0}\right) = sgn\left(\left(\frac{k^{NDC}_{\sigma=0}}{k^{FB}_{\sigma=0}}\right)^{1-\alpha} - 1\right) =$$

$$= sgn\left(-\frac{\tau\theta(1+\alpha\beta)(1-\theta)(1+\alpha+\tau(1-\alpha)))}{(1-\alpha\theta-\tau\theta(1-\alpha))(\theta(1+\alpha)+\alpha\beta(1+\theta)+\tau(\theta(1-\alpha)-\alpha\beta(1-\theta)))}\right) = -$$

Using the logarithm version of (7) and (9), we show that, in absence of idiosyncratic productivity shocks ($\sigma = 0$), an increase in the contribution to the pension system reduces capital under the NDC pension system more than under the FB pension system:

$$\frac{\partial(ln(k^{NDC}_{\sigma=0}) - ln(k^{FB}_{\sigma=0}))}{\partial\tau} =$$

$$-\frac{\theta(1+\alpha\beta)(1-\theta)[(1-\alpha)(\theta+1)(1-\tau)\alpha\beta\tau + (\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau\theta(1-\alpha))(1+\alpha(1-\tau)+\tau))]}{(1-\alpha)(1-\tau)(\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau\theta(1-\alpha))(\theta(1+\alpha)+\alpha\beta(\theta+1)+\tau(\theta(1-\alpha)-\alpha\beta(1-\theta)))(1-\alpha\theta-\tau\theta(1-\alpha))} < 0$$

We compute the change of the capital stock with respect to the variance of the idiosyncratic risk using (7) and (9):

$$\frac{\partial k^{1-\alpha}}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)}{l}\frac{\partial s}{\partial\sigma^2} = \frac{A(1-\alpha)(1-\tau)\theta\beta}{l(\theta+\beta\Gamma)^2}\frac{\partial\Gamma}{\partial\sigma^2}$$

The relationship above holds for both the FB and the NDC economy.

We determine the sensitivity of Γ with respect to the size of the idiosyncratic shock by taking a second order Taylor expansion around ($E(z_i) = 1, \tau$) and differentiating with respect to σ^2:

$$\frac{\partial\Gamma^{FB}}{\partial\sigma^2} \approx \left.\frac{\partial^2\Gamma^{FB}}{\partial z_i^2}\right|_{\sigma=0} = \frac{\alpha}{1-\alpha}l^{FB}\frac{(1-\tau)^2}{\left(\frac{\alpha}{1-\alpha}l^{FB}+\tau l^{FB}+1-\tau\right)^3} > 0 \quad (21)$$

$$\frac{\partial\Gamma^{NDC}}{\partial\sigma^2} \approx \left.\frac{\partial^2\Gamma^{NDC}}{\partial z_i^2}\right|_{\sigma=0} = \frac{\alpha}{1-\alpha}l^{NDC}\frac{1}{\left(\frac{\alpha}{1-\alpha}l^{NDC}+1+(1+n)\tau\right)^3} > 0 \quad (22)$$

$$\frac{1}{l^{FB}}\frac{\partial\Gamma^{FB}}{\partial\sigma^2} - \frac{1}{l^{NDC}}\frac{\partial\Gamma^{NDC}}{\partial\sigma^2} \approx \frac{\alpha\tau P(\theta)}{(1-\alpha)(1-\tau)(1+\tau+\alpha(1-\tau))^3} \quad (23)$$

where

$$P(\theta) = 1 - 3\theta + 3\theta^2(1 - (1-\tau)(1-\alpha)^2) + \theta^3(-3\alpha(1-\tau)(\alpha(1-\tau)+\tau) - \tau^2 + \alpha^3(2 - 3\tau + \tau^2))$$

From relations (21) and (22), we obtain that capital accumulation increases with the level of idiosyncratic risk in both economies.

It is straightforward to show that:

$$P(0) = 1 > 0, P(1) = -(1-\alpha)^3(2-\tau)(1-\tau) < 0$$
$$P(\theta) = 0 \text{ has discriminant } \Delta = -27(1-\alpha)^6(1-\tau)^2\tau^2 < 0$$

Consequently $P(\theta) = 0$ has one real solution θ^* and $\theta^* \in (0, 1]$.

We distinguish 2 cases:

- $\theta < \theta^*$: $P(\theta) > 0$ and capital increases faster in the FB economy than in the NDC economy when the size of idiosyncratic risk increases. Hence for these values of labor supply elasticity, the NDC pension system crowds out capital formation more than the FB pension system at any level of idiosyncratic productivity risk.
- $\theta > \theta^*$: $P(\theta) < 0$ and capital in an NDC pension system economy increases faster than in an FB pension system economy. Hence, although $k^{NDC}(0, \tau) < k^{FB}(0, \tau)$, a sufficiently high increase in the level of idiosyncratic risk may lead to a lower capital to labor ratio in the case of the FB pension system.

Proof of Proposition 3

We consider a simplified measure of consumption inequality:

$$c_{inequality} = \frac{c_H}{c_L}$$

where $z_H - z_L = 2\sigma$.

Using the closed for solutions for c^H and c^L obtained in the proof of Proposition 2, we obtain the following relations:

$$c^{FB}_{inequality} - c^{NDC}_{inequality} = -\frac{2(1-\alpha)\sigma\tau\theta(\tau+1+\alpha(1-\tau))}{(1+\tau+\alpha(1-\tau)-\sigma(1-\alpha\theta))(1+\tau+\alpha(1-\tau)(\sigma\theta+1)+\theta-\sigma(1-\tau))} < 0$$
$$c^{FB}_{inequality} - c^{FB}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1+\theta)}{(1+\alpha-\sigma+\alpha\sigma\theta)(1-\sigma+\tau\sigma\theta+\alpha(1-\tau)(1+\sigma\theta))} < 0$$
$$c^{NDC}_{inequality} - c^{NDC}_{inequality}|_{\sigma=0} = -\frac{2(1-\alpha)\sigma\tau(1-\alpha\theta)}{(1+\alpha-\sigma+\alpha\sigma\theta)(1+\alpha-\sigma+\tau-\alpha\tau+\alpha\sigma\theta)} < 0$$

Proof of Proposition 4

We write all the macroeconomic variables in terms of the τ, k and s:

$$a = swz(1-\tau) = sz(1-\tau)(1-\alpha)k^{\alpha} \Rightarrow 1 = \frac{sz(1-\tau)(1-\alpha)k^{\alpha-1}}{l}$$
$$c^y = z(1-s)(1-\tau)(1-\alpha)k^{\alpha}$$

FB	NDC
$c_i^o = \theta(1-\tau)(1-\alpha)k^{\alpha}(sz\alpha k^{\alpha-1} + z^i + \tau sz(1-\alpha)k^{\alpha-1})$	$c_i^o = \theta(1-\alpha)k^{\alpha}(sz\alpha k^{\alpha-1}(1-\tau) + z^i + z\tau(1+n))$
$l_i^o = \theta - (1-\theta)szk^{\alpha-1}\frac{\alpha+\tau(1-\alpha)}{z_i}$	$l_i^o = \theta - (1-\theta)\frac{sz(1-\tau)\alpha k^{\alpha-1}+z\tau(1+n)}{z_i}$

The ex ante utility of an agent is:

$$SWF(\tau, k, s) = U(c^y) + \beta E(U(c^o, l^o))$$

The change in ex ante utility following a marginal introduction of a pension system is:

$$\frac{\partial SWF}{\partial \tau} = \frac{\partial U(c^y)}{\partial c^y}\left(\frac{\partial c^y}{\partial \tau} + \frac{\partial c^y}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^y}{\partial s}\frac{\partial s}{\partial \tau}\right) +$$
$$+ \beta E\left[\frac{\partial U(c^o, l^o)}{\partial c^o}\left(\frac{\partial c^o}{\partial \tau} + \frac{\partial c^o}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial c^o}{\partial s}\frac{\partial s}{\partial \tau}\right) + \frac{\partial U(c^o, l^o)}{\partial l^o}\left(\frac{\partial l^o}{\partial \tau} + \frac{\partial l^o}{\partial k}\frac{\partial k}{\partial \tau} + \frac{\partial l^o}{\partial s}\frac{\partial s}{\partial \tau}\right)\right]$$

We plug in the formulas for c^y, c_i^o, l_i^o. We group the terms of the derivative in order to obtain the term ω_1 of Proposition 4:

$$\omega_1 = \frac{\partial U(c^y)}{\partial c^y}\frac{\partial c^y}{\partial \tau} + \beta E\left[\frac{\partial U(c^o, l^o)}{\partial c^o}\frac{\partial c^o}{\partial \tau} + \frac{\partial U(c^o, l^o)}{\partial l^o}\frac{\partial l^o}{\partial \tau}\right]$$
$$\omega_1^{FB} = -\frac{\theta}{1-\tau} - \frac{\beta\theta}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\Gamma^{NDC}$$
$$\omega_1^{NDC} = -\frac{\theta}{1-\tau} - \frac{\beta\Gamma^{NDC}}{1-\tau} + \beta\frac{1-\alpha}{\alpha}\frac{\Gamma^{NDC}}{l^{NDC}}$$

The formulas for the term ω_2 are:

$$\omega_2 = \frac{\partial k}{\partial \tau}\left(\frac{\partial U(c^y)}{\partial c^y}\frac{\partial c^y}{\partial k} + \beta E\left[\frac{\partial U(c^o,l^o)}{\partial c^o}\frac{\partial c^o}{\partial k} + \frac{\partial U(c^o,l^o)}{\partial l^o}\frac{\partial l^o}{\partial k}\right]\right) +$$

$$+\frac{\partial s}{\partial \tau}\left(\frac{\partial U(c^y,l^y)}{\partial c^y}\frac{\partial c^y}{\partial s} + \beta E\left[\frac{\partial U(c^o,l^o)}{\partial c^o}\frac{\partial c^o}{\partial s} + \frac{\partial U(c^o,l^o)}{\partial l^o}\frac{\partial l^o}{\partial s}\right]\right)$$

$$\omega_2^{FB} = \frac{\partial ln(k^{FB})}{\partial \tau}\left(\theta\alpha(1+\beta) - \beta(\alpha+\tau(1-\alpha))\frac{1-\alpha}{\alpha}\Gamma^{FB}\right) + \frac{\partial ln(s^{FB})}{\partial \tau}\frac{\beta\Gamma^{FB}\tau(1-\alpha)}{\alpha}$$

$$\frac{\partial ln(s^{FB})}{\partial \tau} = (1-\alpha)\frac{\partial ln(k^{FB})}{\partial \tau} + \frac{1}{1-\tau} + \frac{\partial ln(l^{FB})}{\partial \tau}$$

$$\omega_2^{NDC} = \frac{\partial ln(k^{NDC})}{\partial \tau}\left(\theta\alpha(1+\beta) - \beta(1-\alpha)\Gamma^{NDC}\right)$$

Proof of Proposition 5

1. We compute for $\sigma = 0$ the difference between the impact on welfare through consumption reallocation triggered by a marginal increase in the FB and NDC pension system, respectively.

$$(\omega_1^{FB} - \omega_1^{NDC})|_{\sigma=0} = \frac{2\beta\tau\theta P(\tau)}{(1-\tau)^2(1+\alpha(1-\tau)+\tau)(1+\tau+\alpha)} \leq 0 \tag{24}$$

   where $P(\tau) = \tau^2(1-\alpha)^2 + 2\tau\alpha(1-\alpha) - (1-\alpha^2)$. The result follows from the fact that $P(\tau) < 0$ for $\tau \in [-\frac{1+\alpha}{1-\alpha}, 1]$.

2. We compute the difference between the impact on welfare through general equilibrium effects triggered by a marginal increase in the FB and NDC pension system, respectively.

$$\omega_2^{FB} - \omega_2^{NDC} = \frac{\partial ln(k^{FB})}{\partial \tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{FB}\right) - \frac{\partial ln(k^{NDC})}{\partial \tau}\left(\theta\alpha(\beta+1) - \beta(1-\alpha)\Gamma^{NDC}\right) + \beta\tau\Gamma^{FB}\frac{\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))} \tag{25}$$

   From Proposition 2, we know that:

$$\Gamma^{NDC}_{\sigma=0} \leq \Gamma^{FB}_{\sigma=0} \leq \Gamma_{\sigma=0,\tau=0} \Rightarrow \Phi^{NDC}_{\sigma=0} \geq \Phi^{FB}_{\sigma=0} \geq \Gamma_{\sigma=0,\tau=0} > 0 \tag{26}$$

   Also from Proposition 2, we know that the NDC pension system crowds out capital formation more than the FB pension system when $\sigma = 0$:

$$0 > \frac{\partial ln(k^{FB})}{\partial \tau} > \frac{\partial ln(k^{NDC})}{\partial \tau} \tag{27}$$

From relations (25), (26), (27) , it follows that $0 > \omega_2^{FB}|_{\sigma=0} > \omega_2^{NDC}|_{\sigma=0}$.

3. For this proof we consider first the case $\tau = 0$. Equation (25) becomes:

$$(\omega_2^{FB} - \omega_2^{NDC})_{\sigma=0,\tau=0} = \left(\frac{\partial ln(k^{FB})}{\partial \tau} - \frac{\partial ln(k^{NDC})}{\partial \tau}\right)\Phi_{\sigma=0,\tau=0} < 0 \qquad (28)$$

Increasing τ further has the following effects:

- the crowding out effect under the NDC pension system worsens faster than under the FB pension system (see Proof of Proposition 2):

$$0 > \frac{\partial ln(k^{FB})}{\partial \tau} > \frac{\partial ln(k^{NDC})}{\partial \tau} \qquad (29)$$

- however, the impact of the general equilibrium effect on welfare also shrinks faster in the NDC pension system:

$$\Gamma^{NDC}_{\sigma=0} \leq \Gamma^{FB}_{\sigma=0} \leq \Gamma_{\sigma=0,\tau=0} \Rightarrow 0 > \Phi^{NDC}_{\sigma=0} \geq \Phi^{FB}_{\sigma=0} \geq \Gamma_{\sigma=0,\tau=0} \qquad (30)$$

- ω_2^{FB} has an additional component $\beta\tau\Gamma^{FB}\frac{\theta(1-\alpha)^2}{\alpha(1-\alpha\theta-\tau\theta(1-\alpha))}$ that is positive and increasing with the level of pension contributions τ.

From the above we conclude that, when $\Phi_{\sigma=0} < 0$, we can have a higher welfare gain from general equilibrium effects in the NDC system, but only for low levels of τ.