Space, Mortality, and Economic Growth

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Abstract

At present, academic actuarial research involving the mortality modeling of multiple populations mainly focuses on factor-based approaches. This comes with little attention to interpretable models of mortality that take patterns across space into consideration. To address this, we propose a family of models that extend the seminal factor-based stochastic mortality modeling framework of Li and Lee (2005) to include spatial patterns. Specifically, in this paper, we study the relationship between economic growth, as represented by the real gross domestic product (GDP), and mortality of the contiguous United States. The proposed spatial lag of GDP with GDP (SLGG) model was used to produce forecasts of mortality rates and annuity pricing for each of the states of the United States and demonstrated the effects which economic growth has on mortality. A comparison of annuity pricing across space revealed that the SLGG model preserves more regional differences when it comes to pricing compared to the Li and Lee (2005) model. In a larger context, this research provides a blueprint for the inclusion of spatial components and economic growth into mortality modeling. Importantly, it establishes an empirical basis for the development of spatial natural hedging techniques.

Keywords: mortality, economic growth, spatial lag model, mortality forecasting, annuity pricing.

JEL classification : C1, C13, C31, C51, C53, J11.

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1 Introduction

With life expectancy in the United States undergoing substantial improvements over recent decades, institutionally managed financial needs of a rapidly aging population are getting more costly (see Pitacco et al. (2009)). The process of aging of a population yields a change in the relative number of retirees compared to the number of active workers, which given political constraints of taxation, can create financial uncertainty for these institutions (see Lee and Tuljapurkar (1997)). That is why the ability to model human survivorship is essential for the actuarial practice, especially for annuity pricing (see Pitacco (2016)), hedging (see Cox and Lin (2007), Zhu and Bauer (2014)) and social security affordability (see Soneji and King (2012)). Considering all of the above it is important to better understand how aging correlates with economic growth. For example, if there is a positive correlation of economic growth and longevity in the United States, then a program such as the federal Old-Age and Survivors Insurance (OASI) could be more expensive in the long term as a result of the economy performing well and vice versa. That is why this paper focuses on the relationship between mortality and economic growth in the United States and shows its dynamics are by no means trivial. However, by considering stochastic models of economic and mortality patterns across space and time, this relationship can be much better understood.

In particular, in this work, we propose a novel approach to model human mortality which is affected by a multitude of risk factors, and in principle is heterogeneous in its manifestation among different states in the United States. One reason for this heterogeneity is that the economy may grow faster in one state than others, which may lead to better habits in life and better health care, and opposite when an economy declines. For instance, it is well-known that smoking is more prevalent in populations with a declining economy (see Franks et al. (2007)). In fact, the demographic literature recognizes that there exists long-run relationships between economic development and mortality changes in various countries, as demonstrated by the Preston curve (see Preston (1975)) which finds empirical cross-sectional positive dependence between life expectancy and real per capita income. More recently Hanewald (2011) finds significant co-movements between mortality dynamics and the gross domestic product (GDP) per capita.

Considering mortality and GDP in different states of the United States, we investigate the relationship between mortality and economic growth via multi-population mortality and economic growth models. Additionally, to understand which states are most affected by their neighbors, we investigate the short-term spatial effects impacting mortality rates of the contiguous United States. Previously, Niu and Melenberg (2014) and Boonen and Li (2017) added economic growth as a risk factor to the Lee-Carter model (see Lee and Carter (1992)) and to the Li-Lee model (see Li and Lee (2005)) respectively. Also, Seklecka et al. (2019) obtain better mortality forecasts by adding economic growth as a risk factor to O'Hare-Li model (see O'Hare and Li (2012)).

Whereas the recent works of Li and Lu (2017), Doukhan et al. (2017) and Ludkovski et al. (2018) implement spatial considerations into the mortality modeling framework by defining spatial relationships based on closeness in age and time, we investigate spatial patterns from a geographical perspective. Also, we observe that in recent years, multi-population models have gained significant actuarial interest. Their use is widespread, and various institutions such as Dutch and Belgian Actuarial Institute rely on them to provide mortality projections for insurers (see Antonio et al. (2017)). That is why in this work, we propose a multi-population model with economic growth and extend Boonen and Li (2017) to introduce spatial dependence of mortality dynamics in the short-run. Spatial effect parameters account for the spillover effects among the adjacent states. Intuitively, it the context of the United States, it is not difficult to imagine that due to labor mobility or health care systems, an increase in life expectancy or GDP per capita in Massachusetts may have a direct positive impact on the life expectancy in Rhode Island, but no direct effect on the life expectancy in Arizona.

A research objective of this paper is to study and quantify the impacts that the structural inequalities in economic growth across the states of the United States have on mortality in a given state. In particular, we also study the effect that the evolution of mortality of a particular state may have on the evolution of mortality in its neighbouring states. Moreover, we study the effect that the economic growth of neighboring states have on the mortality rates in a state.

Our work may be of significant interest to actuarial academics and practitioners, as well as the broader social science and demographic academic communities. To the best of our knowledge, we are the first to model the connection between structural economic inequality and mortality at the comprehensive level for the case of the United States. This paper thus bridges the actuarial, social sciences and demographic disciplines. For actuarial practitioners, this paper provides a justification for considering spatial components in mortality models of any given state, as well as incorporating spatial economic components for model considerations. Finally, based on our chosen mortality model, we find substantial heterogeneity in annuity prices across the United States and our works serves as an empirical justification for spatial hedging.

The paper is set out as follows. In Section 2, we describe the data that we use in this paper. The mortality models with GDP are introduced in Section 3, and model selection is performed in Section 4. In Section 5, we compare the mortality forecasts. An application in annuity pricing is provided in Section 6. Finally, Section 7 concludes.

2 Data

In this paper, we measure economic growth by the GDP per capita. Data of the real GDP per capita per state in the United States from 1977 to 2016 is collected from the Bureau of Economic Analysis[1,2].

To model human mortality, we focus on the central death rates in every state of the United States (see Li and Lee (2005)) and we consider only the male population. For notational convenience, we label the 48 contiguous states with numbers from 1 to 48. The states Hawaii and Alaska are excluded because they do not share a land-border with another state. The central death rate of a population for a given year is the number of deaths occurring among a population during a given year relative to the number of people that are alive at the beginning of that given year. We let $m_{i,x,t}$ denote the central death rate for age group $x \in \{0, 1-4, 5-9, \ldots, 85+\}$ at time $t \in \{1977, 1978, \ldots, 2016\}$ in state $i \in \{1, 2, \ldots, 48\}$,

[1] The federal district Washington DC is excluded from our analysis, because the GDP per capita in this district is very high, and uninformative as factor of economic growth for this area.

[2] State level real GDP data was obtained from https://www.bea.gov/

to be

$$m_{i,x,t} = \frac{\text{Number of deaths in age group } x \text{ at year } t \text{ in state } i}{\text{Number of people alive in age group } x \text{ at time } t \text{ in state } i}.$$

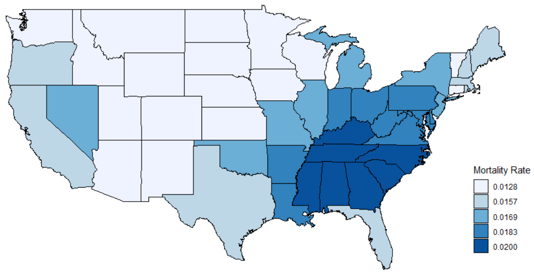
The mortality data that we investigate in this study is obtained from the Centers for Disease Control and Prevention's (CDC) WONDER internet databases. The Compressed Mortality File (CMF), produced by the National Center for Health Statistics, is a national mortality and population database which spans the years 1968-2016. This dataset specifically features the crude mortality rates for each age bracket among the individual counties of the contiguous United States, as identified by their respective Federal Information Processing Standards (FIPS) codes.

As an example, in Figure 1, we display the central death rates for all contiguous states of the age group of 55-64 year, for four different years. To compare the central death rates, we also show in this figure the GDP per state. All plots in Figure 1 display the value of the death rate (in red) and GDP per capita (in blue) relative to the United States average. With a darker shade, a relatively higher value of the death rate or GDP is represented. For instance, the states California and New York have a high relative GDP per capita, while having a low death rate. Each of the maps presents a clear spatial pattern of mortality, with states having similar mortality rates clustered together across the entire country. These are the effects that we capture in this paper.

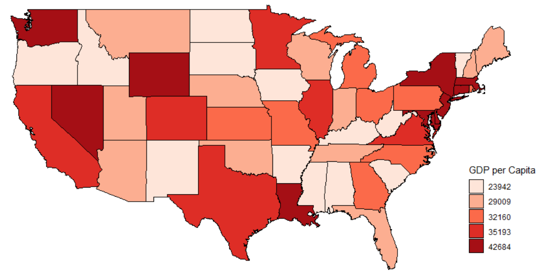
3 Spatial mortality models with state GDPs

Rather than generating separate models for each individual state, Li and Lee (2005) show the plausibility of improving mortality forecasts for individual populations by taking into account the mortality dynamics in a larger group. The Li-Lee model is a multi-population generalization of the Lee-Carter model, where a common mortality pattern is incorporated and a coherence assumption is imposed where forecasts of different populations could not diverge in the long run. The Li-Lee model is given by

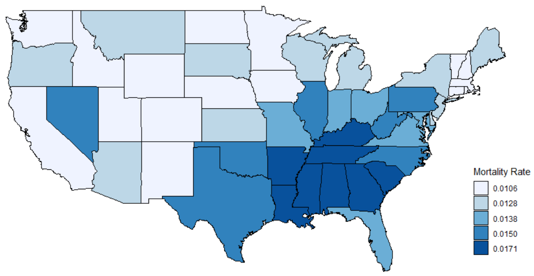
$$\log(m_{i,x,t}) = \alpha_{i,x} + B_x K_t + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}.$$



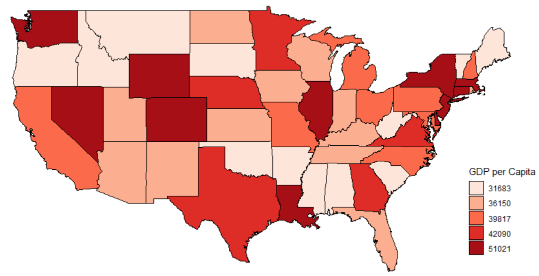
(a) 1986 Mortality Rates for Ages 55-64



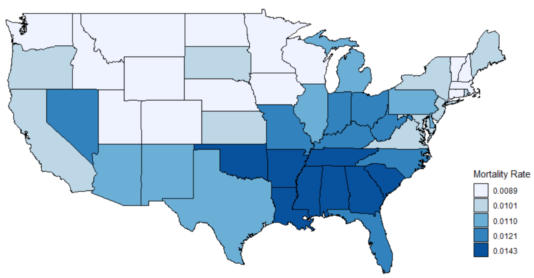
(b) 1986 GDP per Capita



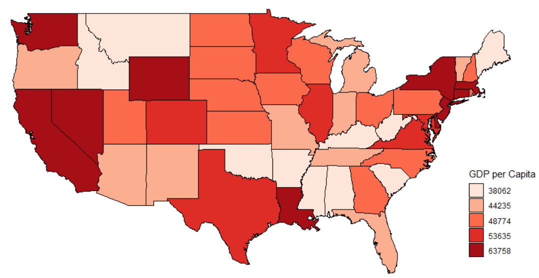
(c) 1996 Mortality Rates for Ages 55-64



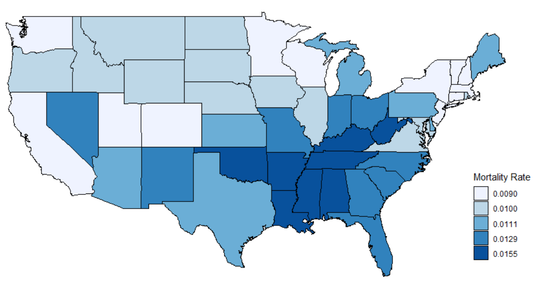
(d) 1996 GDP per Capita



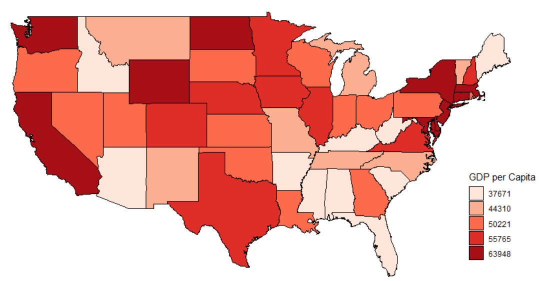
(e) 2006 Mortality Rates for Ages 55-64



(f) 2006 GDP per Capita



(g) 2016 Mortality Rates for Ages 55-64



(h) 2016 GDP per Capita

Figure 1: Comparison between the mortality rates of the 55-64 age group and the GDP per capita for the years 1986, 1996, 2006 and 2016. A darker shade represents a relatively higher death rate or GDP when compared to the national average.

with the following normalization conditions: $\sum_x \beta_{i,x} = 1$ and $\sum_t \kappa_{i,t} = 0$ for all i, and $\sum_x B_x = 1$ and $\sum_t K_t = 0$. The error terms $\varepsilon_{i,x,t}$ are assumed to be i.i.d. Gaussian with mean 0. Here, the parameter $\alpha_{i,x}$ describes the time-average mortality for each state i and age x, the common factor K_t captures the evolution of national mortality rates over time, and the age effect B_x explains this sensitivity to this K_t of the age-specific mortality rates. To ensure coherency, we impose that the $\kappa_{i,t}$ time-series processes are stationary (see Li and Lee (2005)). Li and Lee (2005) propose to estimate the parameters in a two step procedure, by firstly estimating the common parameters B_x and K_t from the combined data for all the populations, and then secondly estimating the remaining population-specific parameters. In both estimation steps, the parameters are estimated by a singular value decomposition.

Starting from the above, to develop a family of multi-population mortality models which combine coherency, macroeconomic variable GDP and spatial effects, we begin by considering the simple extension of the Li and Lee model developed by Boonen and Li (2017)

$$\log(m_{i,x,t}) = \alpha_{i,x} + \gamma_x GDP_t + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t},$$

where $\alpha_{i,x}$, $\beta_{i,x}$ and $\kappa_{i,t}$ are the state population-specific parameters, GDP_t is the demeaned gross domestic product per capita of the United States at time t, with respective loading γ_x. Standard normalization conditions apply, where $\sum_x \beta_{i,x} = 1$ and $\sum_t \kappa_{i,t} = 0$. The error terms $\varepsilon_{i,x,t}$ are assumed to be i.i.d. Gaussian with mean 0. In this base model, the parameters are estimated in two steps. First, parameters $\alpha_{i,x}$ and γ_x are estimated by OLS. Second, the remaining parameters $\beta_{i,x}$ and $\kappa_{i,t}$ are estimated by a singular value decomposition. We refer to Boonen and Li (2017) for a further discussion on the estimation.

To extend this model to the spatial domain, we propose incorporating spatially autoregressive and lagged components to generate a class of models of the form

$$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j \neq i} W_{i,j} \log(m_{j,x,t}) + \psi_{i,x} \sum_{k \neq i} W_{i,k} GDP_{k,t} + \gamma_{i,x} GDP_{i,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}. \tag{1}$$

Again, the standard normalization conditions apply, where $\sum_x \beta_{i,x} = 1$ and $\sum_t \kappa_{i,t} = 0$. The

$\alpha_{i,x}, \beta_{i,x}$ and $\kappa_{i,t}$ are the state-specific parameters, $GDP_{i,t}$ is the demeaned GDP per capita of state i at time t, with respective loadings $\gamma_{i,x}$, and W is a matrix that indicates whether states are neighbors. Specifically, $W_{i,j}$ is non-zero if the two states i and j share a common border, such as California and Arizona, and zero if they are not adjacent, such as California and New York. Rather than treat the United States as an 'island', Canada and Mexico are also included in our spatial models. In Figure 2 we illustrate the construction of the matrix W for California, where a connection line means that the states i and j are adjacent and thus $W_{i,j} > 0$.

Including $\rho_{i,x} \sum_{j \neq i} W_{i,j} \log(m_{j,x,t})$ and $\psi_{i,x} \sum_{k \neq i} W_{i,k} GDP_{k,t}$ into our model allows for the exploration of spatial lag coefficients. Specifically, $\rho_{i,x}$ provides means for an investigation of the effect that mortality rates from neighbouring states have on the mortality of a specific state, while $\psi_{i,x}$ focuses on the relationship between the economic growth of specific states in relation to the mortality rates of their neighbours, which addresses the impact that the economic growth in a state has on the mortality dynamics of states that it shares close proximity with.

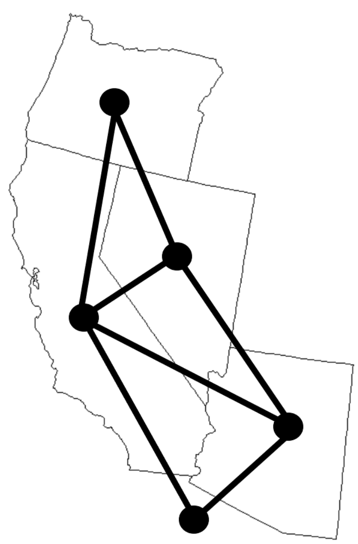


Figure 2: Connectivity map of California and its adjacent states, where Mexico is also included as neighbour. We see that California and Arizona are adjacent as they share a common border.

The parameters of the proposed model are estimated in two steps where we combine the approaches of Haining and Haining (2003) for spatial models and Niu and Melenberg (2014) for mortality models. First, we estimate the parameters $\alpha_{i,x}$ as

$$\hat{\alpha}_{i,x} = \frac{\sum_{t=1977}^{2016} \log(m_{i,x,t})}{2016 - 1977 + 1},$$

and we estimate the spatial parameters and the parameters $B_{i,x}$ by ordinary least-square (OLS). Second, the parameters $\beta_{i,x}$ and $\kappa_{i,t}$ are estimated using a singular value decomposition.

We will refer to the model given by expression (1) as *Spatial 'Everything'*. We also study possible simplifications of this model by setting certain parameters to zero, or by focusing on national GDP rather than state-specific GDP. Table 1 contains an overview of all of the models that we investigate in this study. There are a variety of models which will be investigated. For example, the 'Time Lagged GDP' model assumes that mortality in an individual state at any point in time is only concerned with an unobservable latent factor and the first time lag of the state's individual GDP per capita. The 'Spatial Lag of GDP' model only considers the effects that the GDP of neighbouring states have on mortality of an individual state. The 'Spatial Autoregressive with National GDP' model considers the effects that the mortality of an individual states' neighbours, in combination with the GDP of the entire country, have on mortality rates in the state, etc.

Name	Model Specification
Li-Lee	$\log(m_{i,x,t}) = \alpha_{i,x} + B_x K_t + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Base	$\log(m_{i,x,t}) = \alpha_{i,x} + \gamma_x GDP_t + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \gamma_{i,x} GDP_{i,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Time Lagged GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \phi_{i,x} GDP_{i,t-1} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
GDP with Time Lagged GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \gamma_{i,x} GDP_{i,t} + \phi_{i,x} GDP_{i,t-1} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Time Lagged GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \xi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t-1} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
GDP with Spatial Time Lagged GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \gamma_{i,x} GDP_{i,t} + \xi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t-1} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Autoregressive	$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j\neq i} W_{i,j} \log(m_{j,x,t}) + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Lag of GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \psi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Autoregressive with GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j\neq i} W_{i,j} \log(m_{j,x,t}) + \gamma_{i,x} GDP_{i,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Lag of GDP with GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \psi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t} + \gamma_{i,x} GDP_{i,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial Autoregressive with National GDP	$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j\neq i} W_{i,j} \log(m_{j,x,t}) + \gamma_x GDP_t + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial	$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j\neq i} W_{i,j} \log(m_{j,x,t}) + \psi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$
Spatial 'Everything'	$\log(m_{i,x,t}) = \alpha_{i,x} + \rho_{i,x} \sum_{j\neq i} W_{i,j} \log(m_{j,x,t}) + \psi_{i,x} \sum_{k\neq i} W_{i,k} GDP_{k,t} + \gamma_{i,x} GDP_{i,t} + \beta_{i,x}\kappa_{i,t} + \varepsilon_{i,x,t}$

Table 1: Family of multi-population mortality models that are studied in this paper.

4 Model selection

To evaluate the in-sample fit for model selection, the AIC and BIC ratios are compared for the mortality models. The AIC ratio is introduced by Akaike (1973), and is defined as

$$AIC = -2 \cdot \log(\hat{L}) + 2 \cdot k,$$

where $\log(\hat{L})$ is the log-likelihood of the model and k is the number of its free parameters to be estimated. Moreover, as defined by Schwarz (1978), the BIC ratio is given by

$$BIC = -2 \cdot \log(\hat{L}) + k \cdot \log(n),$$

where n is the number of data points. The number of free parameters, k, is the number of total parameters minus the number of constraints placed in the model. A lower AIC or BIC ratio means that the model has a better in-sample fit. The difference between the AIC and the BIC is that the BIC ratio imposes a higher penalty for the number of free parameters. Multi-population mortality models contain typically many free parameters. A well cited reference that explains the differences between the AIC and BIC ratios is Yang (2005).

Table 2 displays the number of free parameters, the AIC and BIC ratios and the R^2 for each of the models in the study. After fitting procedure, we find that the AIC ratio of the Li-Lee model is the smallest, indicating the best in-sample fit. For the BIC ratio, we observe that the Spatial Lag of GDP with GDP (SLGG) model has the best in-sample fit. While our hypothesis was that the mortality of a particular state may be affected by the evolution of mortality of its neighbouring states, this has implicitly been rejected in the model selection process as the chosen models do not feature any spatial lag of neighbouring mortality rates. However, the main hypotheses that economic inequalities have an impact on mortality still remains to be explored. Based on this, we proceed our investigation by studying the Li-Lee model and the SLGG model.

As an example, Figure 3 displays the fitted Li-Lee model to the observed point values of the age-specific central mortality rates of four selected states in the years 1977 and 2016. The plots display a generally good fit for each of the selected states, with slight underestimation

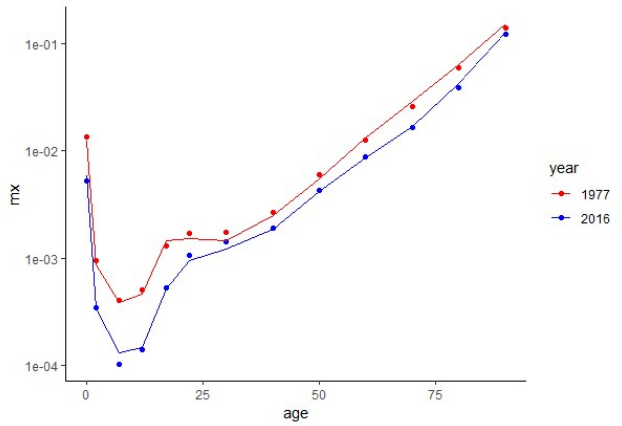
Name	Total Number of Parameters	AIC	BIC	R^2
Li-Lee	3221	-30857.11	**-5490.77**	0.9323
Base	3181	-29868.97	-4803.26	0.9296
GDP	3792	-30671.82	-641.71	0.9347
Time Lagged GDP	3744	-29962.10	-414.35	0.9386
GDP with Time Lagged GDP	4368	-29619.02	4982.95	0.9407
Spatial Time Lagged GDP	3744	-30524.76	-977.02	0.9399
GDP with Spatial Time Lagged GDP	4368	-30706.31	3895.66	**0.9431**
Spatial Autoregressive	3792	-26789.46	3240.65	0.9245
Spatial Lag of GDP	3792	-31259.67	-1229.56	0.9361
Spatial Autoregressive with GDP	4416	-29420.78	5679.34	0.9347
Spatial Lag of GDP with GDP	4416	**-31979.41**	3120.72	0.9407
Spatial Autoregressive with National GDP	3805	-27137.65	2998.09	0.9256
Spatial	4416	-30011.46	5088.67	0.9361
Spatial 'Everything'	5040	-30731.44	9438.71	0.9407

Table 2: The total number of parameters, AIC and BIC ratios and the R^2 for each of the multi-population mortality models. The models are defined in Table 1, and the best models bold-faced for all three model selection criteria.

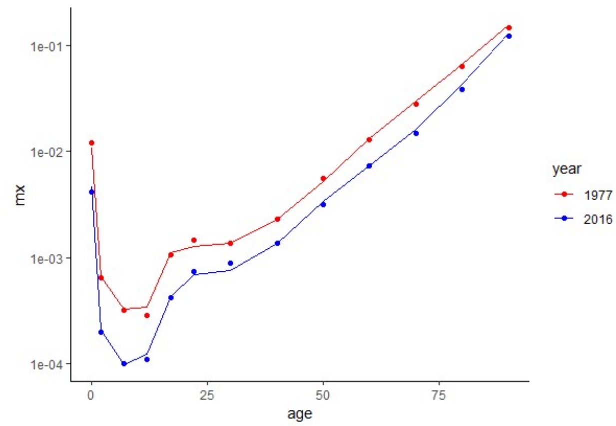
of mortality rates for the ages of 25 to 35 in each of the selected states in the year 2016. Observing the fits of the SLGG model for the same age groups, states and years in Figure 4, we see that the model more accurately fits the observed mortality rates for each state in both years.

To investigate the impact that the economic growth of a state's neighbours has on its mortality, Figure 5 displays a map of the estimates for the state specific parameter $\psi_{i,x}$ obtained by the SLGG model. The parameter $\psi_{i,x}$ represents the spatial spillover effects that GDP of neighbouring states of State i has on the mortality of age-group x in State i. These maps allow us to gain insight into the sensitivity of the mortality with respect to changes in GDP of the neighbouring states. The states coloured in blue represent a negative relationship, indicating a lower mortality experience associated with the improved economic situation of their neighbouring states, and similarly states coloured in red indicate a higher mortality experience associated with the improved economic situation of their neighbours.

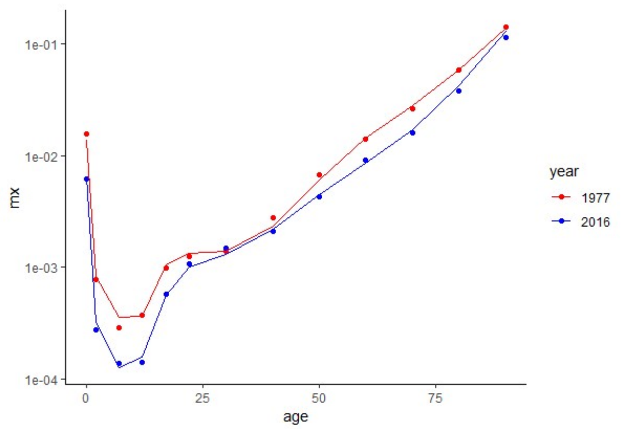
From these maps we observe the strongest associations between mortality rates and neighbouring states' economic situations occur for the 65 to 74 age group, as indicated by the darkest shaded in regions, which contain the peak retirement ages for the majority of the United States. In the northeastern states of Maine, New Hampshire, New Jersey and Connecticut we observe that the improved economic situation of neighbours corresponds with



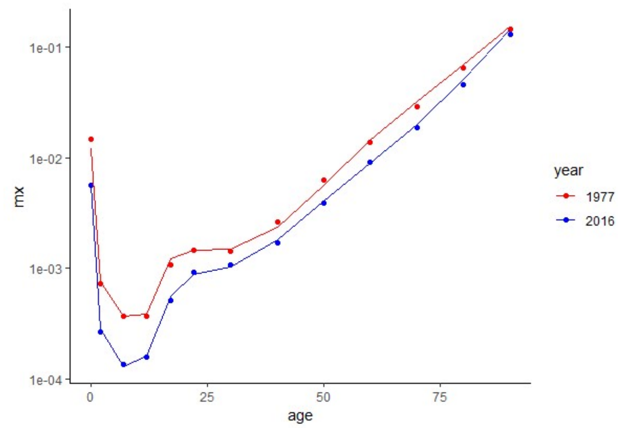
(a) Arizona



(b) California



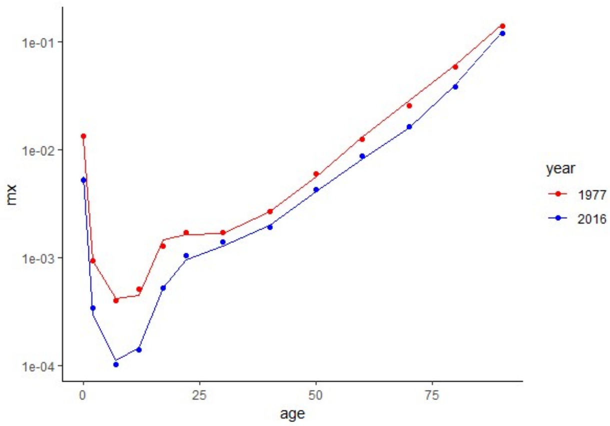
(c) Florida



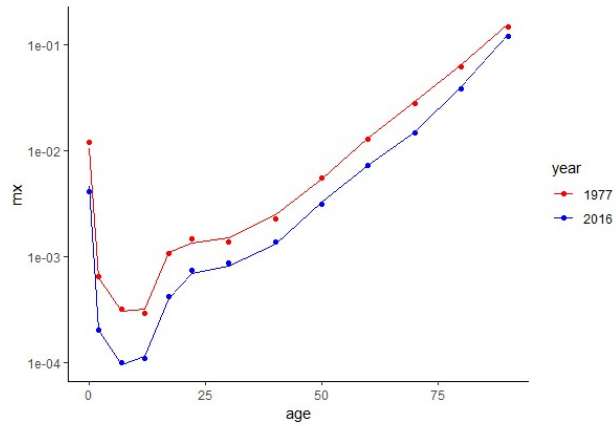
(d) Texas

Figure 3: Comparison of the estimated (line) and observed (dots) value of $m_{i,x,t}$ of the Li-Lee model for the years 1977 and 2016 and the states Arizona, California, Florida and Texas. On the y-axis, a logarithmic scale was used.

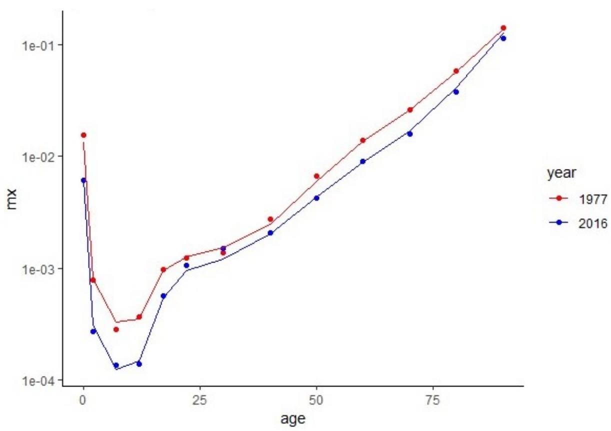
a lower mortality experience. The state of Massachusetts experiences an opposite effect, where the improved economic situation of its neighbours corresponds to higher mortality rates. Further research is needed to investigate whether the higher costs of living in the state of Massachusetts may cause retirees to take advantage of better conditions in neighbouring states when the economic situation is flourishing, and would be valuable for better understanding the mortality experiences of small states that are in very close proximity to each other. It should be noted that influential states such as California and Texas, which have some of the largest GDPs in the United States, are not greatly affected by the economic situation of their neighbouring states.



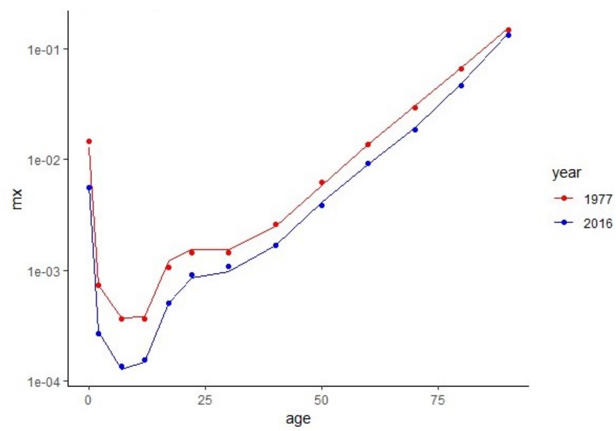
(a) `Arizona`



(b) `California`



(c) `Florida`



(d) `Texas`

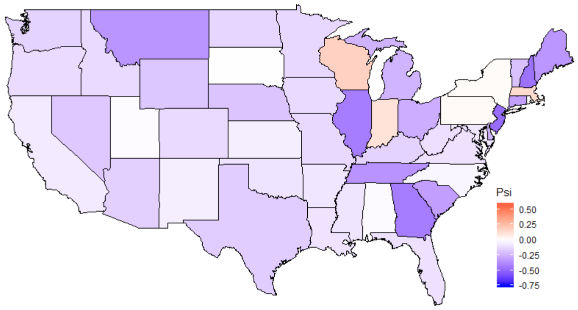
Figure 4: Comparison of the estimated (line) and observed (dots) value of $m_{i,x,t}$ of the SLGG model for the years 1977 and 2016 and the states Arizona, California, Florida and Texas. On the y-axis, a logarithmic scale was used.

5 Forecasting

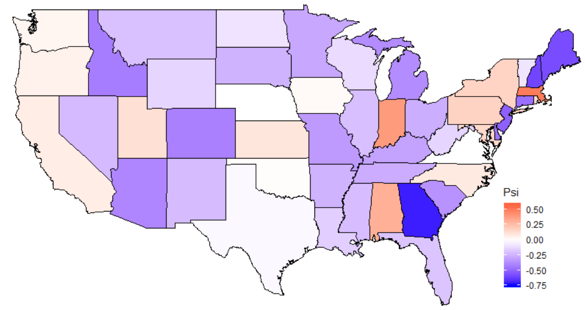
To forecast the time-dependent variables for the selected models, we propose time series models. We fit a random walk with drift to the common latent factor, assuming

$$K_t = K_{t-1} + c + \eta_t,$$

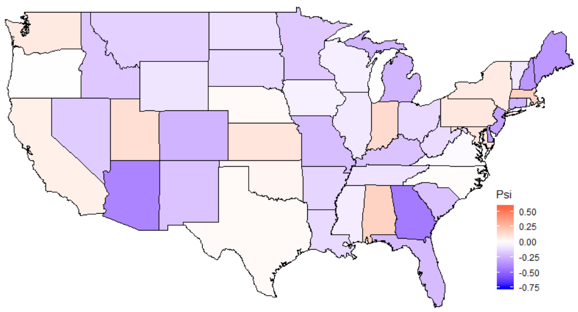
where c is the drift term, and the error term η_t is i.i.d. Gaussian with mean 0. The common factors are assumed to be non-stationary with a linear trend. To allow for stationary of the



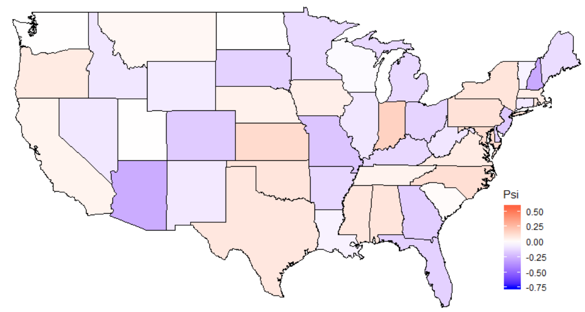
(a) $\psi_{i,x}$, `Ages 55-64`



(b) $\psi_{i,x}$, `Ages 65-74`



(c) $\psi_{i,x}$, `Ages 75-84`



(d) $\psi_{i,x}$, `Ages 85+`

Figure 5: Mapped estimates of the state specific parameter $\psi_{i,x}$ for the three oldest age groups, as obtained from the SLGG model.

population-specific processes $\kappa_{i,t}$, we fit each with an $AR(1)$ specification:

$$\kappa_{i,t} = c_{i,0} + c_{i,1}\kappa_{i,t-1} + \omega_{i,t}$$

where the error term $\omega_{i,t}$ is i.i.d. follows a Gaussian distribution with mean 0.

Forecasting the GDP of each individual state over time requires attention to the dependencies that each of the states have on one other. Vector autoregressive (VAR) models are used to model vectors of variables that are assumed stationary, allowing for lagged relationships between the variables and for the correlations between the variables. A pth-order vector autoregression, VAR(p), based on p lags of the variables is given by

$$\mathbf{y}_t = \mathbf{c} + \mathbf{\Phi}_1\mathbf{y}_{t-1} + \mathbf{\Phi}_2\mathbf{y}_{t-2} + \cdots + \mathbf{\Phi}_p\mathbf{y}_{t-p} + \epsilon_t,$$

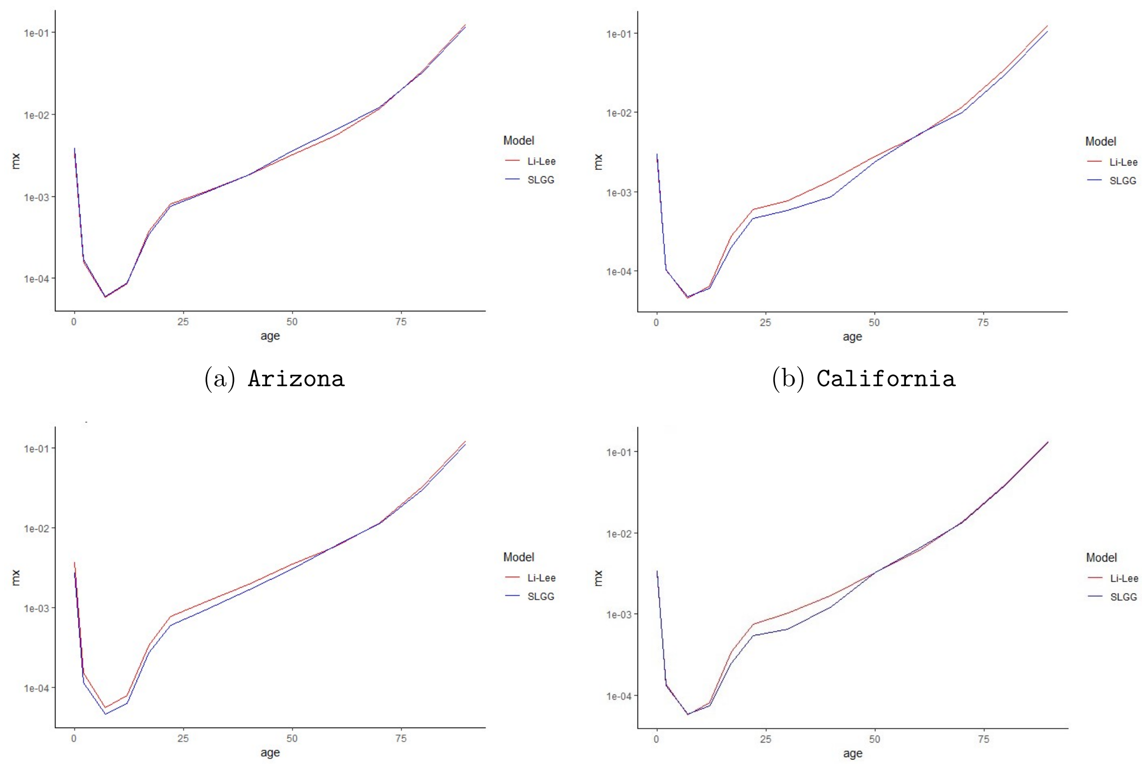
where the GDPs of the individual state being modeled at time t, along with its neighbouring states[3] are denoted by the vector $\mathbf{y}_t$, $\mathbf{c}$ is a vector of constants, $\mathbf{\Phi}_i$ is a matrix of autoregressive coefficients for $i = 1, 2, \ldots, p$ and ϵ_t is the i.i.d. Gaussian error term with mean 0. The lag order of the VAR, p, is determined by using selection criteria such as Akaike's Information Criteria (AIC), Hannan-Quinn Criterion (HQ), Schwarz Criterion (SC) and Final Prediction Error (FRE) (see Akaike (1973), Hannan and Quinn (1979), Schwarz (1978), Hamilton (1994)).[4] For the modelling of the GDP in each of the individual United States these four tests displayed inconclusive results, with the SC consistently indicating a lag order of one, and the AIC, HQ and FRE varying from one to three depending on the state being evaluated. As this study is focused on forecasting mortality, to keep the model simple, a VAR(1) was determined to be the most suitable model for the GDP in each of the individual contiguous United States.

In our setting, specific situations do arise. For example, in order to model the future GDP in California, we also need to consider the economic growth occurring in the neighbouring states: Arizona, Nevada and Oregon, as well as the economic growth in Mexico. The GDP of all these neighbours of California and California itself are jointly forecasted using a VAR(1) model. The same reasoning applies to all states neighbouring either Mexico or Canada.

For each state, we produced forecasts of the age-specific mortality rates 30 years into the future using both the Li-Lee and SLGG models. A comparison of the forecasts for the selected states of Arizona, California, Florida and Texas are displayed in Figure 6. We observe that for each of the selected states, there are similar projections of mortality rates for the younger age-groups below the age of 5, as well as similar patterns in the forecasts of mortality for the age-groups above the age of 75. These plots demonstrate differences between the forecasts of the age-specific mortality rates for the two models among the middle age groups, with lower mortality projections observed for the SLGG model in the states of California, Florida and Texas. To further analyze the differences in the forecasted mortality rates produced by each model, Figure 7 displays maps of the percent differences between the mortality forecasts of the individual states. The corresponding maps illustrate the percent difference obtained

[3]To reduce the model to a reasonable number of parameters, we only incorporate neighboring states.
[4]Tables displaying results of these criteria can be found in Appendix A.

by subtracting the mortality estimates from the Li-Lee model from those produced by the SLGG model. Coloured in dark red are states in which the difference between the forecasts is the greatest, representing higher mortality estimates for the SLGG model. Similarly, the dark blue coloured states are the ones in which the Li-Lee model estimates are greater. From these maps it can be observed that the SLGG model provides higher estimates of age-specific mortality rates in regions of the United States where there is limited economic growth, such as the area in the South with Louisiana, Mississippi, Arkansas and Alabama. Vice versa, we find lower mortality estimates from the SLGG model in states with more economically growth, such as Washington and New York. We observe extreme estimates of mortality for the states of West Virginia and Maryland from the SLGG model, which can be attributed

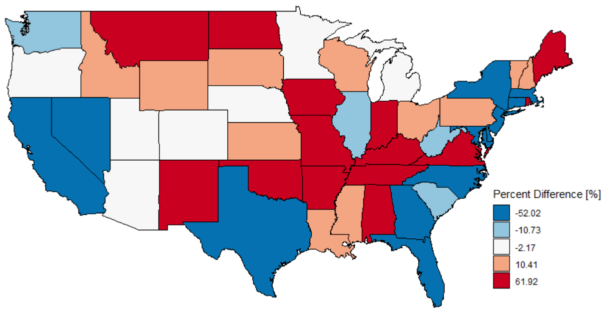


(a) `Arizona` (b) `California`

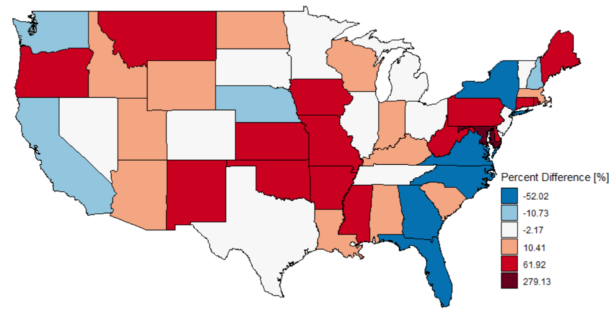
(c) `Florida` (d) `Texas`

Figure 6: Comparison of the age-specific mortality rates as forecasted by the LL and SLGG models for the year 2046 and the states Arizona, California, Florida and Texas. On the y-axis, a logarithmic scale was used.

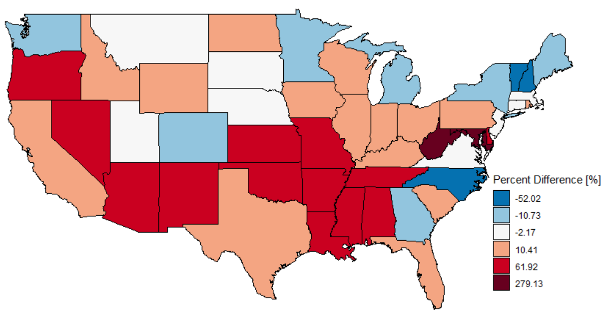
to the exclusion of their influential neighbour, the District of Columbia.



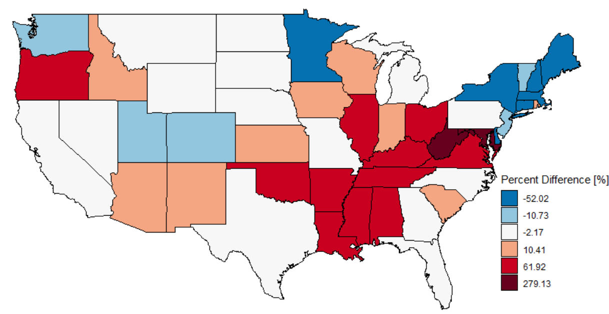
(a) Forecast Difference for Ages 35-44



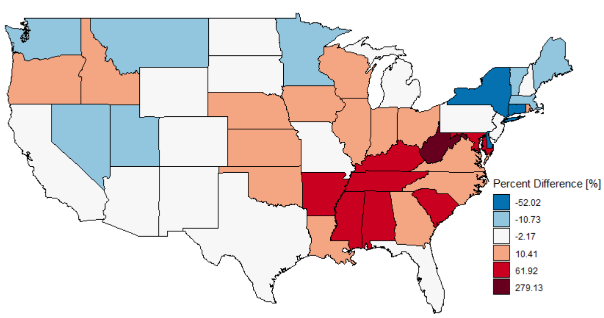
(b) Forecast Difference for Ages 45-54



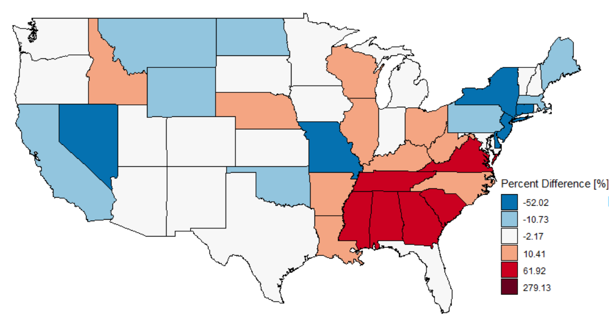
(c) Forecast Difference for Ages 55-64



(d) Forecast Difference for Ages 65-74



(e) Forecast Difference for Ages 75-84



(f) Forecast Difference for Ages 85+

Figure 7: Mapped comparison of the percent difference in the expected mortality rates forecasted by the LL and SLGG models for the year 2046 across the remaining age-groups. A positive difference indicates higher forecasted mortality estimates for the SLGG model.

6 Annuity pricing

Mortality forecasts and their differences have an important impact on mortality linked insurance products, most notable of which are annuities. In this section, we show the impact of the LL and SLGG models on annuity pricing. We use the following well-known definition of a T-year fixed term annuity of an x-year old individual

$$\ddot{a}_{i,x} = \sum_{t=0}^{T} {}_t p_{i,x}/(1+r)^t, \tag{2}$$

where ${}_t p_{i,x}$ is the probability that a (male) life age x from state i survives for t years, i.e.,

$${}_t p_{i,x} = \prod_{k=0}^{t-1}(1 - q_{i,x+k,2016+k})$$

where, starting with year 2016, $q_{i,x+k,2016+k}$ is the time-0 death probability determined from forecasts of $m_{i,x+k,2016+k}$. We calculate the present value of a 45-year fixed term annuity of 1 unit per year commencing at age 65 for a male individual in each of the United States. To obtain $q_{i,x+k,2016+k}$ for each individual age x, we assume a uniform distribution of death (UDD) amongst our 10-year age-group central death rate forecasts (see Bowers et al. (1997)).

Forecasts of mortality rates from the LL and SLGG models were used to calculate ${}_t p_{i,x}$. Table 4 displays values of $\ddot{a}_{i,65}$ as priced with fixed rates r of 1%, 3% and 5%. Importantly, we observe that for both models, the values of $\ddot{a}_{i,x}$ appear to be greatest in wealthier states when compared with the less wealthy states. From the table, we see that under a constant interest rate of 1%, the actuarial value of a 45-year fixed term annuity paying out 1 unit every year until the age of 110 ranges from 13.33 in West Virginia to 17.57 in New York under the SLGG model, and 14.41 in Mississippi to 16.61 in Florida under the LL model. Table 3 displays the summary statistics of mean and standard deviation for the calculated values of $\ddot{a}_{i,x}$, and allows for a comparison of the values across the three fixed rates of r. We observe that while the mean present value of the annuities calculated using the mortality rates of both models are similar, the SLGG model preserves more of the variability amongst these prices than the LL model. As interest rate r increases to higher levels, the SLGG

	LL			SLGG		
	1%	3%	5%	1%	3%	5%
Mean	15.47	12.58	10.49	15.53	12.62	10.51
Standard Deviation	0.55	0.39	0.28	1.04	0.71	0.51

Table 3: Summary statistics of the present value of the annuities from Table 4.

model continues to preserve more regional differences in these annuity prices than the LL model.

To further compare the prices generated from the two models, Figure 8 displays a map of the percent difference in the values of $\ddot{a}_{i,x}$ for each of the individual states at the 1% discount rate. There, we observe the regions of the United States with which the LL and SLGG models reveals their greatest differences in annuity pricing. Observing this map in conjunction with the results of Table 4 we see higher annuity prices in the more economically growing regions of the northeastern United States, and observe that in regions where there is limited economic growth in the South the LL model produces higher values of $\ddot{a}_{i,x}$. We conclude that the pricing of annuities has an underlying spatial dimension to it, and that the SLGG model is better equipped to capture the existing economic inequalities amongst the neighbourhoods of the United States to produce more accurate estimates.

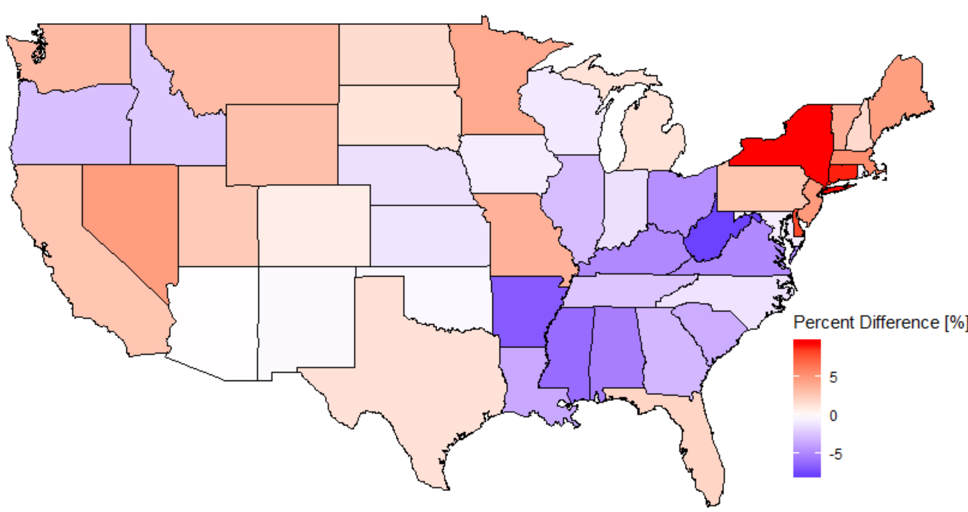


Figure 8: Map of the percent difference in annuity pricing generated by the LL and SLGG models with a discount rate of 1%. A positive difference indicates thats the present value of the annuity was priced higher for the SLGG model.

	LL			SLGG		
State	1%	3%	5%	1%	3%	5%
Alabama	14.58	11.94	10.01	13.76	11.40	9.63
Arizona	16.51	13.28	10.98	16.52	13.26	10.94
Arkansas	14.98	12.23	10.22	13.93	11.50	9.70
California	16.53	13.30	11.00	16.97	13.57	11.16
Colorado	16.05	13.02	10.82	16.19	13.10	10.88
Connecticut	15.97	12.95	10.76	17.40	13.90	11.43
Delaware	15.40	12.54	10.46	16.68	13.37	11.03
Florida	16.61	13.34	11.02	16.94	13.54	11.14
Georgia	14.86	12.14	10.15	14.40	11.86	9.98
Idaho	15.94	12.94	10.76	15.60	12.71	10.60
Illinois	15.27	12.44	10.39	14.86	12.17	10.20
Indiana	14.90	12.19	10.20	14.71	12.05	10.11
Iowa	15.62	12.70	10.59	15.51	12.63	10.53
Kansas	15.58	12.67	10.56	15.42	12.55	10.47
Kentucky	14.43	11.84	9.94	13.68	11.33	9.58
Louisiana	14.71	12.02	10.07	14.16	11.66	9.82
Maine	15.27	12.47	10.42	16.00	12.97	10.78
Maryland	15.37	12.51	10.43	15.26	12.44	10.39
Massachusetts	15.66	12.73	10.60	16.51	13.32	11.02
Michigan	15.21	12.41	10.37	15.42	12.55	10.46
Minnesota	15.99	12.97	10.79	16.65	13.42	11.10
Mississippi	14.41	11.81	9.91	13.47	11.18	9.46
Missouri	15.06	12.30	10.29	15.65	12.67	10.53
Montana	15.72	12.78	10.64	16.24	13.11	10.86
Nebraska	15.61	12.70	10.58	15.41	12.57	10.49
Nevada	15.38	12.52	10.44	16.13	12.96	10.71
New Hampshire	15.65	12.74	10.62	15.94	12.94	10.77
New Jersey	15.58	12.69	10.56	16.34	13.16	10.89
New Mexico	16.23	13.11	10.86	16.20	13.07	10.83
New York	16.09	13.00	10.79	17.57	13.92	11.38
North Carolina	15.10	12.31	10.28	14.92	12.24	10.27
North Dakota	15.92	12.91	10.73	16.19	13.06	10.82
Ohio	15.28	12.45	10.39	14.57	11.97	10.05
Oklahoma	14.83	12.13	10.15	14.79	12.06	10.08
Oregon	15.79	12.82	10.67	15.40	12.55	10.47
Pennsylvania	15.21	12.42	10.38	15.67	12.70	10.57
Rhode Island	15.51	12.62	10.52	15.45	12.59	10.50
South Carolina	15.00	12.23	10.22	14.48	11.93	10.04
South Dakota	15.80	12.83	10.67	16.01	12.97	10.77
Tennessee	14.72	12.05	10.09	14.37	11.81	9.93
Texas	15.46	12.56	10.47	15.69	12.70	10.56
Utah	16.29	13.18	10.94	16.69	13.47	11.15
Vermont	15.47	12.61	10.52	16.09	13.03	10.82
Virginia	15.19	12.39	10.35	14.44	11.91	10.03
Washington	15.93	12.92	10.74	16.46	13.27	10.99
West Virginia	14.53	11.92	10.01	13.33	11.09	9.41
Wisconsin	15.59	12.69	10.58	15.46	12.62	10.54
Wyoming	15.64	12.71	10.59	16.12	13.03	10.80

Table 4: Present value of a 45-year fixed term annuity of 1 unit per year commencing at age 65 with discount rates r of 1%, 3% and 5%, as defined in (2), for each state and for the LL and SLGG models.

7 Conclusion

This study proposes the incorporation of spatial components into the traditional stochastic mortality modelling framework outlined by Li and Lee (2005), and substantially explores the economic growth of the individual states in these models. Our proposed SLGG model was used to produce forecasts of mortality rates and annuity pricing for each of the United States, and illustrated the effects that economic growth has on mortality on a spatial dimension. Whereas the Li and Lee model incorporates latent factors that are not easily identifiable and difficult to project into the future, our model is interpretable as it captures the relationships between GDP and mortality rates in space. This research provides a starting point for more localized studies of mortality, and provides a blueprint for the inclusion of spatial components and economic growth into traditional mortality models. Most importantly for actuaries, with its findings this work establishes empirical justification for development of natural hedging techniques which account for a spatial dimension of mortality.

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A Tables of Selected Criteria

	Criterion			
State	AIC	HQ	SC	FPE
Alabama	1	1	1	1
Arizona	3	3	1	3
Arkansas	3	3	1	3
California	2	1	1	2
Colorado	3	3	1	3
Connecticut	3	1	1	2
Delaware	2	1	1	2
Florida	2	2	1	2
Georgia	3	1	1	3
Idaho	3	3	1	3
Illinois	3	1	1	1
Indiana	1	1	1	1
Iowa	3	1	1	1
Kansas	3	1	1	3
Kentucky	3	3	1	1
Louisiana	3	1	1	3
Maine	1	1	1	1
Maryland	2	1	1	2
Massachusetts	3	1	1	2
Michigan	1	1	1	1
Minnesota	3	1	1	3
Mississippi	2	1	1	1
Missouri	3	3	3	3
Montana	3	3	1	3
Nebraska	3	3	1	3
Nevada	3	1	1	1
New Hampshire	1	1	1	1
New Jersey	1	1	1	1
New Mexico	3	3	1	3
New York	3	3	1	2
North Carolina	1	1	1	1
North Dakota	3	1	1	1
Ohio	3	1	1	1
Oklahoma	3	3	1	3
Oregon	2	2	1	2
Pennsylvania	3	2	1	2
Rhode Island	2	2	1	2
South Carolina	1	1	1	1
South Dakota	3	3	1	3
Tennessee	2	1	1	3
Texas	1	1	1	1
Utah	3	3	1	3
Vermont	1	1	1	1
Virginia	3	3	1	3
Washington	2	1	1	2
West Virginia	3	2	1	2
Wisconsin	1	1	1	1
Wyoming	3	3	1	3

Table 5: The optimal lag order of the VAR selected by the Akaike's Information Criteria (AIC), Hannan-Quinn Criterion (HQ), Schwarz Criterion (SC) and Final Prediction Error (FRE) for the individual states. For the modelling of GDP, a VAR(1) model was determined to be most suitable. (see Akaike (1973), Hannan and Quinn (1979), Schwarz (1978), Hamilton (1994))