

The Tax Benefit of Income Smoothing

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Abstract

A worker can reduce tax liability by contributing to a private pension plan when marginal tax rates are high and withdraw pension benefits when marginal tax rates are low. We quantify the tax benefit of income smoothing through the private retirement system and find that it is negligible. This conclusion is important to households, investment advisers, tax policymakers, and scholars engaged in financial retirement planning.

Keywords: private pensions, life-cycle model, tax progressivity.

JEL Classification Numbers: G11, G18, G23, H24, D91.

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1 Introduction

The United States Congress provides citizens with incentives to save for their own retirement. The two pillars of the private retirement system are tax-exemption of pension plan income and income smoothing. The smoothing benefit is a consequence of tax progressivity. By contributing before-tax income during work years when infra-marginal tax rates are high, and withdrawing funds at retirement when they tend to be low, lifetime tax liability is reduced. Using data from 1979, Ippolito (1986) reaches the conclusion that the two tax benefits are approximately equal and allow a worker to cut lifetime tax liability by as much as 40% together. That the tax-exemption of pension plan income remains large is obvious.¹ However, the tax benefit of income smoothing is not equally clear as it depends on lifetime consumption and savings decisions and all the complications that arise from long-term planning. Many scholars cite the relevance of the smoothing effect, but nobody has updated its measurement since Ippolito (1986).² The objective of our paper is to quantify the tax benefit of income smoothing in recent years. Ascertaining the value of the smoothing benefit should be of interest to both households managing their own retirement accounts, investment advisors and union leaders designing retirement plans and scholars engaged in research on savings and consumption decisions.

Within a life-cycle model extended to include Social Security, we derive an upper boundary of the smoothing benefit under extreme behavioral assumptions that entail lifetime planning from the time the worker enters the job market until death. Adding realism to the life-cycle model, such as delayed start to savings, reduces the estimate. Relative to the attention paid to the smoothing benefit in textbooks, articles, by investment advisers, and by Ippolito (1986), our estimate of the maximum smoothing benefit is surprisingly small. The average worker cannot reduce the average tax rate by more than 3.2 percentage points. The main reason is that the United States income

¹Suppose the tax rate is 40%, the before-tax interest rate 5%, and the investment horizon 30 years. The after-tax balance of \$1 continuously compounded inside a pension plan account equals $\$4.48 \times (1 - 40\%) = \2.69 . This can be compared to regular savings with income taxes paid upfront. The after-tax balance of \$1 before-tax income continuously compounded at the 3% after-tax interest rate equals $\$1 \times (1 - 40\%) \times \$2.46 = \$1.48$. In this example, the net proceeds from saving inside the pension plan are about 80% higher.

²E.g., Munnell (1982), Ozanne and Lindeman (1987), Feenberg and Skinner (1989), Ragan (1994), Burmann, Gale, and Weiner (2001), Horan (2005), Turner (2005), Lankford (2008), Horan (2009), and Nishiyama (2010). See also the tax treatment of pensions in the *Encyclopedia of Taxation and Tax Policy*.

tax system as of 2010 is not sufficiently progressive. With Social Security income, the maximum tax rate reduction resulting from lifetime income smoothing decreases to 1.7 percentage points. Reasonable parameter estimates of real interest and income growth bring down the smoothing benefit measure even further. For example, a real interest rate and an income growth rate of 1% each reduces the smoothing benefit to 1.4%. We believe that such a small magnitude is unlikely to motivate the type of disciplined savings necessary to obtain the maximum smoothing benefit.

As an example of the potential smoothing benefit, consider a worker with annual taxable income \$300,000.³ Using the 2010 federal income tax table, the standard deduction, and two exemptions, the average tax rate on this income is 23.3%. If the worker could split his income equally between work years and retirement years, the taxable income would decrease to \$150,000 and the average tax rate drops to 16.4%. Accordingly, the smoothing benefit equals the 6.9 percentage point reduction in the average tax rate. This is a considerable tax benefit worth paying attention to, but many real-world features reduce it. *(i)* Time spent in the work force tends to exceed the number of retirement years, so annual before-tax income cannot be cut in half. *(ii)* Social Security income fills up lower-income brackets with non-labor income. *(iii)* Additional realism including real interest on retirement funds, income growth, uncertainty, contribution limits, and borrowing restrictions reduce the estimate further. We illustrate how some of these real-world features bring down the smoothing benefit from 6.9% in the example to 1.4%.

Historical smoothing benefit calculations from 1950 conclude our analysis. Ippolito (1986) provides his estimate of the smoothing benefit using 1979 data right before the tax law changes in the 1980s that reduce progressivity and make Social Security income taxable.⁴ He argues that the smoothing benefit is one of the reasons for the growth of the private pension system in the United States. As the smoothing benefit is quite small for middle-income earners throughout most of the post-war period with the exception of a brief period around the time of Ippolito's study, we doubt that the tax benefit of income smoothing is a significant factor in the growth of union led retirement plans. This conclusion does not rule out Ippolito's other assertion that tax exemption

³This example is inspired by the example of income shifting between spouses and children from Stiglitz (1988a).

⁴The Economic Recovery Tax Act of 1981 (ERTA 1981), The Social Security Amendments of 1983 (SSA 1983), and The Tax Reform Act of 1986 (TRA 1986).

of investment income has played a major role in the creation of the private retirement system.⁵

The income smoothing problem in our paper adds to the literature on tax shifting within a progressive system. There are papers studying the tax implications of shifting income across spouses, unmarried couples, from parents to children, and across households.⁶ The tax code itself permits income smoothing. There are carry back and carry forward provisions for corporations (operating losses) and households (capital losses). Farmers and fishermen can pay tax on income averaged over the current year and the three past years and, during a twenty-year period before TRA 1986, income averaging is available to the general tax payer.

The rest of the paper is organized as follows: Section 2 provides the reader with a short summary of the main features of the private and public pension systems in the United States. In the next Section 3, we derive the tax implications of income smoothing using a tax table with only three income brackets and large tax rate jumps. In Section 4, we apply the model to personal income taxation in the United States 2010. The section ends with the analysis of the historical United States time-series. Section 5 concludes the paper. In Appendix A, we analyze two examples of income smoothing and uncertainty. In one example, uncertainty is irrelevant and, in the other example, uncertainty decreases the smoothing benefit. We conclude from these examples that uncertainty is largely irrelevant to income smoothing. When uncertainty matters, it unambiguously decreases the smoothing benefit.

2 Institutional Background

In the United States, there is an array of retirement savings options. We describe the most relevant options to our study. We classify options according to the tax provisions that attach to them. Other important attributes include the sponsor, the party who bears the risk of investment performance, and the option of not participating.

A large class of retirement savings options allows for pre-tax contributions to savings, tax-free

⁵See Rydqvist, Spizman, and Strebulaev (2013) for a statistical test of this hypothesis in a cross-country and time-series panel data set.

⁶Stephens and Ward-Batts (2004) analyze spouses, Eissa and Hoynes (2000) study unmarried couples, and Stiglitz (1988b) discusses shifting income from parents to children. Green and Rydqvist (1999) study interpersonal netting of stock market gains and losses through trading in lottery bonds.

growth, and taxed withdrawals. Options in this class include 401(k), 403(b), 414(h), and 457(g) employer-sponsored retirement savings plans. Qualifying contributions to traditional Individual Retirement Accounts (IRAs) are also included. The difference between IRAs and the other options is that IRAs are self-initiated, while 401(k) products must be provided by employers. Also, the statutory limits on IRA contributions are relatively small. Defined benefit pension plans also belong to this class. Although workers may not have to contribute explicitly to their pension plan it is often the case that pension benefits are granted in lieu of wage increases (Lowenstein (2008)). Hence defined benefit pension plans offer implicit pre-tax retirement savings. The important distinction between defined contribution plans such as 401(k)s and defined benefit plans is the party that bears the risk of investment performance. In the former it is the worker, in the latter it is the employer.

A second option for retirement savings is an after-tax contribution that offers tax free growth and withdrawal of earnings. Options in this class are employer-sponsored Roth 401(k)s and self-initiated Roth IRAs. As in the pre-tax case, the Roth 401(k) allows for significantly higher statutory limits on contributions than the Roth IRA. It is also possible to convert traditional IRAs and 401(k)s to Roth plans. Under a flat-tax system, Roth and traditional plans yield identical tax savings. The Roth account also has the advantage of higher effective contribution limits (see Burmann, Gale, and Weiner (2001)).

A third option, that is generally not of great importance, are after tax contributions that offer tax free earnings growth but not tax free withdrawal. Most notable in this class are contributions to traditional IRAs that do not qualify for pre-tax treatment. For individuals covered by a retirement plan by their employers, the income limit excludes many individuals who would have the financial resources for savings from qualifying for pre-tax treatment of contributions. For individuals without an employer-sponsored plan there is no income limit, however there are significant limits on the size of contributions relative to 401(k) plans. United States Savings Bonds have similar provisions in that they are after-tax savings vehicles that accrue earnings tax free but are taxed upon withdrawal. One difference between savings bonds and IRAs is that savings bond income is not taxable at the state and local level.

By far the most important source of retirement income in the United States is Social Security.

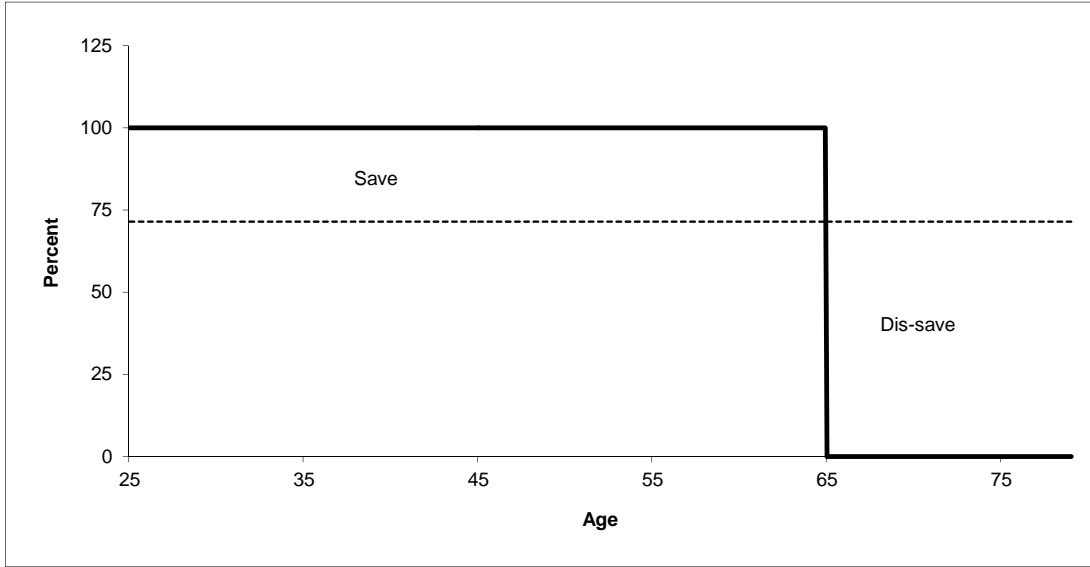
Unlike all of the options described above, Social Security is mandatory for most workers in the United States. Overall Social Security accounts for about 40% of retirees' income and virtually 100% of income for the bottom third of retirees (Reno and Lavery (2007)). Social Security shares some tax attributes with all three retirement savings options described above. Contributions from workers are made after tax, there are no taxes paid during accumulation of benefits and, for many retirees, benefits are not taxed upon withdrawal. In this sense, Social Security acts like a Roth IRA or Roth 401(k). Beginning in 1984, individuals who have significant income in retirement other than Social Security may have some of their Social Security benefits taxed as regular income. For these individuals Social Security acts, in part, like non-qualified contributions to traditional IRAs. Of particular note is that withdrawals from 401(k) plans and similar products are considered other income while withdrawals from Roth products are not. Therefore, savings for retirement pre-tax may come at a cost of increased taxes on Social Security benefits. Finally, half of Social Security premiums are paid by employers. To the extent that these premiums substitute for taxable wages, Social Security resembles a private pension and, hence, provides some smoothing benefits (Vroman (1974) and Ippolito (1986)).

3 Income Smoothing

We analyze the consumption and savings decision of a worker who can save before income tax in a retirement account that is either a company pension, a 401(k)-type account, or an Individual Retirement Account (IRA). The worker contributes to the retirement account over n work years to support consumption over $m - n$ retirement years. A numerical example is provided in Figure 1 assuming certainty, zero interest, no income growth, and no Social Security income. The worker has an annual income of 100 during work years and zero income during retirement years, he begins working at age 25, he retires at age 65, and he dies at age 81. In this example that entails perfect income smoothing over the life cycle the before-tax savings rate is 28.6%.

The model generalizes this example. We consider a worker who maximizes his after-tax income by solving a tax-minimization problem and, then, uses the capital market to reach his desired life-time consumption path. The capital market is represented by a Roth-style savings account

Figure 1: Life-Cycle Model



The figure shows the annual before-tax income (solid line) and the annual before-tax income smoothed over adult life time (dotted line). We assume 40 work years and 16 retirement years.

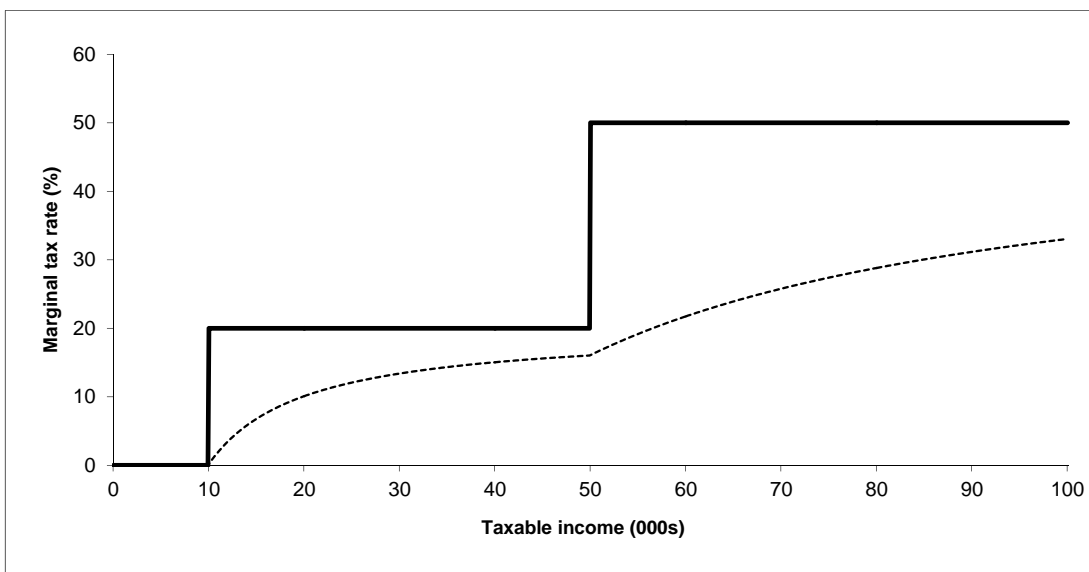
that preserves all the tax benefits of the regular retirement account except the smoothing benefit. We also assume that the worker can save as much for retirement as he wants (no contribution limit), he can withdraw funds from either account without penalty, and he can borrow any amount he wishes against future income. These assumptions allow us to analyze the tax-minimization problem independently of the utility-maximization problem.⁷ In what follows, we explicitly solve the tax-minimization problem only, and we abstain from parameterizing the life-cycle consumption path. Our approach makes the smoothing benefit the largest possible. Realistic modeling of life-cycle consumption behavior with capital market restrictions on borrowing and lending would only reduce the smoothing benefit relative to the tax-minimizing benchmark.

One tax-minimizing solution is to smooth before-tax income perfectly over the life cycle. Perfect income smoothing implies a smooth after-tax consumption stream, which is supported by standard utility functions with time impatience equal to the market discount factor. Perfect income smoothing is a solution in the base case model with interest and income growth, but it is not tax-minimizing with Social Security.

⁷The corresponding assumption that allows the researcher to separate the tax problem from the investment problem is used by Constantinides (1983), Huang (2008), and others.

We begin analyzing the tax-minimization problem with zero interest, no income growth, no Social Security income, and full certainty over future income and tax liability, as in Figure 1. After we have presented the base case model in Subsection 3.1, we extend the analysis to include the effects of positive interest and income growth in Subsection 3.2, and Social Security income in Subsection 3.3. In Appendix A, we study two examples of uncertainty. While we believe that uncertainty is a first-order determinant of life-cycle consumption behavior, we conclude from these examples that uncertainty is inessential to the tax-minimization problem.

Figure 2: Simple Tax Table



This figure plots marginal and average tax rates for a progressive tax table with three income brackets and marginal tax rates at 0%, 20%, and 50%. The breakpoints between income brackets are \$10,000 and \$50,000, respectively.

Our plan is to numerically illustrate the properties of the model using the tax table in Figure 2. The tax table is progressive. It is a step function with three income brackets and marginal tax rates at 0%, 20%, and 50%. The breakpoints between income brackets are \$10,000 and \$50,000, respectively. Within each income bracket the marginal tax rate is constant. The average tax rate curve plotted below (dotted line) is an increasing function with kinks at the breakpoints between income brackets. The tax table has few income brackets as is typical after TRA 1986, but the tax rate increments between income brackets have been exaggerated to bring out the model properties.⁸

⁸Tax tables before TRA 1986 often have more than 30 income brackets. Such tax tables would be better approxi-

3.1 Base Case

The worker earns labor income over n years and lives m years. With constant annual income, we can reduce the problem to one where the worker chooses a before-tax savings rate that is equal for all working years. Hence, the worker's objective is to choose a before-tax savings rate ϕ that minimizes lifetime tax liability:

$$\min_{\phi} \mathcal{T} = nT((1 - \phi)Y) + (m - n)T\left(\frac{n\phi Y}{m - n}\right), \quad (1)$$

where Y is annual income and $T(\cdot)$ is the tax liability function. The first term on the right hand side is the tax liability on work income, and the second term is the tax liability on retirement income. The first-order condition for a minimum requires that the marginal tax rates during work years and retirement years are equal:⁹

$$T'((1 - \phi)Y) = T'\left(\frac{n\phi Y}{m - n}\right). \quad (2)$$

One specific solution that we refer to as perfect income smoothing is when before-tax income during work years equals before-tax income in retirement:

$$\phi^* = \frac{m - n}{m}. \quad (3)$$

Generally, the tax-minimization problem has multiple solutions because the tax liability function is piecewise linear. As long as marginal tax rates are different during work years and retirement years, tax liability can be reduced by shifting income, but when marginal tax rates are equal, shifting income between work years and retirement does not change tax liability.

The reduction of lifetime tax liability divided by lifetime income is a natural measure of the tax

mated by a continuous function. Interestingly, continuous tax tables are used in Germany, where income tax liability is determined by a higher-order polynomial.

⁹The first-order condition is not defined right at the breakpoint between income brackets.

benefit of income smoothing. Specifically, we define the smoothing benefit as:

$$\text{SMOOTH} = \frac{\text{Non-Smooth} - \text{Smooth}}{nY}. \quad (4)$$

The non-smooth term is obtained from the objective function (1) evaluated at $\phi = 0$, and the smooth term from the function evaluated at one of the optimal savings rates ϕ^* . Tax liability divided by before-tax income is the definition of the average tax rate, so the smoothing benefit measure can be interpreted as a reduction of the average tax rate. The smoothing benefit measure increases with the slope of the average tax rate curve, which depends on tax progressivity.¹⁰ The average tax rate curve is upward-sloping when the tax is progressive, it is flat when the tax is proportional, and it is downward-sloping when the tax is regressive. Accordingly, the worker benefits from income smoothing if the tax is progressive, income smoothing has no effect on tax liability if the tax is proportional, and income smoothing hurts the worker if the tax is regressive.¹¹ The smoothing benefit measure is also a function of the number of work years n relative to the number of retirement years $m - n$. An increase in the number of work years relative to time in retirement reduces relative smoothing space, while an increase in retirement years relative to work years increases it.

Figure 3 plots SMOOTH as a function of before-tax income (dotted line; right scale) along with the marginal tax rate function (solid line; left scale). The SMOOTH function has a local minimum at the breakpoint between income brackets, and it has a local maximum to the right of each breakpoint, where the tax-minimization problem has a unique solution.¹² Not surprisingly, the tax benefit of income smoothing is most pronounced to the right of the breakpoints between income brackets where the worker can just shift all income out of the higher bracket.¹³ For the same reason,

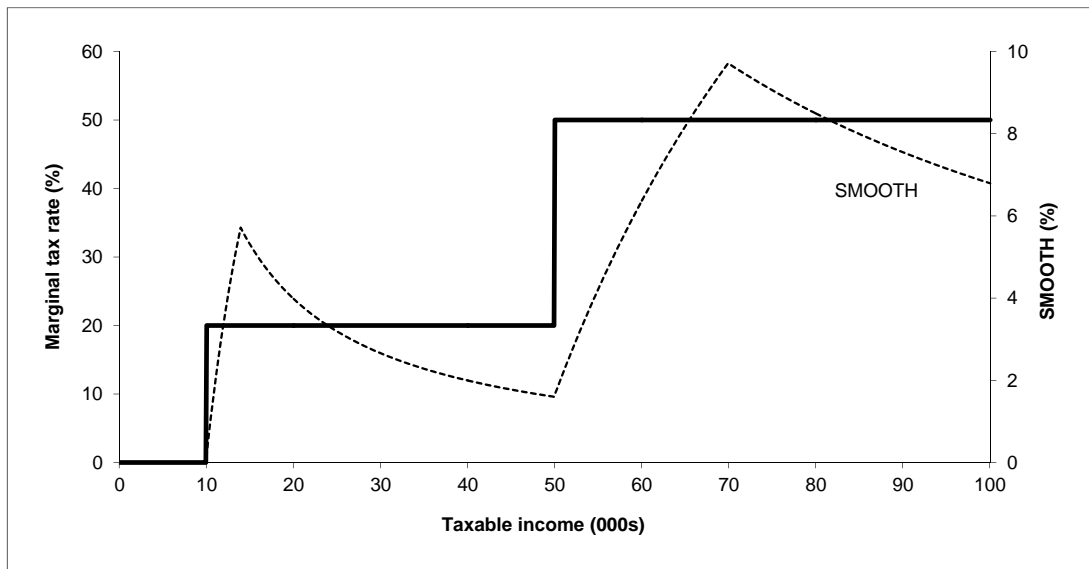
¹⁰Slitor (1948) proposes to measure tax progressivity as the instantaneous slope of the average tax rate curve. Other common measures of tax progressivity are based on the difference between the marginal tax rate and the average tax rate (see Røed and Strøm (2002)).

¹¹The savings rate ϕ drops out from the tax-minimization problem (1), whenever the tax liability function $T(\cdot)$ is a proportional tax rate. Income tax tables are generally progressive. One interesting exception is the taxation of Social Security income (see Subsection 3.3).

¹²The seemingly close connection between the uniqueness of the tax-minimization solution and the local SMOOTH maximum is not general. The SMOOTH function is either monotonically increasing, monotonically decreasing, or hump shaped within each bracket. Depending on bracket width and the marginal tax rate difference, SMOOTH may or may not peak at the uniqueness point.

¹³See Milligan (2003) for a related discussion (page 260, footnote 11).

Figure 3: Tax Benefit of Income Smoothing



This figure plots the marginal tax rate function (solid line; left axis) along with the tax benefit of income smoothing (dotted line; right axis) using the progressive tax table, 40 work years, and 16 retirement years.

the smoothing benefit is particularly small right at each breakpoint where there is significantly more income in the highest bracket than can be shifted into lower brackets in retirement.

3.2 Interest and Income Growth

In this subsection, we investigate how real interest and income growth influence the tax benefit of income smoothing. We analyze real as opposed to nominal interest and income growth because we are interested in long-term retirement planning. In accordance with current practice, we assume that tax tables are indexed to inflation.

3.2.1 Interest

With positive interest, the worker’s objective is to minimize present value of taxes. The full-certainty problem has a simple analytical solution. At age 25, the worker enters an agreement with his employer to swap an N -year working-life annuity for an M -year life-long annuity. Interest generates investment income that fills up smoothing space in lower income brackets. Since the

worker saves less, the tax benefit of smoothing labor income unambiguously decreases.

Consider the interest rate $r > 0$. Each year, the worker pays tax and consumes the net proceeds from $(1 - \phi)Y$, and he saves the residual ϕY for retirement. The future value of the retirement account after n work years equals $F_n \phi Y$, where $F_n = [(1 + r)^n - 1]/r$ is the future-value annuity factor. The worker's decision problem is to choose a before-tax savings rate that minimizes present value of taxes:

$$\min_{\phi} \mathcal{T} = A_n T((1 - \phi)Y) + PV_n A_{m-n} T\left(\frac{F_n \phi Y}{A_{m-n}}\right), \quad (5)$$

where $PV_n = 1/(1 + r)^n$ is the present value factor and $A_n = (1 - 1/(1 + r)^n)/r$ is the present value annuity factor. The first-order condition for a minimum requires that marginal tax rates are equal during work years and retirement:

$$T'((1 - \phi)Y) = T'\left(\frac{F_n \phi Y}{A_{m-n}}\right), \quad (6)$$

Interest enters the smoothing problem inside the right hand side of the first-order condition. It raises the future-value annuity factor in the numerator, and it lowers the present-value annuity factor in the denominator. Both effects are potentially large. A one percent real interest rate raises the future-value annuity factor from $F_n = 40$ to $F_n = 48.9$, which means that one percent real interest compounded over 40 years is equivalent to being able to save for retirement over 8.9 more work years. Furthermore, the continued earning of real interest at the one percent rate during retirement reduces the present-value annuity factor by a little more than one year down from $m - n = 16$ retirement years at zero interest to $A_{m-n} = 14.7$ with one percent real interest.

As previously, perfect income smoothing is one solution that is attained as follows: When the worker enters the labor market, he swaps an N -year working-life annuity for an M -year life-long annuity with his employer.¹⁴ The annuity swap implies the tax-minimizing savings rate:

$$\phi^* = \frac{A_{m-n}}{A_{m-n} + F_n} = \frac{A_m - A_n}{A_m}. \quad (7)$$

¹⁴With uncertainty, the employer may not agree to the annuity swap, and the employee must accumulate funds inside a retirement account that is subject to contribution and withdrawal limits. The effects of such imperfections is to reduce the smoothing benefit.

This expression means that the savings rate decreases with the interest rate. Using the base-case parameters of 40 work years and 16 retirement years, a real interest rate of $r = 1\%$ reduces the savings rate from 28.6% to 23.1%, and a real interest rate of $r = 4\%$ reduces it to 10.9%. Since the worker saves less, the smoothing benefit decreases.

3.2.2 Income Growth

The base case model assumes that labor income is constant before retirement. To explore a realistic feature that labor income grows over time, we assume that work-year income grows geometrically at a constant rate. Geometric growth is an approximation of the observed life-cycle income path that follows an inverse U-shaped pattern, where income peaks around age 50 rather than age 65 (Browning and Crossley (2001)). However, the geometric growth model approximates the data if we reshuffle the income years, so that the income peak occurs at age 65 instead of age 50. We also assume that the present value of life-time income with growth equals the present value of life-time income without growth. This assumption implies that life-time income with growth is a mean-preserving spread of life-time income without growth. It ensures that any differences between the growth and the non-growth cases in our analysis are due to growth and not changes on overall income levels.

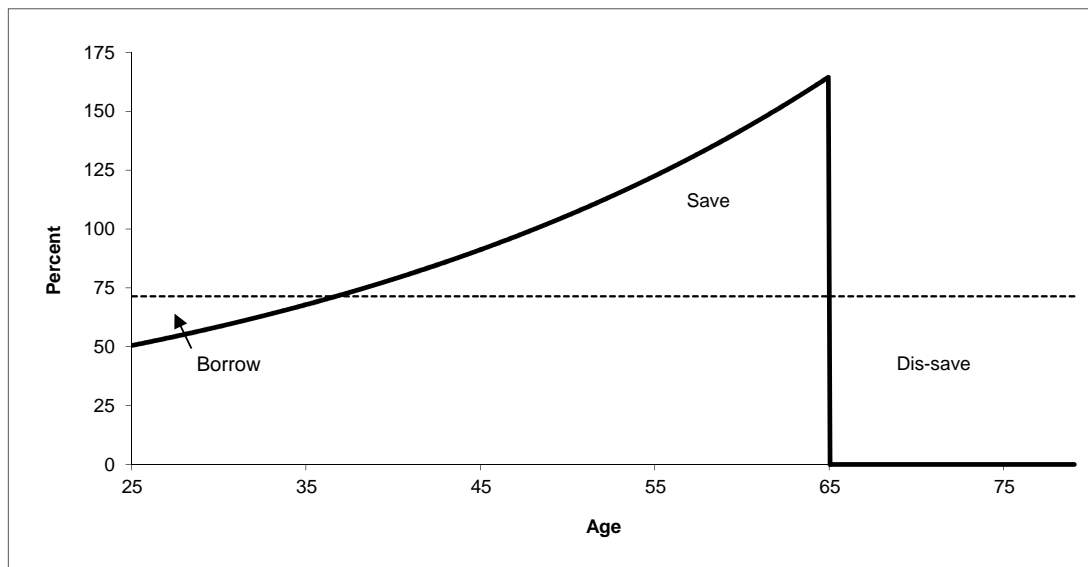
We solve the model with income growth the same way we solve the model with interest. At age 25, when the worker enters the labor market, he swaps an N -year growth annuity for an M -year fixed annuity with his employer. Formally, we define the future-value annuity factor of income growth with zero interest as $G_n = [(1 + g)^n - 1] / g$, and we let the starting income level be Y_0 . Then:

$$PV_n G_n Y_0 = A_n Y = A_m (1 - \phi) Y. \quad (8)$$

The equality to the left follows from assuming that the growth annuity is a mean-preserving spread of the fixed working-life annuity, and the equality to the right is the annuity swap. Perfect lifetime income smoothing is the best the worker can do to minimize tax liability. It implies that marginal tax rates are equal all working years and all retirement years. The non-smooth term of Equation (4) increases as a result of income growth, while the smooth term is the same as without growth. A

mean-preserving spread unambiguously raises the tax benefit of income smoothing because the tax liability function is convex.

Figure 4: Income Growth



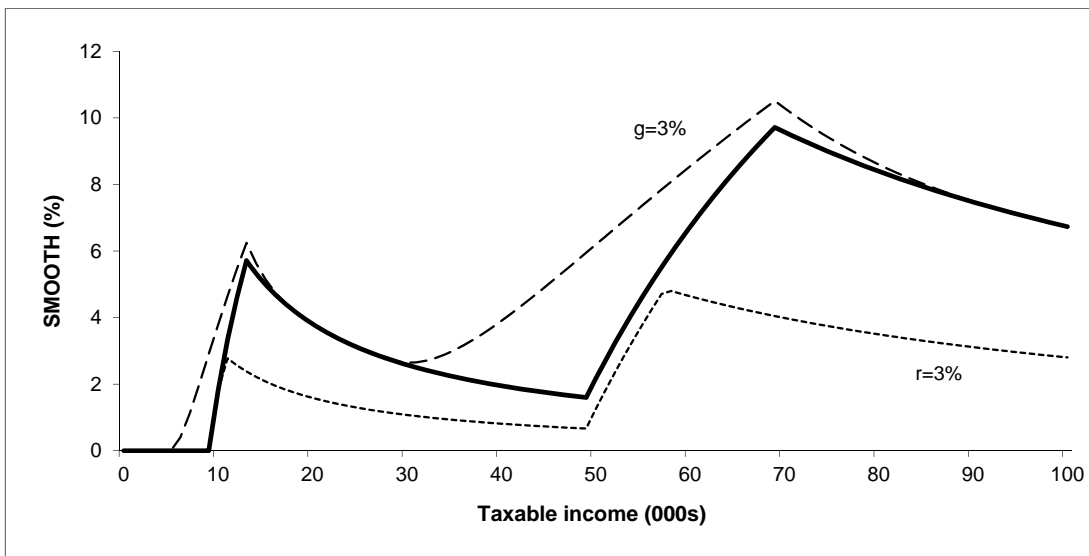
The figure shows the annual before-tax income (solid line) and the annual before-tax income smoothed over adult life time (dotted line). The growth rate is 3%.

A numerical example with constant growth and perfect income smoothing can be seen in Figure 4. The worker borrows (from his employer) from age 25 to 35, he saves from age 36 to 64, and he dis-saves from age 65 to 81. In practice, the worker may not be able to borrow on a before-tax basis during early work years, and contribution limits may restrict how much he can save before tax during late years. Borrowing and lending constraints reduce the positive impact of income growth on the smoothing benefit.

3.2.3 Smoothing with Interest and Income Growth

Interest and income growth move the smoothing benefit in opposite directions. The net effect depends on parameters and income level. In Figure 5, we plot the smoothing benefit against taxable income assuming zero interest and no income growth (solid line), real interest rate $r = 3\%$ (dashed line below), and real growth rate $g = 3\%$ (dashed line above). The smoothing benefit decreases with interest, and it increases with income growth. The effect of interest is most pronounced at

Figure 5: Smoothing with Interest and Income Growth



The figure plots the smoothing benefit as a function of taxable income assuming zero interest and no growth (base case; solid thick line), 3% interest (dashed line below), and 3% growth (dashed line above). The calculations are based on the progressive tax table, 40 work years, and 16 retirement years.

each local SMOOTH maximum. The effect is also large at high income levels because high income earners save more and interest on larger savings fill up fixed smoothing space in lower income brackets, thus preventing the worker from smoothing labor income. The effect of income growth is the largest at the local SMOOTH minima. Elsewhere, at very low income levels, income growth does not push the worker into the higher bracket and, at very high income levels, taxable income quickly fills up fixed smoothing space.

3.3 Social Security

Social Security is a government-sponsored program in the United States that provides support for individuals in times of hardship such as death of a spouse, disability, and old age (retirement). The program is supported by a 6.2% payroll tax paid by the employee and a 6.2% payroll tax paid by employers, both capped at wages of \$106,800 in 2010. In our analysis, we ignore the contribution component and treat the income received from the Social Security system as exogenous. To the extent that Social Security income is taxable, it fills up smoothing space with non-labor income

thereby reducing the tax benefit of income smoothing. The effect of taxable Social Security on the smoothing benefit is qualitatively similar to the effect of real interest.

The taxation of Social Security income differs from the taxation of ordinary income. Up to 85% of Social Security income is taxable under current tax laws. The percentage depends on the amount of other income received in retirement and Social Security income. To determine the taxed portion of Social Security income, we first define combined income:

$$\hat{Y}(\phi) = \frac{n\phi Y}{m-n} + 0.5Y_s, \quad (9)$$

where the first term is the amount of private retirement income given by our model (with zero interest and no income growth), and the second term is half the Social Security income. The taxed portion of Social Security income is determined by the following function:

$$Y_s^\tau(\phi) = \begin{cases} 0, & \text{if } \hat{Y}(\phi) \leq \$32,000, \\ \min[0.50(\hat{Y}(\phi) - \$32,000), 0.50Y_s], & \text{if } \$32,000 < \hat{Y}(\phi) \leq \$44,000, \\ \min[0.85(\hat{Y}(\phi) - \$44,000) + \$6,000, 0.85Y_s], & \text{if } \hat{Y}(\phi) > \$44,000. \end{cases} \quad (10)$$

Social Security income is tax free at low income levels (first row), and it is 85% taxable at high income levels (third row). Between the floor and the cap, the taxed portion of Social Security income increases linearly at the rate of 50 cents per dollar between \$32,000 and \$44,000, and at the rate of 85 cents per dollar above \$44,000. From these definitions, we derive the income-tax basis with private retirement income and Social Security income:

$$Y_r(\phi) = \frac{n\phi Y}{m-n} + Y_s^\tau(\phi) \quad (11)$$

These equations show that saving privately for retirement changes the taxed portion of Social Security income.

The worker solves the following tax-minimization problem with income from Social Security:

$$\min_{\phi} \mathcal{T} = nT((1-\phi)Y) + (m-n)T(Y_r(\phi)), \quad (12)$$

with first-order condition given by:

$$T'((1 - \phi)Y) = T'(Y_r(\phi)) + \left(\frac{(m - n)Y_s^{\tau'}(\phi)}{nY} \right) T'(Y_r(\phi)), \quad (13)$$

where $Y_s^{\tau'}(\phi) = \{0, 0.50, 0.85\}$. With Social Security income, the optimality condition is not given by the simple rule that marginal tax rates during working years and retirement are equal.¹⁵ The worker picks a tax-minimizing solution ϕ^* that determines the tax benefit of income smoothing as:

$$\text{SMOOTH} = \left[\frac{nT(Y)}{nY + (m - n)Y_s} \right] - \left[\frac{nT((1 - \phi^*)Y) + (m - n)T(Y_r(\phi^*))}{nY + (m - n)Y_s} \right]. \quad (14)$$

The first term on the right hand side is lifetime tax liability evaluated at $\phi = 0$. Without private retirement income, Social Security income is tax free, $Y_r(0) = Y_s^{\tau}(0) = 0$. The second term is lifetime tax liability evaluated at one of the tax-minimizing savings rates ϕ^* . The smoothing benefit is scaled by lifetime income including Social Security. The calculations of tax liability under the Social Security program are specific to the United States. Hence, we abstain from illustrating the effects of Social Security income using the simple tax table of Figure 2.

4 Income Smoothing in the United States

4.1 United States 2010

In this section, we apply the model to the institutional setting of the United States 2010. We replace the simple tax table with the 2010 federal income tax table for a married couple filing jointly taking the standard deduction and two exemptions.¹⁶ The effects of progressive state and local taxes that vary widely across the United States are not considered.¹⁷ Many households can do better than the standard deduction by itemizing mortgage interest, state and property taxes, etc., and they can also claim exemptions for children. These tax deductions are household and time

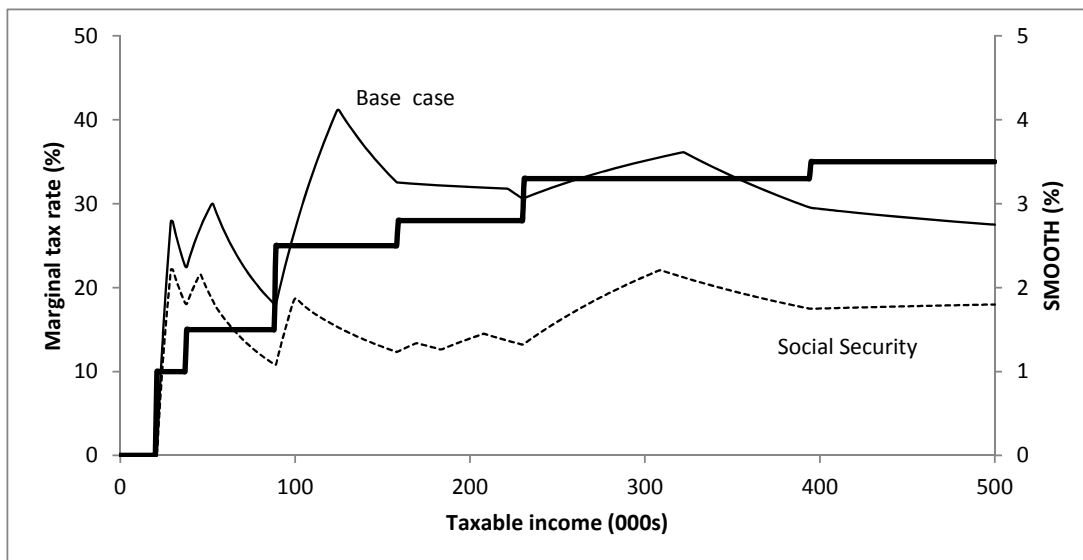
¹⁵We solve the tax-minimization problem numerically. Curiously, in a middle-income zone, where the worker manages his private savings such that Social Security income is either fully or 50% exempt, the tax-minimizing solution is unique over a wide income range.

¹⁶The calculations take into account that the personal exemption is higher for elderly of age 65 and above.

¹⁷By ignoring state and local taxes, we ignore the option to move from a high-tax state during work years to a zero-tax state in retirement.

specific. Contribution limits that tend to be household specific are also ignored.¹⁸ The effect of contribution limits and borrowing restrictions is to reduce the smoothing benefit. For our numerical calculations of Social Security income, we use the benefits estimator found on the Social Security web site. As noted by the Social Security Administration, “The benefit computation is complex and there is no simple method or table to tell you how much you may receive”. We retrieve the benefit amount for wages in increments of \$5,000 and interpolate the values between. In order to be conservative, we assume the married couple has a single income and credit the spouse with 50% of the wage earners benefits.¹⁹

Figure 6: Income Smoothing in the United States 2010



This figure plots the federal tax rate schedule for a married couple filing jointly taking the standard deduction and two exemptions (solid line; left axis) and the tax benefit of income smoothing assuming 40 work years, 16 retirement years, no interest, and no income growth. The base case is without Social Security income.

The federal tax rate schedule up to \$500,000 (left axis) along with the smoothing benefit assuming can be seen in Figure 6. The smoothing benefit without Social Security income (base case)

¹⁸Contributions to defined benefit plans are subject to benefit limits and non-discriminatory rules. Contributions to 401(k) plans are usually stated in percent of before-tax income subject to the maximum 415 limit. Contributions to elective deferrals and IRAs are subject to a limit that applies equally to all workers below or above age 50 (catch-up provisions).

¹⁹For married couples, the spouse with the lower earned benefits receives the greater of actual earned benefits or 50% of the spouse’s earned benefits. With two equal incomes the household receives 200% of the Social Security benefit of the single income earner. Due to the progressivity of Social Security benefits, married couples may receive more than twice the benefits of a single individual with the same income.

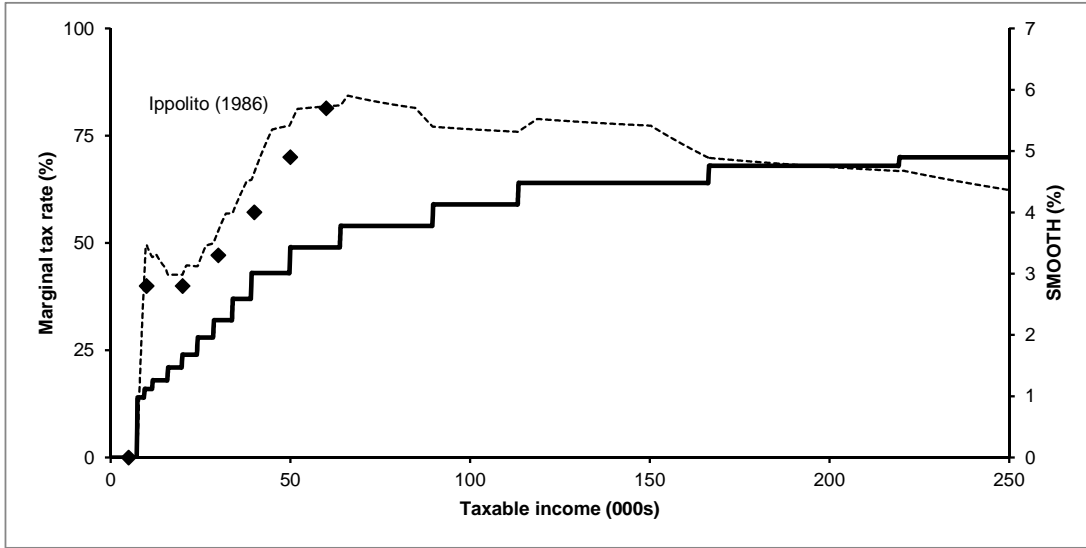
is about two percent at low income levels, and it fluctuates between three and four percent from \$100,000 to \$500,000. The average smoothing benefit in this income range is 3.2%. The smoothing benefit is sensitive to the income level around the breakpoint between the 15% and 25% income brackets. Otherwise, it is approximately constant. Maximum SMOOTH occurs at annual income \$125,000, where the worker can shift all his income from the 25% bracket into the two lower brackets and the tax-exempt smoothing space created by the standard deduction and personal exemptions. Social Security income reduces the tax benefit of income smoothing relative to the base case by approximately one to two percentage points. The smoothing benefit averaged from \$100,000 to \$500,000 decreases from 3.2% without Social Security income to 1.7% with such income. The main reason is smoothing space reduction. Taxable Social Security income fills up the lower infra-marginal brackets that otherwise would have been available for private retirement income. At lower income levels, a large portion of Social Security income is tax exempt and, therefore, does not fill up smoothing space.

The real interest rate and growth rate are not easily estimated, and we limit ourselves to examples. Suppose the long-term real interest and growth rates both equal 1%, then the average smoothing benefit with Social Security equals 1.4%. If we raise the interest and growth rates to 2%, the average smoothing benefit decreases further to 1.2%. The bottom line of these numerical calculations is that the reward for lifelong planning is a tax reduction of between one and two percentage points.

4.2 United States 1950–2010

Ippolito (1986) bases his smoothing benefit calculations on actual taxes paid according to Statistics of Income rather than the Federal tax rate schedule. He assumes that work begins at age 25 and that the worker retires at age 65 but, since life expectancy is shorter in 1979, the worker dies at age 78. His smoothing benefit calculations are marked as diamonds in Figure 7, where we also plot the Federal marginal tax rate schedule up to \$250,000 and our smoothing benefit function using his demographic parameters. Our smoothing benefit estimates are a little higher than those of Ippolito

Figure 7: Income Smoothing in the United States 1979



This figure plots the federal tax rate schedule for a married couple filing jointly taking the standard deduction and two exemptions (solid line; left axis) and the tax benefit of income smoothing assuming 40 work years, 13 retirement years, no interest, and no income growth (dashed line; right axis). Ippolito (1986) bases his calculations on the Statistics of Income (filled diamonds).

(1986), but the increasing pattern is similar.²⁰ The average smoothing benefit is generally higher than in 2010 because the 1979 tax table is more progressive. The smoothing benefit averaged over the income range \$20,000 to \$120,000 is 5.2%. This income range corresponds in real terms to the income used in Figure 6 for 2010 data. The smoothing benefit does not depend on Social Security income, which is tax free in 1979.

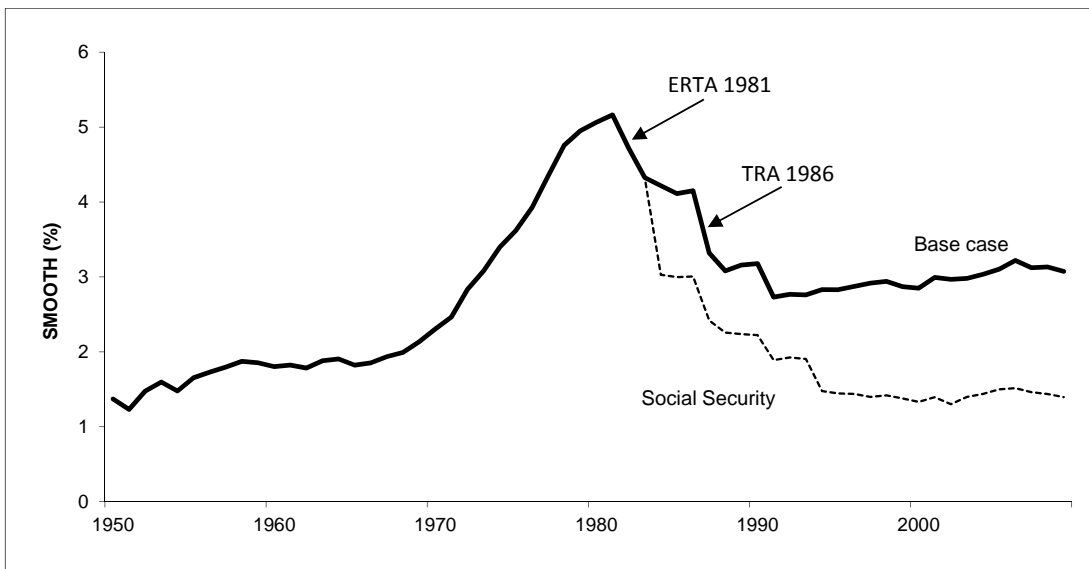
Next, we compute the post-war time-series of the smoothing benefit of a worker with constant before-tax income. Tax liability is evaluated assuming an annual income of five times GDP per capita, which is approximately \$60,000 in 1979 and \$230,000 in 2008 (the most recently available). This income level is relatively high so that saving privately for retirement matters, but it is not so high that standard pension plans become marginal. It is at the margin of the 401(k) contribution limit and statutorily feasible.²¹ We compute the smoothing benefit with and without Social Security. A worker with income equal to five times GDP per capita receives the maximum Social

²⁰We infer these estimates from the information provided by Ippolito (1986) in Table 2-1.

²¹Contribution limits to 401(k) plans are stated in The Economic Recovery Tax Act of 1981. See Gokhale, Kotlikoff, and Warshawsky (2004) for the history of contribution limits.

Security benefit. GDP-per-capita time-series are taken from the International Financial Statistics Browser provided by the International Monetary Fund. Time-series of life expectancy statistics are from the Human Mortality Database, the average of male and female conditional on age 25.²²

Figure 8: Smoothing Benefit History



The solid line above is the base case smoothing benefit assuming 40 work years and time in retirement equal to life expectancy minus 65. Tax liability is evaluated at an income of five times GDP per capita for a married couple filing jointly taking the standard deduction and two exemptions. The dashed line below adjusts for Social Security income assuming that the married couple receives 150% of the wage earner’s Social Security benefit. We have marked The Economic Recovery Tax Act of 1981 (ERTA 1981) and The Tax Reform Act of 1986 (TRA 1986). We assume that the worker cannot predict these and other tax law changes.

The post-war time-series of smoothing benefits of a worker with before-tax income of five times GDP per capita is displayed in Figure 8. The solid line above represents our base case, and the dashed line below the smoothing benefit with Social Security. We have also marked a few major tax reforms that, according to the assumptions of our model, the worker cannot predict. The time-series path is hump shaped with a positive time trend. Increasing life expectancy drives the time trend. The hump is the result of bracket creep and the regulatory response to combat it.²³ The

²²University of California, Berkeley (USA) available at www.mortality.org.

²³Income tax tables are essentially unchanged from 1950 to 1980. While personal income taxes are nominally fixed during this long thirty-year period, nominal income per capita increases approximately six times. As a result of income growth, five times GDP per capita grows from \$10,000 in 1950 to \$60,000 in 1980. The effect on the smoothing benefit can be seen in Figure 7 above. The smoothing benefit associated with \$10,000 income is about

peak occurs right before the tax reforms that reduce progressivity (ERTA 1981 and TRA 1986). Inflation indexing prevents further bracket creep. Social Security reduces the smoothing benefit, but the effect of Social Security changes over time. Social Security income reduces the smoothing benefit from 1984 when it becomes partly taxed (SSA 1983). The gap between the cases with and without Social Security widens in 1994 when the taxed portion increases and fills up more smoothing space.

5 Conclusion

The deferral principle of The Tax Revenue Act of 1921 is meant to stimulate private retirement savings. The exemption of investment income taxes appears to be a strong incentive to save for retirement. The magnitude of the smoothing benefit is less apparent. In this paper, we have computed an upper boundary of the smoothing benefit in a simplistic life-cycle model. Contrary to a widely-held belief and, particularly, that of Ippolito (1986), we conclude that the smoothing benefit is small. Our estimates of the upper boundary of the smoothing benefit for the United States in 2010 are 3.2% without Social Security and 1.7% adjusted for Social Security. These estimates ignore many real-world features that tend to make the estimates smaller.

Whether tax incentives stimulate private savings or simply shift savings from one form to another is debated. We have not studied what economic agents actually do. Especially, we have not discussed the effects of smoothing on individuals' savings behavior or the rationale for tax policies related to pension and retirement savings. However, our calculations suggest that the tax benefit of income smoothing is too small to explain the growth of the US private pension system, as claimed by Ippolito (1986). At best, the smoothing benefit can have inspired retirement planners and labor unions bargaining for pensions over wages during a short time period in the 1970s, when the smoothing benefit was substantially larger than today.

The deferral principle of the US private pension system is the bearing tax principle in many countries. It would be interesting to see numerical calculations of the smoothing benefit elsewhere,

3% (1950), while the smoothing benefit of \$60,000 is close to 6% (1980). Together with increased life expectancy, bracket creep raises the smoothing benefit from 1% in 1950 to above 5% in 1980 as seen in Figure 8.

in particular, how the smoothing benefit depends on the integration of public and private pension systems, the evolution of the smoothing benefit over time, and its impact on private retirement savings.

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A Appendix: Uncertainty

In this Appendix, we analyze uncertainty in the base case model. We construct two examples. *(i)* In the middle of his career, the worker experiences an income shock. After 20 years, income increases or decreases by 50% with equal probability. *(ii)* At retirement, the worker finds out whether he is terminally ill or lives longer than expected. Life expectancy is either 66 or 97 with equal probability. If he is terminally ill, he immediately pays income tax on all his retirement savings and consumes the after-tax proceeds. The two examples are symmetric, so that the average across the two states equals the base case. The numerical results of the experiments are summarized in Table A1, and the details of the calculations are explained below.

Table A1: Income Smoothing and Uncertainty

	Average tax rate		Smoothing benefit	
	Without smoothing (1)	With smoothing (2)	Certainty (3)	Uncertainty (4)
Base case (one state):	18.8	14.0	4.779	n.a.
Experiments (two states):				
Labor income	21.9	16.9	5.028	5.028
Life expectancy	20.9	16.6	4.329	3.607

The table reports average tax rates and smoothing benefits in the base case model using the simple tax table, 40 work years, and 16 retirement years. Average tax rates are evaluated for each increment of \$1,000 and averaged over the income range \$0 to \$100,000. In the first experiment, labor income jumps up or down by 50% after 20 years and, in the second experiment, life expectancy at retirement is either 65 or 97. Each state is equally likely. Under full certainty, the worker knows the state of the world when he enters the work force. We report the average across the two states. Column (3) equals the difference between Column (1) and (2). Uncertainty means that the worker learns the state of world the after 20 years.

The tax rates in Column (1) and (2) show that variability raises tax liability under a progressive tax system, and the smoothing benefits in Column (3) verify that income smoothing reduces tax liability. In each experiment, variability is higher than in the base case, but each path is not a mean-preserving spread of the base case, so the smoothing benefit may increase or decrease. Whether the worker knows the state of the world from the beginning or finds out what it is upon realization matters in the example of uncertain life expectancy, but it makes no difference in the

example of uncertain labor income. When uncertainty matters, it unambiguously decreases the smoothing benefit.

A.1 Uncertain Lifetime Income

We assume that, after k work years, income jumps up or down by a parameter $\lambda \in [0, 1]$ with equal probability, such that:

$$\begin{aligned} Y_u &= Y(1 + \lambda), & \text{probability} &= 0.50, \\ Y_d &= Y(1 - \lambda), & \text{probability} &= 0.50. \end{aligned} \tag{A1}$$

First, we establish a full certainty benchmark against which the effects of uncertainty can be compared. Suppose the worker knows at age 25 whether his income after k work years will jump up or down. The tax-minimization problem with a known income path can be written as:

$$\min_{\phi_s} \mathcal{T} = kT((1 - \phi_s)Y) + (m - k)T\left(\frac{k\phi_s Y + (n - k)Y_s}{m - k}\right), \quad s = u, d. \tag{A2}$$

This is a natural extension of the baseline model, Equation (1). The worker chooses one savings rate for the first k working years and then he smooths income perfectly over the remaining $m - k$ life years. Lifetime tax-liability with and without smoothing, averaged across the two income paths, are equal to:

$$\begin{aligned} \text{Non-Smooth} &= kT(Y) + E[(n - k)T(Y_s)], \\ \text{Smooth} &= E\left[kT((1 - \phi_s^*)Y) + (m - k)T\left(\frac{k\phi_s^* Y + (n - k)Y_s}{m - k}\right)\right]. \end{aligned} \tag{A3}$$

The average smoothing benefit equals the difference between these two terms divided by average lifetime income, $n(Y_u + Y_d)/2 = nY$. This is our new full certainty benchmark.

Next, we introduce uncertainty. Suppose the worker knows that income will jump up or down after k work years, but he does not know in advance whether the jump will be positive or negative.

The corresponding tax-minimization problem with uncertainty is:

$$\min_{\phi} \mathcal{T} = kT((1 - \phi)Y) + (m - k)E\left[T\left(\frac{k\phi Y + (n - k)Y_s}{m - k}\right)\right]. \tag{A4}$$

As previously, the worker chooses a before-tax savings rate during the first k working years and, then, smooths income perfectly after the state of the world has been revealed. Facing uncertainty, the worker cannot condition the before-tax savings rate ϕ on the state of nature. The two terms of the smoothing benefit measure are:

$$\begin{aligned} \text{Non-Smooth} &= kT(Y) + E[(n-k)T(Y_s)], \\ \text{Smooth} &= kT((1-\phi^*)Y) + E\left[(m-k)T\left(\frac{k\phi^*Y+(n-k)Y_s}{m-k}\right)\right]. \end{aligned} \tag{A5}$$

The smooth term in Equation (A3) depends on ϕ_u^* and ϕ_d^* , respectively, while the smooth term in Equation (A5) depends on ϕ^* . However, there are parameter ranges where uncertainty is irrelevant. Specifically, let the range of tax-minimizing solutions in respective state be $[\phi_{min}^*(Y, s), \phi_{max}^*(Y, s)]$, and suppose that the range of tax-minimizing solutions conditional on being in the up-state overlaps with the range conditional on the down-state for all income levels Y such that:

$$\phi_{min}^*(Y, d) \leq \phi^* \leq \phi_{max}^*(Y, u), \quad \forall Y, \tag{A6}$$

Then, the worker can choose ϕ in the intersection, where tax liability is minimized regardless of which state of the world occurs, and uncertainty is irrelevant.

In Figure A1, we plot the smoothing benefit for the base case (solid line), known income variability (A3) (dashed line), and uncertainty (A5) (same dashed line) assuming a jump parameter of $\lambda = 0.5$ and $k = 20$ years. The sizable jump after 20 years makes only a small difference to the smoothing benefit, and uncertainty is irrelevant.

A.2 Uncertain Life Expectancy

Suppose life expectancy can be either high or low with equal probability:

$$\begin{aligned} m_u &= m + \lambda, & \text{probability} &= 0.50, \\ m_d &= m - \lambda, & \text{probability} &= 0.50, \end{aligned} \tag{A7}$$

Figure A1: The Smoothing Benefit with Uncertain Labor Income



The figure shows the smoothing benefit as a function of income using the simple tax table, the average smoothing benefit with certain but variable income, and the expected smoothing benefit with uncertainty. We assume 40 work years and 16 retirement years. Income increases or decreases after 20 years with equal probability.

where $\lambda = 1, 2, 3, \dots$ is an integer. The tax-minimization problem with known life expectancy is:

$$\min_{\phi_s} \mathcal{T} = E \left[nT((1 - \phi_s)Y) + (m_s - n)T \left(\frac{n\phi_s Y}{m_s - n} \right) \right], \quad s = u, d, \quad (\text{A8})$$

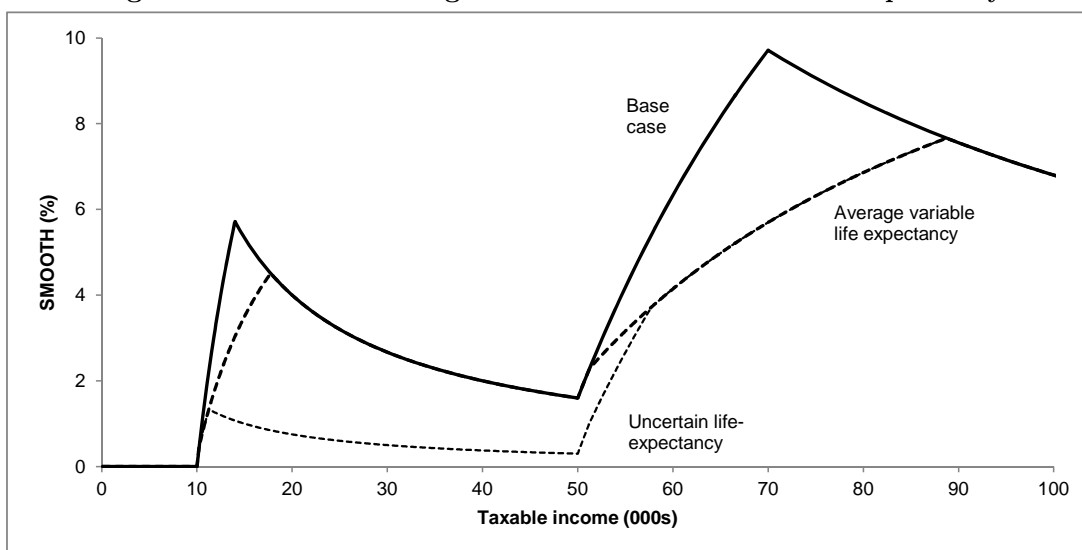
and the corresponding tax-minimization problem with uncertainty being resolved at retirement after n work years is:

$$\min_{\phi} \mathcal{T} = nT((1 - \phi)Y) + E \left[(m_s - n)T \left(\frac{n\phi Y}{m_s - n} \right) \right]. \quad (\text{A9})$$

In Figure A2, we plot the smoothing benefit of the base case model (solid line), the average smoothing benefit with known life expectancy (derived from Equation (A8)), and the smoothing benefit with uncertain life expectancy (solution to Equation (A9)). The base case parameters are 40 work years and 16 retirement years, and the jump parameter is $\lambda = 15$ years. When life expectancy decreases from age 81 to 66, we assume that the worker immediately withdraws all his retirement funds, pays income tax, and consumes the after-tax proceeds during his remaining year alive.

Variable life expectancy reduces the average smoothing benefit around each local smooth max-

Figure A2: The Smoothing Benefit with Uncertain Life Expectancy



The figure plots the smoothing benefit in the base case model (solid line), the average smoothing benefit with variable, but known life expectancy (thick dashed line above), and the smoothing benefit with uncertain life expectancy (thin dashed line below). We assume 40 work years and an average of 16 retirement years. Life expectancy can increase to 96 or decrease to 66 with equal probability.

imum. Uncertainty reduces the smoothing benefit further in the region around the breakpoint between the second and the third income brackets. Otherwise, at higher income levels uncertain life expectancy is irrelevant. The effect of uncertain life expectancy is asymmetric, it can only decrease the smoothing benefit, it cannot increase it. The gain in smoothing space from living long does not compensate for the loss of smoothing space resulting from early death.