

# **Trading Death: The Implications of Annuity Replication for the Annuity Puzzle, Arbitrage, Speculation and Portfolios**

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## **Abstract**

Annuities are perceived as being illiquid financial instruments, and this has limited their attractiveness to consumers and inclusion in financial models. However, short positions in annuities can be replicated using life insurance and debt, permitting long positions in annuities to be offset, or short annuity positions to be created. The implications of this result for the annuity puzzle, arbitrage between the annuity and life insurance markets, and speculation on expected longevity are investigated. It is argued that annuity replication could help solve the annuity puzzle, improve the price efficiency of annuity markets and promote the inclusion of annuities in household portfolios. However it runs counter to current UK government policy.

Keywords: Annuities, annuity puzzle, arbitrage, longevity, speculation, life insurance

JEL: G12, G22, G23

## **1. Introduction**

It is widely understood that annuities cannot be liquidated or traded, and so are highly illiquid assets. This paper explores the uses of a strategy to replicate a short annuity position which allows annuitants to offset a long annuity position, in contradiction to the accepted view. The implications of this replication strategy for the annuity puzzle<sup>1</sup>, arbitrage between the annuity and life insurance markets, speculation on a change in an annuitant's longevity and constructing household portfolios are investigated. This is one of the first studies to conduct a detailed analysis of annuity replication and to investigate annuity arbitrage and longevity speculation. It also argues that annuity replication allows annuities to be integrated into the well-known models of mainstream finance such as portfolio theory.

Section 2 covers how annuities are priced. Section 3 explains the reasons for the illiquidity of annuities, and section 4 presents the relationship between the actuarially fair prices of annuities and life insurance. Section 5 shows how short positions in annuities can be replicated using life insurance and debt, and section 6 considers the implications of annuity replication for the annuity puzzle and argues that, by encouraging consumers to buy annuities, it could help solve this puzzle. Section 7 has a detailed analysis of annuity arbitrage, which is possible for annuity underpricings, but not for annuity overpricings. Examples are provided from a range of countries of situations where annuities have been substantially under-priced, followed by an examination of the effects of variations in the annuity and life insurance load factors on arbitrage profits. This indicates that annuity arbitrage appears, on occasion, to be profitable. Section 8 demonstrates how annuity replication can be used to speculate on a change in an annuitant's expected longevity. The profits from such speculation are available in cash once the increase in expected longevity has been recognised. Section 9 argues that the ability to replicate short positions in annuities means that annuities can take a more prominent role in household portfolio decisions. Section 10 has the conclusions.

## **2. Annuity Prices**

Annuity prices depend primarily on the expected longevity of the annuitant and interest rates expected over the remaining life of the annuitant. Annuity prices may also be affected by load

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<sup>1</sup> The theory indicates that consumers should annuitize most of their wealth, but actual rates of annuitization are much lower, and this widespread under-annuitization creates the annuity puzzle.

factors such as adverse selection<sup>2</sup>, administration, regulatory and marketing costs, profit, the risk premium etc, but these effects are usually of modest size and exhibit only modest variation over time. Studies for a range of countries have found the money's worth ratio<sup>3</sup> is usually not far below one and reasonably stable over time, James and Song (2001), James and Vitas (2000), Murthi, Orszag and Orszag (2000), Finkelstein and Poterba (2002), Cannon and Tonks (2004, 2008, 2009, 2011), and Von Gaudecker and Weber (2004). Therefore load factors are not of major importance, and the prices of annuities are determined primarily by longevity and interest rate expectations.

The actuarially fair price for a single premium, instantaneous, level, single life annuity in the absence of load factors is:-

$$V_x = \sum_{i=1}^n \frac{A.P_{xi}}{(1+r)^i} \quad (1)$$

where  $V_x$  is the current price or value of the annuity for an annuitant aged  $x$  years.

$i$  is the number of years since the annuity was purchased

$r$  is the rate of interest for the life of the annuitant (a non-stochastic interest rate with a flat term structure is assumed)

$A$  is the constant annual annuity payment

$n$  is a number greater than the remaining years of life of the annuitant

$P_{xi}$  is the probability that an annuitant aged  $x$  when the annuity was purchased survives for at least  $i$  years.

As longevity expectations increase, annuity prices increase, while as interest rate expectations increase annuity prices decrease. The term  $P_{xi}$  in equation (1) shows that the price of an annuity depends, not on the expected longevity, but on the cumulative probability distribution of expected longevity. Therefore, any change in the longevity distribution which alters the values of  $P_{xi}$  will

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<sup>2</sup> Those who subsequently have lives of above average length are more likely to buy annuities, and those whose lives are shorter than average are more likely to buy life insurance, which produces adverse selection against the insurance company. In response insurance companies raise their prices to offset the effects of the adverse selection.

<sup>3</sup> The money's worth ratio is the present value of the expected annuity payments, divided by the price of the annuity. A money's worth ratio of one means that the annuity has an actuarially fair price. Money's worth ratios are based on the mortality table chosen by the insurance company, and the set of future interest rates used by the insurance company as discount factors.

affect the annuity price. For example, it is possible that, although some values of  $P_{x_i}$  drop, others rise, so that overall the annuity price rises.

### **3. Illiquidity and Annuities**

In the 18<sup>th</sup> and 19<sup>th</sup> centuries speculators were allowed to purchase annuities on the lives of nominees, rather than on their own lives. This permitted wide scale adverse selection against annuity providers. For example in the early 1770s banks in Geneva created lists of Genevan young girls from families with a record of health and longevity who could serve as nominees. The preference was for girls between the ages of five and ten years who had survived smallpox (Velde and Weir, 1992). These annuities and the income they brought could be traded while the nominee remained alive. To reduce the opportunities for adverse selection, annuities can now only be purchased on the life of the annuitant, and cannot be surrendered back to the provider or traded. For example, if allowed, annuitants would seek to sell their annuity just before their death (Brown, 2002). Browne, Milevsky and Salisbury (2003) suggest that the inability to surrender annuities is because insurance companies use annuity receipts to make long term illiquid investments.

Defined benefit pensions lead to an annuity supplied by the pension scheme. On retirement, members of UK defined benefit pension schemes can take up to 25% of the total value of their accrued pension liability (depending on the rules of the scheme) as a tax-free lump sum, with the remaining 75% taken as a pension. Subsequently, these annuities cannot be traded or liquidated, and so in this respect are comparable to annuities purchased from an insurance company. Similarly, many people have state pensions which cannot be traded or liquidated, and so face the same illiquidity problems. In the UK at least 75% of defined contribution pension pots must be used to buy an annuity before the age of 75 (although there are some exceptions). Therefore, due to defined benefit pensions, state pensions and defined contribution pensions, part of the wealth of most people is tied up in an annuity that cannot be liquidated or traded. Annuitants cannot borrow using their annuity payments as collateral, as annuity payments are conditional on the annuitant being alive.

### **4. The Relationship Between Annuity and Life Insurance Prices**

There is a widespread view that annuities cannot be reversed, and so are illiquid assets, which discourages their purchase. However it is possible to replicate the cash flows of a short position in

an annuity, and so effectively offset or liquidate an annuity. The strategy for replicating annuities relies on the relationship between the prices of annuities and life insurance. The purchaser of an annuity makes a large initial payment, and then receives a stream of payments until death, while the purchaser of life insurance makes a stream of life insurance premia payments until their death, at which time their estate receives a large terminal payment. So, apart from the large payment being made at the end rather than the start, the pattern of cash flows of a life insurance policy is the inverse of those of an annuity.

Ignoring load factors, the actuarially fair price of life insurance (i.e. the annual life insurance premium,  $Y$ ) occurs when the following condition is met:-

$$\sum_{i=1}^n \frac{D_{xi} X^*}{(1+r)^i} = \sum_{i=1}^n \frac{P_{xi} Y^*}{(1+r)^i} \quad (2)$$

where  $X^*$  is the sum insured, the \* superscript denotes an actuarially fair price or amount,  $D_{xi}$  is the probability that a person aged  $x$  when the life insurance policy is purchased dies in year  $i$ , and

$$\sum_{i=1}^n D_{xi} = 1 \quad .$$

Insurance companies sell both life insurance policies and annuities, which has the advantage that the risks of these two types of business tend to be offsetting. For example, underestimating longevity means that annuities are underpriced, while life insurance is overpriced, Cox and Lin (2007). It is assumed that insurance companies use the same longevity estimates when pricing both life insurance and annuities. Therefore, in a competitive environment with no load factors, the prices of both annuities and life insurance are driven to the prices in equations (1) and (2), and a fixed relationship exists between the prices of annuities and life insurance for policies on the life of the same person. For such actuarially fair prices with no load factors, this relationship is:-

$$\bar{a}_x^* (y_x^* + r) = 1 \quad (3)$$

where  $\bar{a}_x^*$  is the actuarially fair cost today of an annuity of £1 for an annuitant aged  $x$ , and  $y_x^*$  is the annual actuarially fair insurance premium for life insurance of £1 for an individual aged  $x$ , Milevsky (2006, pp. 145-6), Promislow (2011, pp. 69-70).

## 5. Annuity Replication

Yaari (1965), Bernheim (1991) and Brown (2001) have shown that debt and life insurance policies can be used to replicate a short annuity position. A long position in an annuity that is level<sup>4</sup> can be offset by borrowing a sum of money now at a fixed rate of interest. At the same time the annuitant insures their life for an amount equal to the size of the loan. When the annuitant dies the life insurance payout is used to pay off the loan, and all payments cease. If equation (3) is satisfied, the size of the loan (which is also the sum insured) can be set so that each year the sum of the interest and life insurance payments (assumed constant) are equal to the annual annuity payment. That is:-

$$A = X^*r + Y^* \quad (4)$$

There are two reasons why the annuity payments are big enough to cover both the interest on the loan and the life insurance payments. First, the loan occurs at the start, while the payout on the life insurance policy occurs at the end. Second, the mortality credit<sup>5</sup> means that the rate of return on the annuity exceeds the rate of return on a bond. If the current price of the annuity is equal to the loan (and sum insured) i.e.  $V^* = X^*$ , then, since  $Y^* = y_x^*X^*$  and  $A = X^*/\bar{a}_x^*$ , equation (4) becomes  $X^*/\bar{a}_x^* = X^*r + y_x^*$ , or  $\bar{a}_x^*(y_x^* + r) = 1$ , and so equation (4) is satisfied. Example 1 shows how a short position in an annuity can be replicated so that it exactly offsets a long position in an annuity.

*Example 1*<sup>6</sup>. Two years ago Orsino Thruston bought an annuity for £100,008 ( $V_{65}^*$ ). Consistent with equation (1), this yields an annuity payment of £8,628 per year ( $A$ ) until his death. At the age of 67 he now wishes to offset this annuity, and so he uses equation (1) to compute the current price of an annuity of £8,628 p.a. at £93,641. Setting  $V_{67}^* = X^*$ , he borrows £93,641 ( $X^*$ ) at a fixed rate of 4% ( $r$ ). This loan requires interest payments of  $(£93,641 \times 0.04) = £3,746$  per year, leaving a net cash inflow of £4,882 per year. Orsino uses this £4,882 p.a. to pay the annual premiums (computed using equation (2) or (4)) on a life insurance policy on his life worth £93,641. For simplicity and brevity,

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<sup>4</sup> Most annuities purchased from insurance companies are not index linked or escalating.

<sup>5</sup> The mortality credit is the reduction in the price of an annuity, relative to the cost of purchasing bonds, to give the same annual payment. This is due to the probability of death each year of the annuitant and the resulting cessation of the annuity payments. The higher is the probability of death, the greater is the mortality credit.

<sup>6</sup> This example is based on a life annuity for a male aged 65 using English actuarial tables for 2007-2009 from the Office of National Statistics. There are no survivor benefits or other complications.

it is assumed that Orsino dies at the end of the fourth year. So Orsino has effectively liquidated his annuity now for £93,641, its actuarially fair value.

Table 1 shows that, by surviving until year two, Orsino has benefited from a mortality credit with a present value of £2,841. This is the difference between investing the £100,008 in bonds and in an annuity. Investment in bonds at 4% would mean that in year two he would have a bond worth £100,008, together with two year's interest worth  $(£4,000(1.04)+£4,000) = £8,160$ : a total of £108,168. Investment in the annuity generates two annuity payments worth  $(£8,628(1.04)+£8,628) = £17,600$  in year two, plus an annuity with an actuarially fair replacement cost of £93,641; which gives a total of £111,241. Therefore investment in the annuity produces a gain due to the mortality credit of  $(£111,241-£108,168) = £3,073$  in year two, which has a present value in year zero of £2,841.

Year	Transaction	Undiscounted Cash Flows		Discounted Cash Flows	
		Inflows	Outflows	Inflows	Outflows
0	Cost of annuity	-	£100,008	-	£100,008
1	Annuity	£8,628	-	£8,296	-
2	Annuity	£8,628	-	£7,977	-
2	Borrow	£93,641	-	£86,576	-
2	Insure life for £93,641	-	-	-	-
	Mortality Credit	-	-	-	£2,841
Totals				<u>£102,849</u>	<u>£102,849</u>
3	Annuity	£8,628	-	£7,670	-
3	Interest payments (£93,641@4%)	-	£3,746	-	£3,330
3	Life insurance premia	-	£4,882	-	£4,340
4	Annuity	£8,628	-	£7,375	-
4	Interest payments (£93,641@4%)	-	£3,746	-	£3,202
4	Life insurance premia	-	£4,882	-	£4,173
4	Life insurance claim	£93,641	-	£80,045	-
4	Repay loan	-	£93,641	-	£80,045
Totals				<u>£95,090</u>	<u>£95,090</u>

Table 1: Example of Offsetting an Annuity

Charupat and Milevsky (2001) describe what they call a mortality swap, which involves buying both an annuity and life insurance at actuarially fair prices, where  $V^* = X^*$ . Then, by equation (4), this generates a riskless cash flow of  $X^*r$  per year until the annuitant dies. So, while full replication generates a payment now ( $X^*$ ), a mortality swap produces a sequence of cash flows that continue for an uncertain period until the annuitant dies. Charupat and Milevsky (2001) show that, under

Canadian tax laws, a mortality swap produces a tax gain.

Insurance companies usually include various costs (load factors) when pricing annuities - administration costs, marketing costs, adverse selection, profit, longevity risk, interest rate risk and regulatory costs and risk. If life insurance or annuities are not traded at actuarially fair prices, equation (3) will not be met. Equation (4) is still relevant, but the value of the loan ( $X$ ) will no longer be equal to the current value of the annuity ( $V$ ) and there will be a gain or loss on the replication (or offset) of an annuity. This is demonstrated in examples 2 and 3.

*Example 2.* Suppose that annuities have a load factor ( $w$ ) equal to 5% of the actuarially fair price. The initial cost in year zero of an annuity of £8,628 on the life of Orsino Thruston rises by 5% to £105,008. In year two the current price of this annuity of £8,628 p.a. is 5% higher at £98,323, rather than £93,641. Due to the higher price of the annuity, an offset by Orsino no longer gives him the actuarially fair value of the annuity. He can still borrow £93,641, pay interest of £3,746 p.a. and £4,882 p.a. in life insurance premiums on a policy worth £93,641 and so meet equation (4). Orsino has effectively liquidated (sold) his annuity at a price that is  $(£98,323 - £93,641) = £4,682$  below the current market price. Since he initially paid £5,000 above its actuarially fair value, this reduction in value was created when the annuity was purchased in year zero<sup>7</sup>.

*Example 3.* If life insurance also has a load factor ( $p$ ), in addition to the annuity load factor of 5%, the size of Orsino's loan is further reduced. If life assurance prices are above their actuarially fair price by  $p$  percent, the size of the sum insured that satisfies equation (4) is:-

$$X = A/[r+y_x^*(1+p)] \quad (5)$$

Suppose the insurance premium has a load factor of 5% of its actuarially fair price, Orsino can only afford to insure his life for £91,064, rather than £93,641; paying £4,985 p.a. in premiums and £3,643 p.a. in interest on a loan of £91,064. So the life insurance load factor means that Orsino's loss, relative to actuarially fair values, in conjunction with the annuity load factor, has increased by £2,577 to £7,259. More generally the profit ( $IT$ ) on annuity replication is:-

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<sup>7</sup> This loss can also be computed from equation (6) below).

$$\Pi = \frac{Ay_x^* - Y^*(1+w)(r + y_x^*(1+p))}{y_x^*[r + y_x^*(1+p)]} \quad (6)$$

## 6. The Annuity Puzzle and Annuity Replication

This section considers the effect of the ability to replicate short positions in annuities on the annuity puzzle. Yaari (1965) has shown that consumers can solve the dilemma of ensuring that they exhaust their resources on their last day of life by annuitizing all their wealth, with longevity risk passing from themselves to the annuity provider. This solves the household portfolio decision in favour of complete annuitization. Davidoff, Brown and Diamond (2005) have generalized Yaari's model and conclude that less than complete, although still substantial, annuitization is optimal. However, the actual level of annuitization is much lower than the total annuitization indicated by Yaari (1965), or the substantial levels of annuitization of Davidson, Brown and Diamond (2005), leading to the annuity puzzle.

A very wide range of explanations for the annuity puzzle have been proposed. Possible explanations include the load factor, adverse selection, illiquidity, default risk, higher expected returns on equities, the unavailability of index linked annuities, small pension pots, the bequest motive, self-insurance by families, taxation effects, behavioural effects, and biased estimates of longevity and interest rates. However, there is no widely accepted explanation of the annuity puzzle.

One of the reasons that has been widely proposed to explain the annuity puzzle is that consumers are unwilling to buy annuities because they are illiquid irreversible contracts, Brown (2002, 2009), Brown and Warshawsky (2004, 2012). A UK survey of 3,511 people aged 50-64 years found that, of those who said they would never buy an annuity, 74% said this was because they would lose flexibility, and this was the most popular reason by a considerable distance (Gardner and Wadsworth, 2004). In a US survey of 321 US financial planners, 31% gave loss of flexibility and control as the reason for low annuitization. Again this response was far more popular than any other (Brown, 2002).

In the US the possibility of unexpected and substantial health expenses in old age creates a need for liquidity, and this militates against the purchase of annuities (Ameriks, Caplin, Laufer and Van Nieuwerburgh, 2008; Turra and Mitchell, 2008; Peijnenburg, Nijman and Werker, 2011; Pang and

Warshawsky, 2010; Nijman and Werker, 2011; and Sinclair and Smetters, 2004)<sup>8</sup>. Peijnenburg, Nijman and Werker (2011) go so far as to argue that the possibility of unexpected health costs early in retirement, coupled with the illiquidity of annuities, solves the annuity puzzle. Wang and Young (2012a, 2012b) have investigated the demand for what they call a commutable annuity, i.e. an annuity that can be surrendered to the supplier in exchange for a fixed proportion of its actuarially fair price at the time of surrender. They show that as the surrender charge increases, the level of annuitization is reduced, and suggest that the flexibility offered by commutable annuities may solve the annuity puzzle. Similarly Neuberger (2003) has argued that reversible annuities would help to solve the annuity puzzle. In contrast, Sinclair and Smetters (2004) found that, even if annuities were reversible and there are no transaction costs, the annuity puzzle remains.

The implications for the annuity puzzle of the strategy for replicating a short position in an annuity are unclear. Most annuity decisions are made by individuals, who are clearly advised that the purchase of an annuity is irreversible. This is evidenced by the advice and promotional material received by potential annuitants<sup>9</sup>. Many academics have taken a similar position. If the advertised illiquidity of annuities is partly responsible for the annuity puzzle, awareness of the possibility of replication may increase annuity demand, going some way towards removing the annuity puzzle. However, the load factors for annuities and life insurance when replicating an annuity may be large enough to outweigh the desire for liquidity, removing the benefits of annuity replication and leaving the annuity puzzle unaffected.

Ameriks, Caplin, Laufer and Van Nieuwerburgh (2008) show that, if annuities could be liquidated

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<sup>8</sup> Health and long-term care insurance are ways of dealing with such contingencies.

<sup>9</sup> “Once you've bought an annuity there's no going back, so you've got to get it right first time.” Which?

<http://www.which.co.uk/money/retirement/guides/annuities-explained/buying-an-annuity/>

“Once bought, they cannot be changed, transferred or cashed-in.” The Annuity Bureau

<http://www.annuitybureau.co.uk/about-annuities/conventional-annuities.aspx>

“You only have one opportunity to shop around for your annuity. Once you have committed to an annuity provider and started to receive an income, the decision can't be reversed.” Age UK

<http://www.ageuk.org.uk/products/financial-products-and-services/annuities/>

“Make the right decision now, because you cannot reverse it later”. Nationwide

[http://www.nationwide.co.uk/investments/products\\_and\\_services/annuities/default.htm](http://www.nationwide.co.uk/investments/products_and_services/annuities/default.htm)

“Buying an annuity is a long-term decision - once it's set up you can't cash it in or make any changes.” Standard Life

<http://www.standardlife.co.uk/1/site/uk/pensions/our-retirement-and-annuity-products/annuity-compu>  
[lsory-purchase](http://www.standardlife.co.uk/1/site/uk/pensions/our-retirement-and-annuity-products/annuity-compu)

or reversed at any time, this would lead to a substantial rise in demand, with an increase in the willingness to pay of up to 16%. This suggests annuitants are willing to bear load factors of up to 16% in order to get liquid rather than illiquid annuities. Browne, Milevsky and Salisbury (2003) argue that, while investors differ in their personal circumstances and preferences, they might be willing to pay up to about 11% of the annuity price for a liquid annuity. Therefore, if the combined annuity and life insurance load factors are less than about 11%-16%, publicising the possibility of annuity replication may help to remove the annuity puzzle. Conversely, if the apparent illiquidity of annuities is not the reason for the annuity puzzle, removing this perceived restriction will have no effect on the demand for annuities. It is also possible that, rather than solve the annuity puzzle, annuity replication could increase the puzzle. Those who have a defined benefit pension, state pension or defined contribution pension may decide to offset part or all of these pensions. The pensions will still be paid, and so appear in the statistics, but the economic reality will be that, due to annuity replication, those who fully offset their pensions will no longer have an annuity.

A number of empirical studies using US data have examined the extent to which annuities are offset. There are various reasons why a household may simultaneously hold long positions in both annuities and life insurance, i.e. an annuity offset position:- (a) to remove the drop in wealth suffered by a surviving spouse when an annuity on the life of their deceased spouse ceases (Auerbach and Kotlikoff, 1987, 1989), (b) to provide a bequest by converting an annuity into a lump sum (Bernheim, 1991, Brown 2001), (c) to reallocate consumption across time in a way that is incompatible with the income stream from their annuity, (d) to offset an annuity to create a precautionary balance to meet unexpected expenditure shocks, e.g. health expenses, (e) to offset an annuity to provide a ready source of funds to their executors to cover their funeral expenses, and (f) because they continue to hold life insurance when it is no longer required to insure their future earning potential as they have retired (Brown, 2001).

Auerbach and Kotlikoff (1987, 1989) examined the purchase of insurance on the life of a spouse with an annuity income, with the surviving spouse as the beneficiary of the life insurance. Such life insurance is designed to ensure there is no drop in the survivor's consumption after the death of their spouse. However Auerbach and Kotlikoff were unable to find evidence of such behaviour in the US. Bernheim (1991) and Brown (2001) investigated the extent to which US pensioners use life insurance to offset state pensions (i.e. annuities) in order to leave bequests. In Bernheim's sample

36% were both in receipt of an annuity and had life insurance. He found evidence that about a quarter of US pensioners were over-annuitized, and offset roughly 20% of their annuities. Brown (2001) argues that this result may be attributable to data problems. Using different US data Brown (2001) concludes that no more than 2% of his sample had bought life insurance to offset annuities in order to provide for a bequest motive. Empirical investigations of annuity replication using US data have looked for offset amongst married couples, and offset to generate a bequest. There have not been any studies looking for offset due to reallocating consumption, a precautionary motive, provision for funeral expenses or inertia. In addition, as explained below, annuity replication can be used for arbitrage and speculation, but there have been no empirical studies of these additional motives for annuity replication.

## **7. Arbitrage and Annuity Replication**

Arbitrage involves the simultaneous purchase of one asset against the sale of the same or equivalent asset from zero initial wealth to create a riskless profit due to price discrepancies. A short position in an annuity can be replicated using life insurance and debt, while long positions in annuities can be purchased in the market. Therefore if annuities are relatively underpriced, or life insurance is relatively overpriced, arbitrage is possible. Example 4 shows how an annuity underpricing can be arbitrated when there are actuarially fair life insurance prices, and example 5 demonstrates arbitrage when life insurance is overpriced.

*Example 4.* Suppose that Orsino Thruston is offered an annuity of £8,628 p.a. at a price of only £88,000, when its actuarially fair price is £93,641, which implies a negative load factor of 6.0%. Life insurance is being sold at its actuarially fair price. He simultaneously buys this annuity, borrows £93,641 at 4% (or £3,746 p.a.) and insures his life for £93,641 at an annual cost of £4,882. Therefore he has made an instant arbitrage profit of  $(£93,641 - £88,000) = £5,641$  which is immediately available to him in cash.

*Example 5.* Suppose that, as in example 4, Orsino is offered an annuity of £8,628 p.a. at a price of £88,000, when its actuarially fair price is £93,641. Life assurance is now overpriced by 5%, and using equation (5) this corresponds to a loan of £91,064. Orsino can make an instant arbitrage profit of  $(£91,064 - £88,000) = £3,064$  by simultaneously buying the annuity, insuring his life for £91,064, and borrowing £91,064. He pays interest of £3,643 p.a. and life insurance premia of £4,985 p.a. So,

as table 2 shows, even when life insurance is sold at 5% above its actuarially fair price, arbitrage can still be profitable, provided annuities are underpriced. For simplicity, Orsino is assumed to die after two years.

Year	Transaction	Undiscounted Cash Flows		Discounted Cash Flows	
		Inflows	Outflows	Inflows	Outflows
0	Cost of annuity	-	£88,000	-	£88,000
0	Borrow	£91,064	-	£91,064	-
0	Insure life for £91,064	-	-	-	-
0	Profit	-	-	-	<b>£3,064</b>
Totals				<u>£91,064</u>	<u>£91,064</u>
1	Annuity	£8,628	-	£8,296	-
1	Interest payments (£91,068@4%)	-	£3,643	-	£3,503
1	Life insurance premia	-	£4,985	-	£4,793
2	Annuity	£8,628	-	£7,977	-
2	Interest payments (£91,068@4%)	-	£3,643	-	£3,368
2	Life insurance premia	-	£4,985	-	£4,609
2	Life insurance claim	£91,064	-	£84,194	-
2	Repay loan	-	£91,064	-	£84,194
Totals				<u>£100,467</u>	<u>£100,467</u>

Table 2: Example of Arbitrage of an Annuity Underpricing with a Life Insurance Load Factor

The total proportionate change in the cost of insurance is made up of two parts - the increase in price and the reduction in the quantity purchased. Thus  $Y/Y^* = (X/X^*)(y_x/y_x^*) = 0.97248 \times 1.05 = 1.0211$ . The total cost of insurance has increased by only 2.11% because, although its price has risen by 5%, the quantity purchased has fallen by 2.75%<sup>10</sup>.

Due to the presence of load factors, annuity prices tend to be above the actuarially fair price. Life insurance has a similar set of costs, leading to life insurance prices tending to be above the actuarially fair price. When annuities are overpriced, relative to life insurance, arbitrage is not possible because individuals cannot sell annuities, nor can they sell life insurance. However the inability to arbitrage overpricings does not necessarily mean that annuity prices fail to reflect the no-arbitrage condition in equation (3). Annuity overpricings, or life insurance underpricings, can be removed by competition between insurance companies, with consumers choosing to buy annuities and life insurance from the cheapest supplier, or deciding not to purchase an annuity or

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<sup>10</sup> Note that  $5.00\% - 2.75\% > 2.11\%$ , i.e. the combined reduction in the insurance cost is 0.14% greater than the arithmetic difference between the percentage rise in price and the drop in the quantity purchased.

life insurance.

There are three ways of presenting annuity prices<sup>11</sup>: (a) the sum that must be paid by the annuitant to buy an annuity of a specified size ( $V$ ), e.g. £100,008 for an annuity of £8,245 p.a., (b) the money's worth ratio, which is the actuarially fair price divided by the actual price ( $\bar{a}_x^* A/V$ ), and (c) the annuity rate, i.e. ( $A/V$ ). Annuity prices are usually presented as annuity rates, but for arbitrage purposes the price of the annuity ( $V$ ) is preferable. If the annuity price is less than the actuarially fair annuity price ( $\bar{a}_x^* A$ ) there is a mispricing that is potentially arbitrageable. Arbitrage is only profitable if the mispricing is large enough to cover the load factors of both the annuity ( $LFA$ ) and the life insurance ( $LFL$ ). So an arbitrage opportunity only exists if:-

$$\bar{a}_x^* A > V + LFA + LFL \tag{7}$$

Figure 1 shows the actuarially fair price (AF, or  $\bar{a}_x^* A$ ), and the no-arbitrage boundary (NAB, or  $\bar{a}_x^* A - LFA - LFL$ ). Values of  $V$  below the NAB line represent arbitrage opportunities (i.e. inequality (7) is satisfied), while values of  $V$  between the AF and NAB lines are underpricings. However, inequality (7) is not met, and so they are not arbitrage opportunities. For annuity overpricings, competition between insurance companies will tend to push annuity prices down to cost, which is an overpricing of  $\bar{a}_x^* A + LFA$  and shown as the cost boundary line (CB) in figure 1. In the zone between the CB and NAB lines, where arbitrage does not operate, competition in the annuity market will tend to push prices up to the CB line, which represents zero abnormal profit for insurance companies.

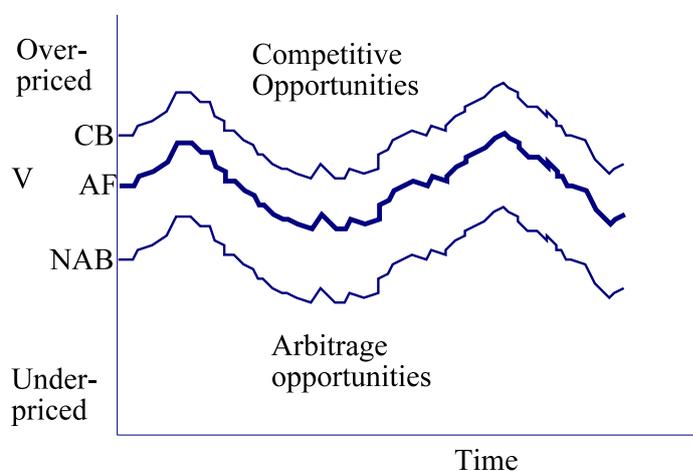


Figure 1: Arbitrage of Underpricings

If insurance companies price annuities at their actuarially fair levels; and then add competitive load factors, annuity prices will tend to fluctuate around the cost boundary line (CB) in figure 1. Therefore prices below the NAB line, i.e. arbitrage opportunities, are likely to be rare. However,

<sup>11</sup> Annuity equivalent wealth will not be considered as it requires knowledge of the annuitant's utility function (Mitchell, Poterba, Warshawsky & Brown, 1999).

there are occasions when annuities are substantially underpriced, relative to the actuarially fair price. For example, the Oregon Public Employees Retirement System offers annuities with a median money's worth ratio of 1.45 (and over 1.60 in more recent years), which is an underpricing of  $(0.45/1.45) = 31\%$  (and in some years over 38%), Chalmers and Reuter (2012). This underpricing is very probably large enough to be an arbitrageable. The purchase of these annuities is restricted to retiring members of this pension scheme, and it is surprising that only 85% of members on average opt for the annuity, with the remaining 15% (about 4,800 people) choosing the lump sum<sup>12</sup>.

Previous researchers have computed the money's worth ratio for various years and a range of annuities in different countries, and a few of these ratios are substantially above one. Fong, Mitchell and Koh (2010) estimated that in Singapore the money's worth ratio for CPF LIFE (refund 75) annuities in 2008 was 1.37 for women and 1.34 for men, while James and Vitas (2000) computed a money's worth ratio in Singapore for both men and women of 1.25. For Chile Rocha and Thorburn (2007) report that for the years 2002-2005 almost all annuities had a money's worth ratio above one, with some annuities having money's worth ratios of 1.28. James and Song (2001) document a money's worth ratio of 1.20 for joint life annuities in Switzerland, while James and Vitas (2000) report Swiss money's worth ratios of 1.17 for men and 1.15 for women. They also found money's worth ratios of 1.16 for Israeli men and 1.24 for Israeli women. Von Gaudecker and Weber computed a money's worth ratio of 1.10 for German men. Cannon and Tonks (2004) have estimated that the money's worth ratio for UK males in 1971, 1990 and 1991 were above 1.20. Finally, Mitchell (2002) found a money's worth ratio of 1.14 for US Hispanic women in 1998. These studies show that, while substantial annuity underpricing is uncommon, it does happen, and arbitrage opportunities can sometimes occur.

There is empirical evidence to support the view that annuity prices do not immediately respond to changes in the main factors determining annuity prices, indicating a market that is inefficient at the semi-strong level. Charupat, Kamstra and Milevsky (2012) studied the speed of response of US annuity prices to changes in interest rates, and found that the response takes several weeks. There is also cross-section evidence of considerable price dispersion. In 2009 a comparison of the prices for annuities in the UK found that a top quartile annuity was 20% cheaper than a bottom quartile

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<sup>12</sup> The perceived lack of liquidity of annuities or lack of knowledge of annuity replication may have deterred this 15%.

annuity ((MGM Advantage, 2009), while Harrison, Byrne and Blake (2006) report that the differential between the highest and lowest UK prices can be up to 39%. Crawford and Tetlow (2012) found a 17% price improvement for those who purchased externally, rather than accepting an annuity from their pension provider. Hueler and Rappaport (2012) analysed US annuity prices and found the average difference between the high and low prices was 8%, although in some cases this difference was 20%. Such variation in annuity prices between suppliers makes arbitrage more likely to be profitable because arbitrageurs can buy from the cheapest supplier. However, arbitrage opportunities in the annuity market are unlikely to be immediately removed in the way that occurs in other financial markets. Arbitrage can be of an unlimited scale in other financial markets, and so a few very wealthy arbitrageurs (e.g. investment banks or hedge funds) are able to remove any arbitrage opportunities. Annuity replication can only be undertaken by individuals on their own lives, and so many arbitrageurs are needed to move prices. These findings indicate that price competition and the effects of arbitrage in the annuity market are weak.

To investigate the relationship between load factors and arbitrage profit, equation (6) was used to compute the profit or loss from arbitrage for various combinations of annuity and life insurance load factors. This was done for an annuity of £8,628 p.a. with an actuarially fair price of £93,641 and with interest rates at 4%. Table 3 shows that when annuities have a negative load factor of -10% (i.e. a money's worth ratio of 1.10), arbitrage shows a profit, even when life insurance has a positive load factor of over 15%. When annuities have a money's worth of 0.95 (i.e. a positive load factor of 5%) arbitrage can still be profitable, provided life insurance has a negative load factor of -8.5% or greater.

Differentiating equation (6) with respect to the annuity load factor ( $w$ ) reveals that a 1% increase in  $w$  leads to a reduction in the arbitrage profit of 1% of the sum insured when there are actuarially fair prices ( $X^*$ ), irrespective of the life insurance load factor ( $p$ ). For the example in table 3, this is £936.

$$\frac{\partial \Pi}{\partial w} = X^* \tag{8}$$

Differentiating equation (6) with respect to the life insurance load factor ( $p$ ) gives:-

$$\frac{\partial \Pi}{\partial p} = \frac{-Ay_x^*{}^3}{(y_x^*r + y_x^*{}^2(1+p))^2} \quad (9)$$

		Life Insurance Load Factor ( $p$ )						
		-15%	-10%	-5%	0	5%	10%	15%
Annuity Load Factor ( $w$ )	-15%	£22,732	£19,663	£16,773	£14,046	£11,470	£9,031	£6,720
	-10%	£18,050	£14,981	£12,091	£9,364	£6,788	£4,349	£2,038
	-5%	£13,368	£10,299	£7,409	£4,682	£1,955	-£333	-£2,644
	0	£8,686	£5,617	£2,727	0	-£2,576	-£5,015	-£7,326
	5%	£4,004	£935	-£1,955	-£4,682	-£7,259	-£9,697	-£12,009
	10%	-£679	-£3,747	-£6,638	-£9,364	-£11,941	-£14,379	-£16,691
	15%	-£5,361	-£8,429	-£11,320	-£14,046	-£16,623	-£19,061	-£21,373

Table 3: Effect of Load Factors on the Arbitrage Profit

Equation (9) reveals that the arbitrage profit is a non-linear function of the life insurance load factor; and that this relationship depends on the parameters  $A$ ,  $y_x^*$ ,  $r$  and  $p$ , but not the annuity load factor ( $w$ ). For the example in table 3, the decrease in arbitrage profit from a 1% increase in the life insurance load factor are shown in table 4.

	Life Insurance Load Factor ( $p$ )						
	-15%	-10%	-5%	0	5%	10%	15%
Profit Change	-£633	-£595	-£561	-£530	-£501	-£475	-£450

Table 4: Effects of a 1% increase in the Life Insurance Load Factor on Arbitrage Profit

Table 4 shows that, as the life insurance load factor increases, its effect on the arbitrage profit diminishes. Throughout the range considered in table 3 the effect of the life insurance load factor on arbitrage profit is smaller than the £936 of the annuity load factor. Hence, in this range, the annuity load factor is more important in creating arbitrage opportunities than the life insurance load factor. Overall, the analysis in this section suggests that arbitrage opportunities occur in annuity markets, and that worthwhile profits can be made<sup>13</sup>.

<sup>13</sup> Arbitrage may also be possible using the prices of deferred annuities and deferred life insurance

## 8. Longevity Speculation and Annuity Replication

There is strong evidence that potential purchasers of annuities and life insurance possess much better information about their own life expectancy than do insurance companies. This information asymmetry leads to adverse selection in both the annuity and life insurance markets, Finkelstein and Poterba (2002, 2004), Fong (2002), Friedman and Warshawsky (1990), He (2009), McCarthy and Mitchell (2010), Mitchell and McCarthy (2002), Walliser (2000).

Not only do annuitants have better information on their life expectancy when they buy an annuity or life insurance, but they are more likely to be aware of changes in their life expectancy that occur after the annuity or life insurance has started. Insurance companies only assess annuitants' life expectancy when the annuity or life insurance is purchased as the payments remain fixed once the contract is traded, and so they are unaware of later changes in health status and life expectancy. This information asymmetry offers an opportunity for consumers to benefit from subsequent changes in their life expectancy<sup>14</sup>. Speculators, who are assumed to already hold their desired stock of annuities, can buy or sell (via replication) an annuity in the expectation that their life expectancy, as assessed by insurance companies, will rise or fall in the future.

Keilman (2007) investigated the accuracy of male longevity forecasts for 14 European countries over the period 1950 to 2001. He found liquidity forecasts 26 years ahead consistently under-predicted actual longevity by almost 5 years (a 19% error). Shaw (2007) obtained similar results for the UK. This shows that, for the male population as a whole, longevity forecasts have been substantially below actual longevity. For some individuals the longevity under-prediction will have been even larger, indicating plenty of scope for speculation on upward revisions in longevity expectations.

Insurance companies assess the annuitant's life expectancy using a variety of pieces of information such as age, gender, post code, health status, occupation, marital status, education, size of the

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policies.

<sup>14</sup> It is possible for US annuitants to speculate on a drop in longevity expectations using just life insurance. This can be done by buying life insurance, and then waiting for longevity expectations to drop. At this time it may be possible to sell the insurance policy in the US traded life settlement market, although trading takes place at a deep discount, Blake and Harrison (2008, 2009) and Braun, Gatzert and Schmeiser (2012). The UK has a well-developed market in life settlements, but this trades exclusively in traditional with-profits endowment contracts, i.e. traded endowment policies (Gatzert (2010).

annuitant's pension pot and behavioural risks (Fong, 2011, Brown and Scahill, 2010, Brown and McDaid, 2003)). The criteria used vary both over time and between insurance companies. The expected longevity of an annuitant, as assessed by insurance companies, may increase for a variety of reasons<sup>15</sup>. For example, consumers may expect that new mortality tables will show a rise in longevity for their gender and age cohort, or that there will be a substantial change in the nature of those living in their post code due to an influx of people with greater longevity than the current residents. They may plan to move to a post code where the residents have a greater longevity than their current post code, or to switch from a high risk to a lower risk occupation. They might intend to give up smoking, lose weight or otherwise change their behaviour in a way that increases their expected longevity, or they may have previously undergone genetic testing (whose results they are not required to disclose to the insurance company) which reveals that their longevity is higher than expected, and then choose to disclose this information when they buy life insurance.

Speculators may expect that estimates of the life shortening effects of their medical condition will be revised downwards due to new medical evidence or treatment, or that they will recover from a life-shortening medical condition, possibly due to the development of a new treatment, so increasing their expected longevity. For example, they may have been assessed as having a high probability of death in (say) the two years after they bought the annuity, but if they survive this period their expected longevity greatly increases, e.g. the cancer treatment has worked. When they buy the annuity the annuitant has information that, in their particular case, the treatment is more likely to be successful than the figure used by insurance companies.

The size of an annuitant's pension pot is used by some insurance companies when setting annuity rates, and the speculator may expect that the size effect on annuity rates for a pot of their size may change, leading to an increase in the annuity price. They may predict that the criteria used by insurance companies changes in a way that increases the annuitant's expected longevity. For example Fong (2011) shows that using marital status and education would improve longevity prediction in the US. They might be planning to undergo gender reassignment from male to female, or expect that annuity and life insurance prices will be legally required to be gender neutral (as in

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<sup>15</sup> Reversing these arguments provides reasons for a decrease in expected longevity.

the European Union), so benefiting female annuitants<sup>16</sup>.

If an individual wishes to speculate on an increase in their expected longevity, as assessed by insurance companies, this can be accomplished by purchasing an annuity, which involves both longevity and interest rate risk. The speculator hedges the interest rate risk implicit in the annuity, e.g. by trading interest rate futures, leaving the longevity risk. Example 6 demonstrates how a speculator can make a profit if their longevity increases, and enjoy the profit when the increase in longevity is recognized by insurance companies. The speculator can buy the annuity from the insurance company with the lowest longevity expectations at that time. Later, when they buy life insurance to offset the annuity, they can buy from the insurance company which then has the highest longevity expectations.

*Example 6.* Continuing with the Orsino Thruston example, suppose that over the next two years Orsino expects his expected longevity to increase by two years. So he buys an annuity now based on the life insurance company's current assessment of his longevity. This annuity is for £8,628 p.a. at a price of £100,008. After two years his expected longevity has, as he expected, increased by two years, so that the price of the annuity of £8,628 on his life is still £100,008. Orsino can now offset the original annuity by insuring his life for £100,008 at a cost of £4,628 p.a. (using the new longevity expectations and assuming no load factors), and borrowing £100,008 at a cost of £4,000 p.a. Table 3 shows that this generates cash flows with a present value of £8,728 in year zero. The mortality credit accounts for £2,841 of this gain (his reward for surviving for two years to reach the age of 67), leaving a profit due to longevity speculation with a present value of £5,887. All of this increase in Orsino's wealth is available to him in year 2, i.e.  $£5,887(1.04)^2 = £6,367$ .

A potential speculator may already own an annuity due to being a member of a defined benefit or defined contribution pension scheme, have a state pension, or have voluntarily purchased an annuity, and be content with their current holding of annuities. They may now realise that insurance company expectations of their longevity have risen, thereby increasing the value of these annuities and their wealth. While they could offset some of their annuities and convert this gain into cash, the increase in their expected longevity means they must now expect to support themselves for

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<sup>16</sup> UK annuity and life insurance prices were required to be unisex as from 21 December 2012 by a ruling of the European Court of Justice on 1<sup>st</sup> March 2011. This ruling does not apply to occupational pensions.

additional years of life. Therefore, along with the additional wealth comes an offsetting liability. Their response to this situation is unclear, and they may decide not to offset any of their holdings of annuities.

Year	Transaction	Undiscounted Cash Flows		Discounted Cash Flows	
		Inflows	Outflows	Inflows	Outflows
0	Cost of annuity	-	£100,008	-	£100,008
0	Open an interest rate hedge	-	-	-	-
1	Annuity	£8,628	-	£8,296	-
2	Annuity	£8,628	-	£7,977	-
2	Borrow	£100,008	-	£92,463	-
2	Insure life for £100,008	-	-	-	-
2	Close the interest rate hedge	-	-	-	-
	Mortality Credit	-	-	-	£2,841
	Profit	-	-	-	<b>£5,887</b>
	Totals			<u>£108,736</u>	<u>£108,736</u>
3	Annuity	£8,628	-	£7,670	-
3	Interest payments (£100,008@4%)	-	£4,000	-	£3,556
3	Life insurance premia	-	£4,628	-	£4,114
4	Annuity	£8,628	-	£7,375	-
4	Interest payments (£100,008@4%)	-	£4,000	-	£3,419
4	Life insurance premia	-	£4,628	-	£3,956
4	Life insurance claim	£100,008	-	£85,487	-
4	Repay loan	-	£100,008	-	£85,487
	Totals			<u>£100,532</u>	<u>£100,532</u>

Table 3: Example of Speculating on an Increase in Expected Longevity

Since life insurance and debt can be used to replicate a short position in an annuity, it is possible to use annuity replication to speculate on a decrease in expected longevity. This is done by buying life insurance and borrowing the sum insured to create a synthetic short position in an annuity. At the same time the interest rate risk of this synthetic short annuity position is hedged. When annuity prices have declined along with longevity expectations, an annuity is purchased at a favourable price, the interest rate hedge is closed, and the speculation is effectively ended.

Continuing with the Orsino Thruston example, and assuming that in year two the price of an annuity of £8,628 p.a. is £88,000, rather than the previously actuarially fair price of £93,641. Just as a long position in an annuity generates a mortality credit, holding a short position in an annuity produces a negative mortality credit. Orsino suffers the loss of a mortality credit with a value of £2,841 in year zero, due to delaying the purchase of the annuity by two years. Table 4 shows that

the decrease in expected longevity and the resulting drop in annuity prices means that Orsino makes a profit in year two, with a present value of £5,215 (or  $£5,215(1.04)^2 = £5,641$  in year two).

Individuals can also speculate on a rise in their longevity by buying an annuity (and hedging the interest rate risk), and then living beyond its break-even age, rather than offsetting the annuity. The resulting gain is only available after the annuitant passes the annuity's break-even age, with the additional annuity payments being received in the final years of the annuitant's life. These payments depend on how long they actually live, rather than their expected longevity when the annuity is purchased, and so are risky. An individual can speculate on a fall in their longevity by insuring their life (and hedging the interest rate risk), and then dying soon afterwards, rather than offsetting their life insurance. In this case their estate, rather than themselves, will benefit from the early payout of the death benefit.

Year	Transaction	Undiscounted Cash Flows		Discounted Cash Flows	
		Inflows	Outflows	Inflows	Outflows
0	Borrow	£100,008	-	£100,008	-
0	Insure life for £100,008	-	-	-	-
0	Open an interest rate hedge	-	-	-	-
1	Interest payments (£100,008@4%)	-	£4,000	-	£3,846
1	Life insurance premia	-	£4,628	-	£4,450
2	Interest payments (£100,008@4%)	-	£4,000	-	£3,698
2	Life insurance premia	-	£4,628	-	£4,279
2	Cost of annuity	-	£88,000	-	£81,361
2	Close the interest rate hedge	-	-	-	-
	Mortality Credit (see table 1)	-	-	£2,841	-
	Profit	-	-	-	<b>£5,215</b>
Totals		-	-	£102,849	£102,849
3	Annuity	£8,628	-	£7,670	-
3	Interest payments (£100,008@4%)	-	£4,000	-	£3,556
3	Life insurance premia	-	£4,628	-	£4,114
4	Annuity	£8,628	-	£7,375	-
4	Interest payments (£100,008@4%)	-	£4,000	-	£3,419
4	Life insurance premia	-	£4,628	-	£3,956
4	Life insurance claim	£100,008	-	£85,487	-
4	Repay loan	-	£100,008	-	£85,487
Totals				<u>£100,532</u>	<u>£100,532</u>

Table 4: Example of Speculating on a Decrease in Expected Longevity

An individual who is not retired may anticipate a rise or fall in their expected longevity during the period before they retire. They can speculate on such a change by buying a single premium deferred

annuity or deferred life insurance policy (coupled with an interest rate hedge) now. This locks in the annuity or life insurance price now using current longevity expectations. There is an extra risk in such trades as the speculator needs to forecast the size of the annuity or pension they will receive on retirement.

Longevity speculation requires the speculator to have access to the capital to buy an annuity in year zero, and involves exposure to longevity expectations risk, as well as incurring various transactions costs. So any profits have to be balanced against the transactions costs, risks and cost of capital. As with most speculative trades, the speculator also has the problem of deciding when to take their profit, or cut their losses. In this case the problem may be slightly easier as the speculation might be on some discrete event, such as a change in policy by insurance companies, the release of new mortality data, or a change in the status of the annuitant.

## **9. Portfolios and Annuity Replication**

The ability to replicate short positions in annuities has important implications for household portfolio decisions, as a substantial part of household wealth is invested in annuities. For example, in 2010 total UK pension liabilities were £7 trillion, or almost five times UK gross domestic product (OECD, 2012). Since households have a large proportion of their wealth in annuities, portfolio models need to incorporate investment in annuities. The demand for annuities is primarily driven by longevity risk, and so the incorporation of annuities in portfolio problems leads to multi-period life cycle portfolio models. The ability to replicate short positions in annuities means they can be treated like other financial assets that can be both purchased and liquidated, and this facilitates their inclusion in portfolio models. Using replication, annuities can be sold short by investors, so models incorporating annuities need not impose a restriction on short sales, although this non-negativity restriction is common in portfolio models. A few researchers have investigated such portfolio models and included investment in irreversible annuities, along with equities and bonds in multi-period portfolio models<sup>17</sup>. However the vast majority of portfolio models do not include annuities as an existing or potential investment, and a recent survey of household finance mentions annuities only in a footnote and appendix (Guiso and Sodini, 2013).

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<sup>17</sup> Babbel and Merrill (2007), Horneff and Maurer (2008), Horneff, Maurer, Mitchell and Dus (2008), Horneff, Maurer, Mitchell and Stamos (2009), Horneff, Maurer and Rogalla (2010), Horneff, Maurer and Stamos (2008a, 2008b), Huang and Milevsky (2008), Kapur and Orszag (1999), Milevsky and Young (2007) and Pang and Warshawsky (2010).

## 10. Conclusions

Annuities are generally viewed as irreversible investments, and this perceived illiquidity may be partly responsible for their neglect by both consumers and finance researchers. This paper has shown that annuities can be treated as liquid financial instruments, and demonstrated how they can be integrated into some of the standard models of finance. Because of the relationship between annuity and life insurance prices, short positions in annuities can be replicated using life insurance and debt. This enables long positions in annuities to be offset, which considerably increases their liquidity. Widespread knowledge of the possibility of annuity offset should increase the willingness of households to buy annuities, in the knowledge that it is not an irreversible position, and this may help to reduce the annuity puzzle.

The inefficiency of annuity markets offers scope for arbitrage between the life insurance and annuity markets, and for speculation on the expected longevity of annuitants. The ability to replicate short annuity positions permits the arbitrage of annuity underpricings, (or life insurance overpricings), and knowledge of how to conduct such arbitrage should increase the low price efficiency of annuity markets. It has been demonstrated that arbitrage can be profitable, even when substantial load factors are present. A long annuity position gives the owner an embedded American style option to liquidate this annuity at a better price sometime in the future, and this permits individuals to speculate on changes in their own life expectancy. Arbitrage and speculation using annuities can only be conducted by the annuitant, as annuities must be purchased by the nominee. So the size of individual arbitrage and speculative transactions is severely limited, although in aggregate the total value of these transactions could be considerable and large enough to move prices.

Households typically hold a sizable proportion of their wealth as long positions in occupational and state pensions and annuities. Given the importance of annuities, recognition of the ability to liquidate these positions makes it straightforward to incorporate annuities in household portfolio models, as additional annuities can be purchased, existing annuities offset, or short positions created. The inclusion of annuities in household portfolio models also highlights the need to use multi-period life-cycle portfolio models for households.

Once the ability to replicate annuities is recognised, financial services companies may offer a

service advising annuitants on offsetting their annuities and pensions. This would increase the knowledge of annuity replication amongst the general public, increasing the volume of such transactions. Finally, if annuity replication becomes widespread it will undermine the UK government policy of compulsory annuitization for those with defined contribution pensions<sup>18</sup>, and will also allow those with defined benefit and state pensions to offset these annuities.

## References

- Ameriks, J., Caplin, A., Laufer, S. and Van Nieuwerburgh, S. (2008) Annuity Valuation, Long Term Care and Bequest Motives. In *Recalibrating Retirement Spending and Saving* edited by J. Ameriks and O.S. Mitchell, Oxford University Press, pp. 251-275.
- Auerbach, A.J. and Kotlikoff, L.J. (1987) Life Insurance of the Elderly: Its Adequacy and Determinants. In *Work, Health and Income Among the Elderly*, edited by G. Burtless, Brookings Institution, pp. 229-268.
- Auerbach, A.J. and Kotlikoff, L.J. (1989) The Adequacy of Life Insurance Purchases, *Journal of Financial Intermediation*, vol. 1, no.3, June, pp. 215-241.
- Babbel, D.F. and Merrill, C.B. (2007), Rational Decumulation, Working paper, University of Pennsylvania, May.
- Bernheim, B.D. (1991) How Strong Are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities, *Journal of Political Economy*, Vol. 99, No.5, October, pp. 899-927.
- Blake, D., Cannon, E. and Tonks, I. (2010) *Ending Compulsory Annuitization: What Are the Consequences?*, The Pensions Institute.
- Blake, D. and Harrison, D. (2008) *And Death Shall Have No Dominion: Life Settlements and the Ethics of Profiting from Mortality*, The Pensions Institute.
- Blake, D. and Harrison, D. (2009) What is a Life Settlement? In *Life Markets: Trading Mortality and Longevity Risk with Life Settlements and Linked Securities*, edited by V.B. Bhuyan, John Wiley, pp. 11-16.
- Braun, A., Gatzert, N. and Schmeiser, H. (2012) Performance and Risks of Open-End Life Settlement Funds, *Journal of Risk and Insurance*, vol. 79, no. 1, pp. 193-229.
- Brown, J.R. (2001) Are the Elderly Really Over-Annuitized? New Evidence on Life Insurance and Bequests. In *Themes in the Economics of Aging*, edited by D.A. Wise, University of Chicago Press, pp. 91-126.
- Brown, J.R. (2002) How Should We Insure Longevity Risk in Pensions and Social Security? In *The Future of Social Insurance: Incremental Action or Fundamental Reform?*, edited by P. Edelman, D.L. Salisbury, and P.J. Larson, National Academy of Social Insurance, pp. 148-183.
- Brown, J.R. (2009) Financial Education and Annuities, *OECD Journal: General Papers*, no. 20, vol. 2008, issue 3, pp. 173-215.
- Brown, J.R. and Warshawsky, M.J. (2012) Longevity-Insured Retirement Distributions: Basic Theories and Institutions. In *Retirement Income: Risks and Strategies*, edited by M.J. Warshawsky, The MIT Press, pp. 57-83.
- Brown, J.R. and Warshawsky, M.J. (2004) Longevity-Insured Retirement Distributions from Pension Plans: Market and Regulatory Issues. In *Private Pensions and Public Policies* edited by W.G. Gale, J.B. Shoven and M.J. Warshawsky, Brookings Institution Press, pp. 332-381.

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<sup>18</sup> Compulsion has the advantage that it ensures there is no adverse selection in the market for compulsory annuities, and this reduces annuity prices. Compulsion also avoids the moral hazard that pensioners will spend all their pension pot, and then fall back on the welfare state. Pension pots have been accumulated without paying tax, while annuities and pension receipts are taxed; so any lump sum withdrawals need to be taxed if taxation is not to be reduced on defined contribution pensions. Finally, if defined contribution pensioners can avoid compulsory annuitization, defined benefit pensioners are likely to want similar treatment (Blake, Cannon and Tonks, 2010).

- Brown, R.L. and McDaid, J. (2003) Factors Affecting Retirement Mortality, *North American Actuarial Journal*, vol. 7, no. 2, April, pp. 24-43.
- Brown, R.L. and Scahill, P. (2010) Issues in the Issuance of Enhanced Annuities, SEDAP Research Paper no. 265, May, McMaster University.
- Browne, S., Milevsky, M.A. and Salisbury, T.S. (2003) Asset Allocation and the Liquidity Premium for Illiquid Annuities, *Journal of Risk and Insurance*, vol. 70, no. 3, September, pp. 509-526.
- Cannon, E. and Tonks, I. (2004) UK Annuity Rates, Money's Worth and Pension Replacement Ratios 1957-2002, *Geneva Papers on Risk and Insurance*, vol. 29, no. 3, July, pp. 394-416.
- Cannon, E. and Tonks, I. (2008) *Annuity Markets*, Oxford University Press, chapter 6.
- Cannon, E. and Tonks, I. (2009) *Money's Worth of Pension Annuities*, Department for Work and Pensions, Research Report no. 563.
- Cannon, E. and Tonks, I. (2011) Compulsory and Voluntary Annuities Markets in the UK. In *Securing Lifelong Retirement Income: Global Annuity Markets and Policy*, edited by O. Mitchell, J. Piggott and N. Takayama, Oxford University Press, pp. 171-194.
- Chalmers, J. and Reuter, J. (2012) How Do Retirees Value Life Annuities? Evidence from Public Employees, *Review of Financial Studies*, vol. 25, no. 8, August, pp. 2601-2634.
- Charupat, N., Kamstra, M. and Milevsky, M.A. (2012) The Annuity Duration Puzzle, Working paper, York University, March. Download from:-  
[http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2021579](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2021579)
- Charupat, N. and Milevsky, M.A. (2001) Mortality Swaps and Tax Arbitrage in the Canadian Insurance and Annuity Markets, *Journal of Risk and Insurance*, vol. 68, no. 2, June, pp. 277-302.
- Cox, S.H. and Lin, Y. (2007) Natural Hedging of Life and Annuity Mortality Risks, *North American Actuarial Journal*, vol. 11, no. 3, July, pp. 1-15.
- Crawford, R. and Tetlow, G. (2012) *Expectations and Experience of Retirement in Defined Contribution Pensions: A Study of Older People in England*, Institute for Fiscal Studies, November.
- Davidoff, T., Brown, J.R. and Diamond, P.A. (2005) Annuities and Individual Welfare, *American Economic Review*, vol. 95, no. 5, December, pp. 1573-1590.
- Finkelstein, A. and Poterba, J.M. (2002) Selection Effects in the United Kingdom Individual Annuities Market, *Economic Journal*, vol. 112, no. 476, January, pp. 28-50.
- Finkelstein, A. and Poterba, J. (2004) Adverse Selection in Insurance Markets: Policyholder Evidence from the UK Annuity Market, *Journal of Political Economy*, vol. 112, no. 1, February, pp. 183-208.
- Fong, J.H.Y. (2011) Beyond Age and Sex: Enhancing Annuity Pricing, Pension Research Council Working Paper 2011-07, University of Pennsylvania, June.
- Fong, J.H.Y., Mitchell, O.S. and Koh, B.S.K. (2010) Longevity Risk Management in Singapore's National Pension System, Working paper, Pension Research Council, WP2010-10.
- Fong, W.M. (2002) On the Cost of Adverse Selection in Individual Annuity Markets: Evidence from Singapore, *Journal of Risk and Insurance*, vol. 69, no. 2, June, pp. 193-207.
- Friedman, B. and Warshawsky, M. (1990) The Cost of Annuities: Implications for Saving Behaviour and Bequests, *Quarterly Journal of Economics*, vol. 105, no. 1, February, pp. 135-154.
- Gardner, J. and Wadsworth, M. (2004) Who Would Buy an Annuity? An Empirical Investigation, Watson Wyatt Technical Paper Series.
- Gatzert, N. (2010) The Secondary Market for Life Insurance in the United Kingdom, Germany and the United States: Comparison and Overview, *Risk Management and Insurance Review*, vol. 13, no. 2, Fall, pp. 279-301.
- Guiso, L. and Sodini, P. (2013) Household Finance: An Emerging Field. In *Handbook of the Economics of Finance* edited by G.M. Constantinides, M. Harris and R.M. Stulz, vol. 2, part B, Science Direct, pp. 1397-1532.
- Harrison, D., Byrne, A. and Blake, D. (2006) *Annuities and Accessibility: How the Industry Can Empower Consumers to Make Rational Choices*, The Pensions Institute, March.
- He, D. (2009) The Life Insurance Market: Asymmetric Information Revisited, *Journal of Public Economics*, vol. 93, no. 9-10, October, pp. 1090-1097.
- Horneff, W.J. and Maurer, R.H. (2008) Deferred Annuities and Strategic Asset Allocation, Working paper, Michigan Retirement Research Centre, no. 2008-178.
- Horneff, W.J., Maurer, R.H., Mitchell, O.S. and Dus, I. (2008) Following the Rules: Integrating Asset Allocation and Annuitization in Retirement Portfolios, *Insurance: Mathematics and Economics*, vol.

- 42, no. 1, February, pp. 396-408.
- Horneff, W.J., Maurer, R.H., Mitchell, O.S. and Stamos, M.Z.(2008) Asset Allocation and Location Over the Life Cycle with Investment-Linked Survival-Contingent Payouts, *Journal of Banking and Finance*, vol. 33, no. 9, September, pp. 1688-1699.
- Horneff, W.J., Maurer, R.H., and Rogalla, R. (2010) Dynamic Portfolio Choice with Deferred Annuities, *Journal of Banking and Finance*, vol. 34, no. 11, November, pp. 2652-2664.
- Horneff, W.J., Maurer, R.H., and Stamos, M.Z.(2008a) Life-Cycle Asset Allocation with Annuity Markets, *Journal of Economic Dynamics and Control*, vol. 32, no. 11, November, pp. 3590-3612.
- Horneff, W.J., Maurer, R.H., and Stamos, M.Z.(2008b) Optimal Gradual Annuitization: Quantifying the Costs of Switching to Annuities, *Journal of Risk and Insurance*, vol. 75, no. 4, December, pp. 1019-1038.
- Huang, H. and Milevsky, M.A. (2008) Portfolio Choice and Mortality-Contingent Claims: The General HARA Case, *Journal of Banking and Finance*, vol. 32, no. 11, November, pp. 2444-2452.
- Hueler, K. and Rappaport, A. (2012) The Role of Guidance in the Annuity Decision Making Process, Working paper, Pension Research Council, WP2012-11, September.
- James, E. and Song, X. (2001) Annuities Markets Around the World: Money's Worth and Risk Intermediation, Working paper no. 16/01, Centre for Research on Pensions, December.
- James, E. and Vittas, D. (2000) Annuity Markets in Comparative Perspective: Do Consumers Get Their Money's Worth?, World Bank, Development Research Group, Policy Research Working Paper, No. 2493, November.
- Kapur, S. and Orszag, J.M. (1999) A Portfolio Approach to Investment and Annuitization During Retirement, Birkbeck Working Paper no 15.
- Keilman, N. (2007) UK National Population Projections in Perspective: How Successful Compared to Those in Other European Countries?, *Population Trends*, Vol. 129, Autumn, pp. 20-30.
- McCarthy, D. and Mitchell, O.S. (2010) International Adverse Selection in Life Insurance and Annuities. In *Ageing in Advanced Industrial States* edited by S. Tuljapurkar, N. Ogawa and A.H. Gauthier, Springer, pp. 119-135.
- MGM Advantage (2009) Launch of MGM Advantage Annuity Index Reveals Average Difference of 22% Between Standard and Enhanced Annuities.  
<http://www.mgmadvantage.com/homepage/media-centre/press-releases/press-release-14.html>
- Milevsky, M. A. (2006) *The Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance*, Cambridge University Press.
- Milevsky, M.A, and Young, V.R. (2007) Annuitization and Asset Allocation, *Journal of Economic Dynamics and Control*, vol. 31, no. 9, September, pp. 3138-3177.
- Mitchell, O.S. (2002) Developments in Decumulation: The Role of Annuity Products in Financing Retirement. In *Ageing, Financial Markets and Monetary Policy*, edited by A.J. Auerbach and H. Herrmann, Springer, pp. 97-138.
- Mitchell, O.S., Poterba, J.M., Warshawsky, M.J. and Brown, J.R. (1999). New Evidence on the Money's Worth of Individual Annuities, *American Economic Review*, vol. 89, no. 5, December, pp. 1299-1318.
- Mitchell, O.S. and McCarthy, D. (2002) Estimating International Adverse Selection in Annuities, *North American Actuarial Journal*, vol. 6, no. 4, October, pp. 38-54.
- Murthi, M., Orszag, M. and Orszag, P.R. (2000) The Value for Money of Annuities in the UK: Theory, Experience and Policy, *Journal of Pensions Management*, May.
- Neuberger, A. (2003) Annuities and the Optimal Investment Decision, Working paper London Business School, May.
- OECD (2012) *Pension Markets in Focus*, no. 9, September, Download from:-  
<http://www.oecd.org/insurance/privatepensions/pensionmarketsinfocus.htm>
- Pang, G. and Warshawsky, M. (2010) Optimizing the Equity-Bond-Annuity Portfolio in Retirement: The Impact of Uncertain Health Expenses, *Insurance: Mathematics and Economics*, vol. 46, no. 1, January, pp. 198-209.
- Peijnenburg, K., Nijman, T. and Werker, B.J.M. (2011) Health Cost Risk: A Potential Solution to the Annuity Puzzle, Working paper, CentER, Tilburg University, April.
- Promislow, S.D. (2011) *Fundamentals of Actuarial Mathematics*, Wiley, Second edition.
- Rocha, R. and Thorburn, C. (2007) *Developing Annuity Markets: The Experience of Chile*, The World Bank, Annex 1, pp. 149-180.
- Shaw, C. (2007) Fifty Years of United Kingdom National Population Projections: How Accurate Have They

- Been?, *Population Trends*, vol. 128, Summer, pp. 8-23.
- Sinclair, S.H. and Smetters, K.A. (2004) Health Shocks and the Demand for Annuities, Technical Paper Series, Congressional Budget Office, July.
- Turra, C.M. and Mitchell, O.S. (2008) The Impact of Health Status and Out-of-Pocket Medical Expenditures on Annuity Valuation. In *Recalibrating Retirement Spending and Saving* edited by J. Ameriks and O.S. Mitchell, Oxford University Press, pp. 227-250.
- Velde, F.R. and Weir, D.R. (1992) The Financial Market and Government Debt Policy in France, 1746-1793, *Journal of Economic History*, vol. 52, no. 1, March, pp. 1-39.
- Von Gaudecker, H.M. and Weber, C. (2004) Surprises in a Growing Market Niche: An Evaluation of the German Private Life Annuities Market, *Geneva Papers on Risk and Insurance*, vol. 29, no. 3, July, pp. 394-416.
- Walliser, J. (2000) Adverse Selection in the Annuities Market and the Impact of Privatizing Social Security, *Scandinavian Journal of Economics*, vol. 102, no. 3, September, pp. 373-393.
- Wang, T. and Young, V.R. (2012a) Optimal Commutable Annuities to Minimise the Probability of Lifetime Ruin, *Insurance: Mathematics and Economics*, vol. 50, no. 1, January, pp. 200-216.
- Wang, T. and Young, V.R. (2012b) Maximizing the Utility of Consumption with Commutable Life Annuities, *Insurance: Mathematics and Economics*, vol. 51, no. 2, September, pp. 352-369.
- Yaari, M.E. (1965) Uncertain Lifetime, Life Insurance, and the Theory of the Consumer, *Review of Economic Studies*, vol. 32, no. 2, April, pp. 137-150.