A Sustainable, Variable Lifetime Retirement Income Solution for the Chilean Pension System[1]

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Abstract. There is a need in pension systems to significantly improve the level and stability of pension payments as pensioners age. Solutions to address increased longevity and longevity risk should be not limited to increasing the take-up rate of annuities – explicit guarantees are costly in a low-interest rate environment, and lock-in of savings may not be in line with members' preferences. Our proposal is to develop a Sustainable, Variable Lifetime Retirement Income Solution in a more flexible and cost-efficient way. Recent developments favoring flexible products that are more suited to satisfy the needs and preferences of members are key for improving the pay-out phase. In this respect, we believe our tontine design proposal is a superior alternative for the Chilean Pension System. Tontine-like arrangements offer a unique value proposition to address the global retirement challenge. Our retirement income proposal provides clear transparency and investment flexibility with higher expected income streams. It does not involve higher costs since there are no explicit guarantees and provides a means to offer longevity insurance even if insurers are unwilling to supply it. Several proposals are analyzed, including deferred pension arrangements and tontine-like solutions combined with existing pay-out products. Our proposal does not distort the annuity market; on the contrary, it complements it, and it is in line with the transition of many countries to include tontine-like longevity-risk sharing and collective elements in their defined-contribution designs.

Keywords: Tontines, Annuities, Programmed Withdrawals, Pensions, Retirement, Longevity Risk

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1. Introduction

This paper examines the merits of introducing a new kind of longevity risk-sharing option into the post-retirement phase of the Chilean pension system. The idea is based on the tontine principle, in which longevity risks are shared among the participants rather than transferred to an insurer. There is no guaranteed level of income, which distinguishes the solution from Chilean annuities. The solution is essentially a hybrid that blends aspects of both programmed withdrawals and annuitization, which can be offered alongside them. It addresses retirement income inadequacy through higher income levels and longevity protection. It offers flexible design features and is practical in that it can be invested in the same set of portfolio options that exist today. Moreover, the solution is both equitable and sustainable.

The main challenge faced by young generations is obtaining adequate pensions, and increased longevity and longevity risk are key threats affecting future retirement income. In this respect, Chile – like many other countries in the world – is entering a period of accelerated aging. According to United Nations (2019), there were 27 old-age dependents for every 100 working-age Chileans in 2020, but this will increase to 59 by 2050 and 86 by 2100. This represents a two-fold increase in the old-age dependency ratio in the three decades ahead and a three-fold increase by the end of the century. Consequently, as Table 1 suggests, the proportion of the population aged 60+ will jump from 17% in 2020 to 40% by 2100. Furthermore, the ratio of individuals aged 80 and over to the working-age population will be three times larger in 2050 than today.

Table 1 Change in the age structure of the Chilean population (1950-2100)

Demographic indicators	Year				
	1950	2000	2020	2050	2100
Median age of population	20.6	28.7	35.3	46.1	51.4
Ageing index (per 100)	14.2	39.4	90.3	222.1	307.7
% of population:					
0-14 years old	38.0	27.3	19.2	14.2	13.1
15-59 years old	56.5	61.9	63.4	54.1	46.7
60 and over	5.4	10.8	17.4	31.6	40.2
60-74 years old	4.2	8.1	12.4	18.2	17.5
75 and over	1.2	2.7	5.0	13.5	22.7
80 and over	0.6	1.3	2.8	8.7	16.9
Dependency ratio (per 100)					
Total	76.9	61.6	57.8	84.8	113.9
less than 15 years old	67.3	44.2	30.4	26.3	27.9
of 60 and over	9.6	17.4	27.4	58.5	86.0

Ageing index = (population 60 and over/population 0-14 years old)*100. Dependency ratio = [(population 0-14 years old + population 60 and over)/population 15-59 years old]*100

Source: United Nations, 2019.

Despite recent collective efforts to improve old age individuals' quality of life, the Chilean society is not yet satisfied, nor does it feel prepared for the upcoming aging process. The most recent survey of Social Exclusion and Inclusion of Old Individuals (Thumala et al., 2015) reveals that 75% of the Chilean population think that the country is not well prepared to deal with the challenges of increasing life expectancy in the next three decades. The authors suggest that this may be due to a lack of satisfaction among Chilean families regarding pensions, health, and/or old-age care in a context of persistent social inequalities.

Due to the Covid-19 pandemic, the Chilean legislature permitted three rounds of early pension withdrawals during the 2020-2021 period. Most members had unconditional access to part or all their pension savings, causing significant outflows from the pension system and an estimated pension decrease of 30% on average for those withdrawing their savings. Total early withdrawals added up to US$ 50 bn, equivalent to 20% of GDP (Fuentes et al., 2021).

Pensions in Chile are insufficient, and the consequences of the Covid-19 pandemic reduced pension adequacy even further. Chile is below the OECD average net pension replacement rate by 24 percentage points for men and 26 percentage points for women.[5] Towards the future, aging worsens this complex picture. Life expectancy is expected to rise almost one year per decade in the coming decades (United Nations, 2019). The Solidarity Pillar improves the situation for low-income earners, whose net replacement rates stand at 52% for men and 50% for women, but these figures are still below the OECD average for this income group. With no commensurate change in the retirement age and other relevant parameters such as the pension system's contribution rate, the ability of the fully funded component to provide an adequate retirement will be severely curtailed by the natural aging process. Using the model developed by the Interamerican Development Bank (Altamirano et al. 2018), we find that replacement rates would be reduced six percentage points by 2050 and a further five percentage points by 2100 relative to its current level, just accounting for the expected increase in longevity.[6]

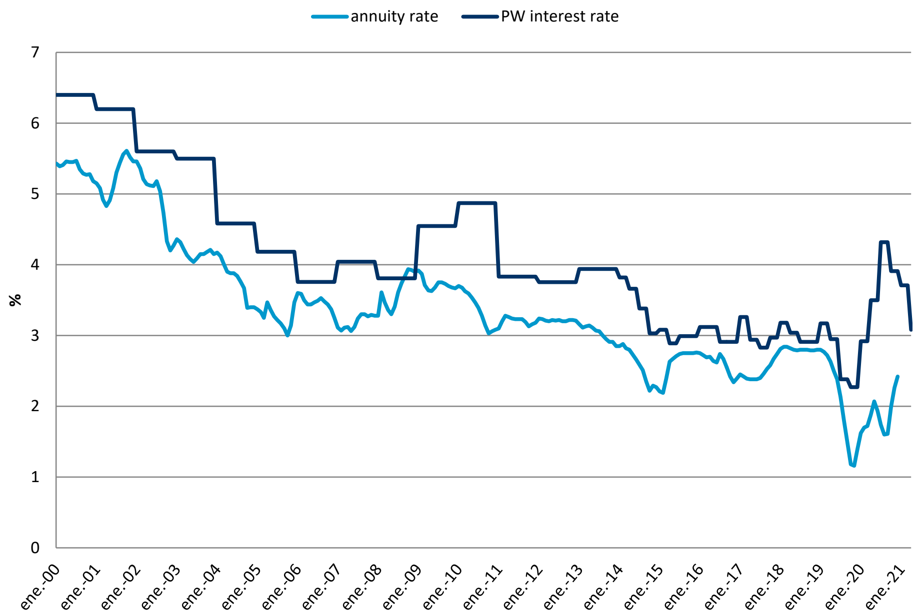
However, the impact of the increase in longevity on pensions is just one side of the coin. At the point of retirement, the value of a pension depends not only on life expectancy but also on the implicit interest rate used by insurance companies to discount future cash flows. This discount rate has been decreasing for many years, increasing the price of annuities. The same is true for programmed withdrawals, where the technical interest rate, representing the long-term return expectation for pension funds, has shown a similar decreasing pattern (see Figure 1). In the Chilean case, a person retiring at the end of 2020 would have gotten 28% less annuity benefit than the same person retiring at the turn of the century, just

[5] The OECD average pension replacement rate is 62.4% for men and 61.3% for women. See OECD (2021).

[6] The figure reports the base case hypothetical worker defined in the IDB report (Altamirano et. Al, 2018), which uses the conservative demographic scenario by the United Nations.

accounting for the lower discount rate. If changes in mortality were also factored in, the annuity benefit would have been 37% lower.[7]

Figure 1 Chile: Annuity rate and Programmed Withdrawals interest rate



Source: Authors' calculations based on Pension Regulator information

After the virtual demise of the annuity markets in Great Britain following the "Pension Freedoms Act" (2014), and in Peru after the 95.5% rule (2016), Chile may be, together with the US and Canada, one of the few countries in the world with a deep and efficiently functioning annuity market. Indeed, around 45% of the stock of Chilean pensioners currently participates in the annuity market. However, the annuity market share has decreased in the past years, giving way to a pension product that combines a temporal withdrawal with a deferred annuity.[8] The use of Programmed Withdrawals has also increased over the last two years. The secular decline in interest rates and rising life expectancy pose a monumental challenge to pension systems, policymakers, and annuity providers. Such challenges call for innovative solutions. It is in this context that modern tontines could play a role.

A modern tontine is an innovative product that allows individuals to share longevity risks in an actuarially fair and transparent manner. A key difference with an annuity contract is that individuals fully benefit from the mutualization of longevity risks with tontines. With annuities, insurers retain all credits above and beyond the implicit guarantee locked-in at the time of purchase as increased revenue (Iwry et al., 2020). In a tontine, group members insure themselves against longevity risk, and longevity credits are distributed in their

[7] The magnitude of these results is consistent with similar computations in the literature for other countries. See Nijman and Soest (2019), Bilsen et al. (2018), Merton and Muralidhar (2020), and Verani and Yu (2021).

[8] The average deferral is 2.7 years.

entirety to the group. Tontines can be sold at a lower cost relative to annuities' price because, unlike an annuity, a tontine offers no guarantees and therefore does not require solvency reserves.

Like annuities, tontines do not tackle interest rate or conversion risks at retirement. However, this issue is mitigated in that tontines do not lock the buyer into a payment amount dependent on interest rates at the time of purchase. Instead, a tontine can be invested as participants desire, with payouts that vary naturally according to the generated returns over time. It is worth noting that proposals to incorporate interest rate hedges into investment strategies are compatible with structuring payments through a tontine.[9] It is also consistent with other innovative proposals, such as the introduction of SeLFIES for retirement put forward by Merton and Muralidhar (2020).

We analyze tontine portfolios as an option for the post-retirement drawdown phase in the Chilean pension system. The analysis uses Monte Carlo simulation to model participant outcomes under various tontine designs, compares those outcomes to the phased withdrawals and life annuity options currently available to retirees, and examines the benefits and drawbacks of each of these options. The main results indicate that our proposal provides real value to pension systems, introducing greater transparency, increased flexibility, and higher expected income streams. This gain is significant when our proposal is compared to programmed withdrawals, the alternative taken by more than 85% of new pensioners in Chile. These results and the relevant policy implications derived from them can easily be applied to other countries' pension systems, and more broadly, the methodology is used to develop more efficient, flexible, and affordable solutions for the pay-out phase.

The rest of the paper is structured as follows. Section 2 summarizes the international empirical evidence on tontine design and applications. Section 3 describes the Chilean pension system's main elements to give context to the analysis and results. Section 4 describes the methodology, tontine assumptions, data used, and parameters for modeling individual participants and heterogeneity. Section 5 shows the estimates and main outcomes, including different tontine design results and output comparisons for different pension retirement options. Section 6 gives the main conclusions and delivers policy considerations.

2. International empirical evidence and tontine technical justification

2.1 The rationale for annuitization, whether via annuities or tontines

Yaari (1965) demonstrated through a random-length life-cycle model that, under certain assumptions, complete annuitization at retirement is utility-maximizing for risk-averse individuals with no bequest motives. He further showed that those with bequest motives

[9] See Mantilla-García et. al (2021) and Chapter IV in Impavido et. al. (2010).

should still typically annuitize a portion of their assets (see also Davidoff et al., 2005). However, it is well documented in the literature that many people do not annuitize, which economists have labeled "the annuity puzzle."[10] Chileans are an exception to the puzzle in that a significant number do annuitize. One reason for this may be that retirees in Chile are given limited options at retirement, and some retirees may find the non-annuitized option (programmed withdrawals) unappealing because the expected payout declines sharply over time.

Many valid reasons have been given for the annuity puzzle, but relevant here is that Yaari's model assumed that individuals could annuitize at an actuarially fair price.[11] However, insured annuities are not actuarially fair because they offer guarantees, which require the insurer to hold reserves and otherwise incur risk hedging costs.[12] So it can be argued that the annuity (or "actuarial notes") that Yaari assumes in his model does not represent a real-world guaranteed annuity but rather something closer to an actuarially fair tontine annuity invested to minimize payout variability. Also relevant to the annuity puzzle here is that insured annuities are opaquely priced – buyers know how much income they will receive but do not see the level of embedded fees that the insurance company earns. Thus, asymmetric information may cause some retirees to suspect whether the deal is better for the insurer than for them.

Tontines resolve these issues in that they can be transparent and actuarially fair – or at least nearly so since there would be some administration costs – because tontines incur no guarantee costs. Of course, some people may prefer an insured annuity if they desire certainty and perceive that the cost of a guarantee is worth it. However, others may choose a tontine annuity that provides greater transparency, more investment flexibility, and a higher expected income stream.

2.2 Tontine history and practice

Tontine finance was invented in Europe in the middle of the 17th century in the form of a proposed French government fundraising scheme.[13] That initial proposal was never implemented, but the idea took root, and tontines soon began appearing in France, England, Denmark, Germany, and other countries. By the 19th century, tontines had spread to a number of countries in South America.[14]

Insurance companies in the United States started offering a product called "tontine insurance" in 1868, which quickly became enormously popular. By the year 1905, it is estimated that over nine million individual tontine insurance policies were in force in the

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[10] A brief review of annuity puzzle is given in Lloyd (2014). See also Benartzi et al. (2011) and Ramsay and Oguledo (2018) for recent surveys.

[11] The term *actuarially fair* is defined in terms of the expected present value given probabilities as defined by an appropriate cohort mortality table and discounted using current bond yields.

[12] The level of actuarial unfairness can be expressed as a "money's worth ratio" as in Mitchell et al. (1999), Cannon and Tonks (2009), Wettstein et al. (2021), and Poterba and Solomon (2021).

[13] Lorenzo Tonti is generally credited with inventing the tontine in 1653, although scholars have since uncovered evidence that Nicolas Bourey came up with the concept twelve years earlier in 1641. Refer to Milevsky (2015) and Hellwege (2018).

[14] See chapter 14 titled "Tontines in Latin America" by Nasser in Hellwege (2018).

US when the nation totaled approximately 18 million households.[15] The value of these policies represented about 7.5% of the nation's wealth and 64% of all life insurance in force at the time.[16] It is fair to say that tontine insurance was instrumental to the US insurance industry's early creation.

The year 1905 is significant in tontine history because, sadly, US insurance companies' executives were caught misappropriating tontine assets. Here, a distinction between the general concept of "tontine finance" and the 19th-century product called "tontine insurance" becomes relevant. Issued as *insurance* products, tontine investments were held as assets of the issuing insurance company. However, these assets were never offset by a liability on the company books nor accounted for separately, making them available for use or misuse as the companies desired.

When the fraud was finally exposed, regulators stepped in to sort out the mess, banning certain practices as a result. Notably, the regulators did not ban tontines outright, but only certain tontine insurance features they viewed as problematic. Still, the effect halted tontine insurance sales and damaged their reputation. Insurance companies, eager to put the scandal behind them, quickly pivoted to another type of product—annuities.

Of course, there is no reason that a modern tontine cannot account for investor assets properly and accurately. Moreover, pure tontines need not be issued by insurance companies at all (although they could be) because pure tontines do not embed a guarantee or involve a contract of risk transfer. Rather, a tontine is simply a risk-*sharing* device so that investor money can be held in trust for the investors' sole benefit in the same way as traditional retirement investments.

The 1905 insurance scandal devastated an industry, but it did not end tontine finance completely. Quite the contrary. It persisted in a number of places, often hiding in plain sight under a different name. For example, in Sweden, the public pension system is explicitly tontine-like in that payments adjust automatically with shifting economics and demographics to keep the program financially balanced. Denmark also features pension schemes with tontine-like characteristics that adjust payouts in response to shifting circumstances.[17]

Tontine-like longevity risk-sharing schemes appear to be expanding in the second and third pillars as well.[18] This trend is furthest along in Europe and North America. European Union (EU) member states have allowed commercial tontine offerings under insurance regulation since 1990.[19] The EU is also expected to begin allowing pure tontine products under the new Pan European Personal Pension (PEPP) regulation starting in 2021, which would allow

[15] U.S. Bureau of the Census (1975)

[16] Random and Sutch (1987)

[17] To learn more about tontines and collective arrangements, see Turner et al. (2021). To dig into tontine design in the Swedish first pillar, see Diamond (2020).

[18] As Price et al. (2021) put it, these examples show that tontines are "innovative but not untested."

[19] Hellwege (2018): Council Directive 90/619/EEC of 8 November 1990 on the coordination of laws, regulations and administrative provisions relating to direct life assurance, 29 Nov. 1990, 50–61.

issuance more broadly by other non-insurance firms. In addition, the United Kingdom is moving toward tontine-like collective defined contribution schemes (CDC). Such plans are already in place in the Netherlands, Iceland, Israel, and Japan.[20]

In North America, a large second pillar tontine has existed in the United States since 1952, issued by the Teachers Insurance and Annuity Association of America (TIAA). TIAA refers to its product as a variable pooled annuity, and it operates on the tontine principle. Also, in the United States, the public pension plan offered by the State of Wisconsin operates much like a tontine and, not surprisingly, maintains a funded status remarkably close to 100% – a rarity among US state and local pensions, many of which are grossly underfunded. Tontine schemes also exist in a few places in Canada, such as the University of British Columbia's defined contribution plan where it goes by the name "variable payment life annuity (VLPA)." The plan has been operative since 1967. The Canadian legislature is currently evaluating the idea of allowing these plans nationally.[21]

Elsewhere, South Africa has permitted tontines to be sold in the informal economy since 2017. The Singaporean Central Provident Fund has offered a tontine payout core product called CPF LIFE since 2009.[22] Recently in Australia, QSuper, a pension fund based in Brisbane and serving Queensland government employees, introduced a new tontine-like pension product named QSuper Lifetime Pension.[23] This new option will complement the basic pension and the income account as an additional source of retirement income. QSuper Lifetime Pension promises higher income rates, a balanced investment strategy, and pension payments for life.

2.3 Pension reform proposals in Chile

In Chile, three proposals have emerged over the years, addressing the decreasing path of pension payments offered by Programmed Withdrawals and the longevity risk of outliving total savings under this pension product in the context of increased longevity (see section I).

Critics point to the lack of income stability for those pensioners with Programmed Withdrawals due to the decreasing path of payments and the yearly fluctuations in the pension value due to the annual update of payments following an actuarial formula. The population does not consider it a social security product since it does not provide secure and stable payments over time and because the income received falls significantly over time, even if it starts at a low initial value. The argument is that if adequacy is low at retirement, it is even lower in the medium to long term under a Programmed Withdrawal.

Larraín (2014) proposes a pay-as-you-go (PAYG) pillar for advanced ages that splits the retirement period into two stages, the third and fourth ages, financed by different

[20] To learn more about tontines in Israel, see Gronau and Spivak (2021), Milevsky (2015), and Haran (2017). About the Netherlands, see Bovenberg et al. (2017), Bonenkamp and Westerhout (2014), and Cui et al. (2011). For Japan, see Willis Towers Watson (2020).

[21] See Munnell and Sass (2011), Sanders (2017), and MacDonald et al. (2021).

[22] To learn more about the tontine design in the Singaporean CPF LIFE see Price et al. (2021).

[23] In Australia, tontine-like group self-annuitization schemes, also called pooled annuity schemes, were first proposed by Piggott et al. (2005).

instruments: savings and insurance. The savings sub-stage maintains the same current structure, but the disbursement of funds is carried out only up to the average life expectancy (measured at retirement). Subsequently, longevity insurance would finance the affiliates' pensions with an additional contribution or premium charge on active workers' labor income, creating a pay-as-you-go element administered by the State. In the author's calculations, the proposal allows for a short-term increase in pensions of current and future pensioners by an average of 24%, with an additional contribution borne by workers estimated between 2% to 4%.

Berstein and Morales (2021) propose a longevity insurance mechanism in which each worker, throughout his/her working life, contributes, for example, an additional percentage point, which would not go to the individual capitalization account but rather to a collective fund to finance longevity insurance. At retirement, the individual has the right to longevity insurance coverage, which becomes effective from age 85. The payment of benefits can be operationalized through one of two instruments: (i) a life annuity provided by the State or (ii) through a cohort-based mutualization of risks. Considering a payment equivalent to 70% of the initial pension payable at age 85 and onwards, the insurance cost would be around 1.2% of taxable income for men and 1.6% for women. According to the authors, the pensions for programmed withdrawals increase from 20% to 27% under their proposal. This product does not consider survival pensions or the possibility of leaving inheritances. The transition could involve a temporary pay-as-you-go element.

Valdés (2017) proposes a “Protected Programmed Withdrawal.” Unlike the previous two proposals, the Protected Programmed Withdrawal does not require a temporary (as in Berstein and Morales, 2021) or a permanent (as in Larraín, 2014) PAYG element for its implementation. In the author's calculations, when an individual survives to age 80, his pension benefit has halved relative to its initial amount, implying excessive bequests paid to survivors. The Protected Programmed Withdrawal splits the funds in the individual account to allocate a part of the pension savings to longevity insurance, which would start rendering at the fourth age (82-85 years), and where the amount of the fourth age pension would not be less than 80% of the initial pension payment that could have been obtained by acquiring a fixed life income from insurance companies.[24]

One of the potential issues with longevity insurance is that insurers must be willing to supply it at a reasonable price. Following up on the study by Mitchell et al. (1999), Wettstein et al. (2021) show that the money's worth ratio of retail fixed immediate annuities in the United States has remained relatively stable over time despite dramatic changes in mortality and interest rates. They conclude that 65-year-old retail annuity buyers selected randomly from the population receive only about 80% of the premium they pay. The extra 20% represents the cost of the insurance that is provided. Moreover, the authors found the money’s worth of longevity insurance (annuities with payouts deferred to age 85 and no death benefit if death occurs before then) in the United States to be far lower – just 50%. They justify this by noting that deferred annuities have greater “insurance value” than do immediate

[24] See Blanchett and Finke (2021) for a similar argument for the case of the United States.

annuities. Poterba and Solomon (2021) conduct a similar analysis but using a mortality table that accounts for adverse selection to estimate the money's worth received by actual annuity purchasers rather than people selected at random from the entire population. Their study computes a money's worth ratio of about 92 cents on the dollar for immediate annuities and about 72% for deferred income annuities. Yet regardless of whether one computes the money's worth of longevity insurance to be 50% or 72%, a product that offers such a high insurance cost could represent a significant drain on the pension system because every dollar or peso that the insurer keeps is a dollar or peso that does not go to the participants. Participants could benefit greatly if this kind of longevity protection was available at lower cost and higher money's worth.

Deferred annuities are offered in Chile, but they are typically only deferred by two to three years and thus do not serve as proper longevity insurance. Deferred annuities are offered as a combined product, in which pensioners choose the portion of total savings to allocate between a temporal rent and a deferred annuity. Temporal rent refers to scheduled withdrawals over a specific, fixed time horizon. By regulation, the ratio of the temporal rent to the deferred annuity can be between ½ and 2. In general, the effective selection is a temporal rent twice as large as the deferred annuity. It is unclear whether the reason that longer deferral periods are not offered is due to a supply or demand constraint, but insurance companies might not be willing to supply extended deferral periods (to age 80 or above) given that this concentrates the longevity risk exposure, increasing the guarantee costs relative to the initial premium that may be charged and reducing either the product's profitability or competitiveness. This would be consistent with the empirical studies by Wettstein et al. (2021) and Poterba and Solomon (2021). In this respect, a deferred tontine could effectively offer what insurers may be unwilling to supply.

We believe our tontine design proposal would be a superior alternative to these three schemes since: (i) it provides clear transparency and investment flexibility with higher expected income streams; (ii) it is easier to implement; (iii) it does not involve higher costs since there are no explicit guarantees; (iv) it does not distort the annuity market but could be complementary and supplementary to it; (v) it provides a means to offer a form of longevity insurance even if insurers are unwilling to supply it; and (vi) it is in line with the transition of many countries to include tontine-like longevity-risk sharing and collective elements in their defined-contribution designs.

3. The Chilean Pension System and its decumulation phase

3.1 The main components of the Multi-Pillar pension system

The Chilean pension system has three components: a redistributive first tier, a mandatory defined contribution with individual accounts second tier, and a voluntary third tier. Table 2 summarizes the main elements for each pillar, considering their objective, funding, and benefits.

Table 2 Chilean Pension System

	1st Pillar: Solidarity Pillar	2nd Pillar: Mandatory DC	3rd Pillar: Voluntary
Objective	To prevent poverty	To smooth consumption between the accumulation and decumulation phase	To complement mandatory savings, improving the final pension
Funding	General taxes	Individual savings with tax exemptions	Individual savings with tax incentives and state matching
Benefits	A basic pension and a pension top-up to individuals with low or null participation in the pension system. Benefits are means-tested.	Contingent on individual total final savings	Contingent on individual total final savings

Source: Authors

All formally employed individuals must contribute 10% of their salary to an individual defined contribution pension up to a limit of 80.2 UF[25] (3,290 US dollars[26]). The legal retirement age is 65 years for men and 60 years for women. Individuals are not required to stop working to claim a pension benefit, and it is possible to defer pension claiming after the legal retirement age. The average effective retirement age is 66 for men and 62 for women. Early retirement is allowed at any age in the defined contribution scheme if the account's capital is sufficient to finance a pension above a certain threshold.

Upon retirement, individuals have the choice of transforming their accumulated assets into (i) a programmed withdrawal,[27] (ii) an inflation-indexed life annuity, (iii) some combination of (i) and (ii), or (iv) a deferred annuity combined with a temporary income. If a deferred annuity option is chosen, the annuity benefit must be at least 50% but no higher than 100% of the first payment under the temporary income product. If accumulated assets are insufficient to purchase an annuity higher than the basic pension level[28], individuals must draw down their assets as a programmed withdrawal.

An electronic market quotation called "SCOMP" is in place for those affiliates with enough savings to choose a pension product, where members can request, receive, and select pension offers from all market providers. The SCOMP was introduced in 2004 to bring higher competition, transparency, and information to the pension system. Table 3 shows the pension product selection for all old-age new pensioners between 2018 and 2021. It shows the pension product selection through the SCOMP system and the percentage of new pensioners with a programmed withdrawal by default, i.e., those unable to choose due to low savings. The percentage of pensioners receiving a programmed withdrawal by default is large and above 55% for old-age pensions, indicating that a large group is exposed to longevity risk and would face a significant decrease in their pension payments as they age.

[25] *Unidad de Fomento (UF)* is a unit of account indexed to inflation.

[26] All figures in dollars use the exchange rate as of end of 2020.

[27] An amount of 15 UF (615 US dollars) is taken from the account at retirement to cover funeral expenses.

[28] This threshold is equal to the Basic Solidarity Pension, with a value of US$240 as of December 31, 2020.

Table 3 Pension product selection for the payout phase 2018-2021

Pay-out phase products	Total	Women	Men
Immediate annuity	6%	4%	9%
Temporal rent + Deferred annuity	11%	7%	16%
PW by choice	24%	16%	33%
PW by default	58%	73%	42%
	100%	100%	100%

Source: Authors' calculations based on Pension Regulator information

3.2 Mechanisms in place to cover longevity risk

The main objectives of a pension system are to smooth consumption and prevent poverty at retirement. Pension products must be aligned with these objectives. In Chile's defined contribution system, savings accumulated are converted at the point of retirement into a stream of pension payments using an actuarial formula that factors in the life expectancy of the pensioner population and a long-term interest rate expectation to mitigate the risk that the savings are not depleted during the retirement period. The participant faces multiple risks at the time of withdrawal, mainly longevity risk, investment risk, and interest rate risk.

Table 4 summarizes the characteristics for different types of products for retirement (a lump sum, the programmed withdrawal, the simple life annuity, and a deferred annuity) in terms of desirable features such as the liquidity of the pension product, protection against the main risks faced at and during retirement, and the possibility of leaving an inheritance.

Table 4 Main characteristics of retirement products

	Liquidity/ Ownership	Protection against longevity risk	Protection against investment risk	Protection against interest rate risk	Bequest	Products allowed by the Chilean regulation
Lump-sum	Yes	No	No	No	Yes	Very limited under strict requirements
PW	Yes	No	No	No	Yes	Yes. Default product for low balances
Annuity	No	Yes	Yes	No	No	Yes, if self-financed pension > PBS
Deferred Annuity	No	Yes	Yes	No	No	Yes, as a combined product.

Source: Authors

While the lump sum and programmed withdrawal options provide liquidity and maintain the ownership of one's savings, they leave the affiliate exposed to market and longevity risk. On the other hand, while life annuity products lack liquidity, they protect against market and longevity risks. A design feature of the Chilean system is that the first payment under a programmed withdrawal will be higher than the replacement rate offered by a life annuity. As a result, participants face the trade-off of either increasing their initial replacement rate (but accepting a decreasing payment schedule thereafter) or covering their market and longevity risks.

The current design of the Chilean pension system includes three elements to protect members against longevity risk. First, qualifying individuals may choose an annuity. Second, for those eligible for the Solidarity Pillar, the solidarity pension contribution covers the pensioner for life, conditional on satisfying the means-tested requirements, with a top-up that brings the final pension to an annuity-equivalent payout.[29] Third, the Solidarity Pillar also benefits advanced aged pensioners who were not eligible for the social benefit at younger ages with a solidarity supplement that brings the pension to a level of at least the basic solidarity pension (PBS).

4. Model Description and Methodology

4.1 The tontine principle

A tontine is an investment scheme in which the individual longevity risks of the investors are pooled and shared. Tontines are like regular investments, with two crucial differences. One difference is that tontine investments are typically irrevocable. This is to enforce the risk-sharing nature of the tontine pool. The other difference is that when tontine members die, their account balances are not left to their heirs but are redistributed to the other tontine members who are still alive. We refer to these redistributions as survivor credits. Tontine survivor credits are similar in concept to the "mortality credits" typically associated with annuities. While the two terms are sometimes used interchangeably, we note that tontine survivor credits are precisely quantified and added to the surviving tontine members' account balance. In contrast, annuity mortality credits are only estimated in aggregate and are not accounted for transparently.[30]

[29] The Solidarity Pillar (redistributive first tier) is a means-tested benefit. This pillar's main objective is to provide a safety net for individuals who cannot contribute to the system at all or regularly. The basic solidarity pension (PBS) is a non-contributive pension for individuals without other types of benefits. The PBS is payable from the age of 65 to the poorest 60% of the population, with benefit receipt conditional on having at least 20 years of residency and on being resident in at least four of the five years before the claim. There is also a complement to the contributory pension named Solidarity Pension Payment (APS), which is targeted at individuals with low pensions. Pensioners may claim this benefit if their defined contribution pension is lower than the maximum welfare pension (PMAS). The qualifying conditions for this benefit are the same as the qualifying conditions to claim a PBS. The APS value depends negatively on the contributory pension and positively on the PBS and PMAS levels. Currently, the value of the PBS is US$240 and US$707 for the PMAS.

[30] To learn more about the value of longevity risk sharing see Milevsky and Huang (2018) and Millard et. al. (2021).

Under the tontine principle, longevity risk is pooled and shared by all the tontine members. This pooling serves to diversify the member's *idiosyncratic* longevity risk in the same manner as insured annuities – through the law of large numbers. Tontine members also share and bear *systematic* longevity risk, which is not diversifiable. This makes them distinctly different than insured annuities, in which the insurance company bears the systematic longevity risk.[31]

4.2 Fair tontines

A "fair" tontine is one in which the *expected* value of the survivor credits that each and every member receives while living is equal to the expected value that they will forfeit at death. Each member gets a "fair bet" in the probabilistic sense. Some will die relatively early and forfeit more than they receive in survivor credits, while others will live long lives and receive more in survivor credits than they forfeit when they finally die. Compared to regular investments, investing in a tontine changes the conditional distribution of outcomes in a useful way because those who live long lives require their retirement accounts to last longer, and that is precisely what a tontine delivers.

Ensuring fairness requires that the forfeited account balances of deceased members be transferred to the surviving members in an actuarially neutral way, considering each member's relative stake in the pool (as measured by their account balance) as well as their probability of dying. Methods for achieving this have been cited in Sabin (2010), Donnelly et al. (2014), and Sabin and Forman (2016). Furthermore, Fullmer and Sabin (2019a) show that fairness can be achieved when retirement accounts are accounted for individually, and members are allowed to make their own investment decisions, as is the case of the Chilean retirement system.[32]

Tontines do not guarantee a specific payout level. Instead, each tontine member's payout varies over time as a function of A) the performance of the member's selected investment fund and B) the mortality experience of the entire tontine pool. Like annuities, tontines are likely to be subject to adverse selection, meaning that those who perceive their health to be better than average may be more likely to purchase them. As with annuities, then, the mortality rates used to design tontines should account for this adverse selection.[33]

4.3 Simulation model

We model different investment and payout options for retirees in the Chilean retirement system. The investment options are modeled on the characteristics of funds C, D, and E. The payout options include programmed withdrawals, temporal withdrawals, immediate insured annuities, immediate tontine annuities, and deferred tontine annuities. The model uses Monte Carlo simulation to model individual member accounts and outcomes by

[31] See Gemmo et al. (2020).

[32] See also Chen and Rach (2022), Chen et al. (2021b), Winter and Planchet (2021), Weinert J.H. (2017a), and Milevsky and Salisbury (2016).

[33] Adverse selection could be mitigated by including mortality risk factors that go beyond age and gender to include socioeconomic status or other demographic factors.

randomly selecting members from a population, randomly selecting their death dates using mortality tables, and randomly selecting investment returns from a distribution.

4.3.1 Tontine model

We model an account-based, heterogeneous, open-ended tontine system, meaning one that:

- operates on a set of individual accounts in which investors can make their own investment decisions,
- accepts members of different ages and genders,
- continually accepts new members, and
- runs in perpetuity.

We assume that the tontine option becomes available on the first day of 2021. We simulate the first 55 years of operation, covering the years 2021 through 2075, inclusive. The tontine is designed using the CB-2014 mortality table for men and the RV-2014 table for women. The base year of these tables is 2014, and each is projected forward from year to year to account for mortality improvement. This means that an individual's probability of death depends not only on age and gender but also on the year of birth.

The expected survivor credit yield r (the survivor credit expressed as a percentage of a member's balance) for a given year is readily computed from these tables as $r = q/(1 - q)$, where q represents the member's probability of dying during the year as assigned by the (projected) mortality table.

Tontine Membership Pool Size. Membership in the tontine system can be expected to increase over the first few years of its operation as new cohorts of individuals retire each year. The size of the membership pool is important because it affects the level of longevity risk diversification that the members enjoy. This gives us two choices for how to conduct our analysis. One method is to model the expected growth in the tontine membership pool, but this would require us to assume the total tontine enrollment each year, and the effect of this assumption would be difficult to assess unless we isolated it. For this reason, we instead chose to isolate the effect by holding the membership size constant in our primary analysis and then running a separate analysis later to isolate the impact of tontine membership pool size.

Tontine Enrollment. To begin, we assume that the tontine starts each year with a membership size of 10,000 investors. The first 10,000 are enrolled at the beginning of the first year of operation. At the beginning of each subsequent year, one new member enrolls for every member that died in the previous year.

Each member who joins is randomly assigned parameters as follows:

- Gender: male or female with equal probability.
- Age: a randomly assigned integer in the range of 60 to 65, inclusive, for women and 65 to 70, inclusive, for men, with all ages equiprobable. By "age," we mean the age on January 1 of the year of entry.

- Account balance at retirement: a value ranging from 1,000 to 10,000 UF and selected according to a log-uniform distribution. That is, the initial balance was selected as $10^{\mathrm{U}+3}$, where U is a uniform random number in the range of 0 to 1. The log-uniform distribution results in a large number of members that retire with relatively smaller amounts. For example, roughly one-third of the new members enter retirement with less than 2,200 UF, and approximately two-thirds enter retirement with less than 4,700 UF. Only a tiny fraction of members enter retirement with amounts near 10,000 UF.
- Portfolio: one of three portfolios, each equiprobable: Fund C, Fund D, or Fund E.
- Payout option: one of two choices, each equiprobable: monthly payouts that begin immediately or monthly payouts that commence in the future at some advanced age. Details of each are given below.

Immediate lifetime payouts. This option makes monthly payouts in the amount of $s\ /\ a$, where s is the member's account balance at the end of the prior year, and a is the member's current "annuity factor." The annuity factor a is the expected present value of $1 paid this year and every subsequent year for the duration of his lifetime, with future payments discounted to the present using an assumed annual interest rate equal to the expected return on the selected investment portfolio.[34] Suppose it happens that the member's investments earn exactly the expected return every subsequent year and that the tontine's mortality experience likewise turns out exactly as predicted by its mortality table. In that case, the member's payout will have the same value $s\ /\ a$ every year. Of course, future investment returns will not match the expected return exactly each year, and future mortality experience will not turn out exactly as expected. Thus, future payouts will not be constant but will fluctuate according to actual investment return and actual mortality experience.

Deferred lifetime payouts. This option is the same as the immediate option, except that payouts do not begin until the month that the member turns some given age, such as 75 or 85. The payout is again computed as $s\ /\ a$, except in this case, a represents a deferred annuity factor. There is no death benefit, meaning that a member receives nothing if death occurs before payouts commence.

Operation. The number of members is initially set to zero. Beginning with the year 2021, the logic for each year of a simulation run is as follows:

1. New members are enrolled at the start of the year.
2. At the end of the year:
    - The tontine accounts of all members who were alive at the start of the year are credited with the portfolio return outcome for their selected investment choice

[34] The formula for the annuity factor at age x is $a = \sum_{t=1}^{\infty} v^t\ {}_tp_x$, where ${}_tp_x$ is the probability of surviving to age $x + t$ given that the member is alive at age x, and $v = 1/(1 + i)$ is the discount factor, with i the assumed discount interest rate. The value of ${}_tp_x$ is calculated from the mortality table. The mortality table used here has a terminal age of 110, meaning there is zero probability of surviving to ages greater than 110, and so the sum in the formula has a finite number of terms.

(Fund C, D, or E). These accounts are also charged with a 15 basis point annual tontine administration fee.

- Forfeiture processing occurs for any member that died during the calendar year. Forfeited balances are apportioned and redistributed to the surviving members in the form of a survival credit for the period.[35] Each surviving member's account balance is updated to reflect the credit.
- The payout for each surviving member is computed and deducted from that member's balance. The payout is zero in the case of deferred payouts that have not yet commenced.
- The account balance of members who died during the year is set to zero, and their accounts are effectively removed from the tontine membership.

We performed 10,000 simulation runs, each run spanning the 55 years from 2021 to 2075. In each simulation run, random fund returns for funds C, D, and E were generated for each of the $55 \times 12 = 660$ months using a distribution with parameters as shown in Table 5.

Table 5 Investment return parameters (annual)

			Correlation				
	Arithmetic Mean	Standard Deviation	Fund C	Fund D	Fund E	Annuity Rate	Mean Tontine Return*
Fund C	4.17%	4.52%	1.00				4.02%
Fund D	3.65%	3.03%	0.91	1.00			3.50%
Fund E	3.20%	2.36%	0.61	0.87	1.00		3.05%
Annuity Rate	3.24%	0.26%	-0.38	-0.26	-0.12	1.00	N/A

Source: Author's estimates based on Berstein et al. (2013).

* The mean tontine return for each fund refers to the arithmetic mean of the fund's return less the 15 basis point tontine administration fee.

At the start of each year, new members are enrolled with randomly selected parameters (gender, ownership, age, amount, portfolio, and payout option) as described above. Each member is assigned a random year of death, which changes with every simulation run. The year of death is randomly generated using the associated mortality table's probability rates and two random parameters to account for systematic mortality risk. The first parameter represents an error term applied to the base (2014) mortality rate, multiplying it by a normally distributed random value with mean zero and standard deviation of 0.005. The second parameter represents an error term applied to the mortality improvement factor, multiplying it by a normally distributed random value with mean zero and standard deviation of 0.001. These systematic mortality error terms change with each simulation run.

[35] Our model uses the "nominal gain method" described by Sabin and Forman (2016) as the method of forfeiture reallocation, which is desirable for its relative simplicity and explanatory properties. Fullmer and Sabin (2019a) demonstrate the practical applicability of this method in the case of individual accounts such as is used in the Chilean system.

4.3.2 Annuity model

We model the price of annuities in an analogous way as for tontines, except that we use a random discount rate drawn from a distribution with characteristics described in Table (the "annuity rate"). The annuity pays a fixed amount (in UF terms) every month for as long as the annuitant is alive. The same mortality tables used to model tontines are used to price annuities. An implicit assumption, then, is that the level of adverse selection would be similar in both products.

4.3.3 Programmed withdrawals

We model programmed withdrawals under each of Fund C, Fund D, and Fund E. The required programmed withdrawal rate is computed each calendar year as an annuity factor as previously described, but with a random discount rate of 60 basis points higher than the one used to calculate annuity payouts. In other words, the discount rate applied to programmed withdrawals is the randomly-drawn "annuity rate" plus 0.006. This 60-basis-point spread is roughly comparable to the average historical spread shown in Figure 1.

5. Simulation Results

In this section, we present results for a male who is 65 years old at the beginning of the year 2021 and elects to invest in Fund D. All the data are presented per 100 UF of account balance, so the initial payouts in the first year may be considered as initial payout ratios when thought of in percentage terms. Confidence intervals are shown between the 5th and 95th percentiles. Data for other age/gender cohorts and other investment fund options appear in the Appendix.

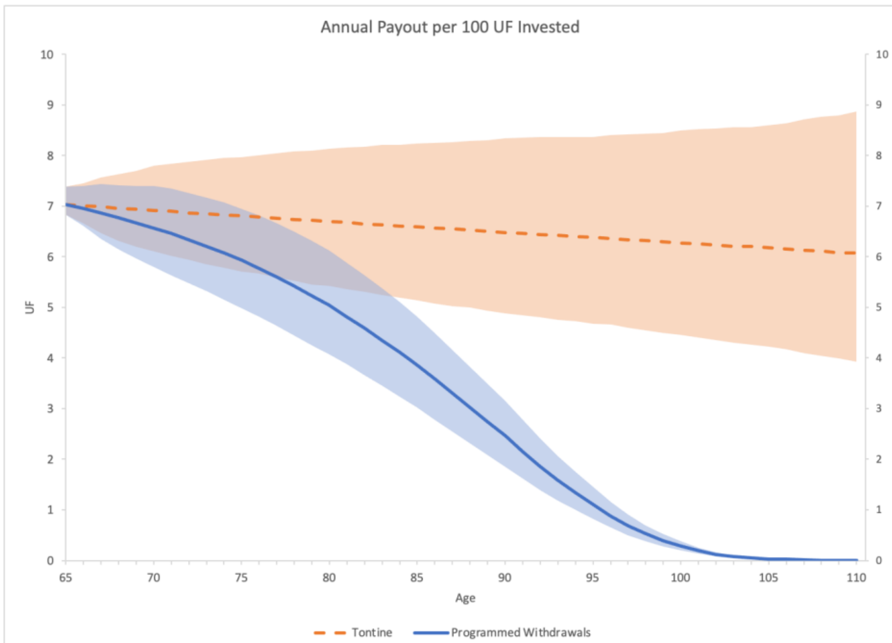
5.1 Comparison of tontine payouts to programmed withdrawal payouts

In Chile, programmed withdrawals are designed to have a higher initial payout than an annuity. However, programmed withdrawal payouts generally decline steeply over time because this high payout rate is not sustainable. Conversely, tontines are much better able to sustain this payout level because tontines offer not only investment returns but also survivor credits that increase with age even as the investor's account balance is systematically drawn down.

Tontines allow many degrees of freedom with respect to how their payouts are designed, whether rising, declining, or flat in expected terms. To facilitate the most direct comparison to programmed withdrawals, we model tontine payouts using the same discount rate as the "programmed withdrawal rate" at each age. In other words, the discount rate used to compute the tontine payouts is likewise the current "annuity rate" plus 0.60%. In this way, the initial payout in the first year is the same for both the tontine and the programmed withdrawal. Payouts in subsequent years will be higher in the tontine due to the survivor credits received by tontine participants each year. These survivor credits grow exponentially with age, thus contributing substantially to the income-generating potential of the tontine.

Figure 2 compares the evolution of payouts for Fund D investors per 100 UF invested. The solid lines represent the expected (average) payout across the simulation runs, while the shaded areas represent 90% confidence ranges between the 5th to 95th percentiles. The expected value of the initial payout is 7.03 UF in both cases.[36] The programmed withdrawal payouts fall substantially in subsequent years, while the expected tontine payout falls at a much more gradual pace.[37] As a result, programmed withdrawals provide little income at advanced ages, while a tontine invested in the same underlying fund provides significantly higher income at advanced ages. The tontine boosts the retiree's income and protects against longevity risk to the advanced age of 110.[38]

Figure 2 Payout of Programmed Withdrawal versus Tontine



Source: Authors estimates

Notes: Tontine designed to use the same assumed interest rate as for programmed withdrawals

The source of each programmed withdrawal payout can be attributed to two components: 1) a partial return of the participant's account balance (return of capital) and 2) investment returns that continue to be earned during the retirement years. Tontine payouts can likewise be attributed to these sources *plus* a third component – the survivor credits. Figure

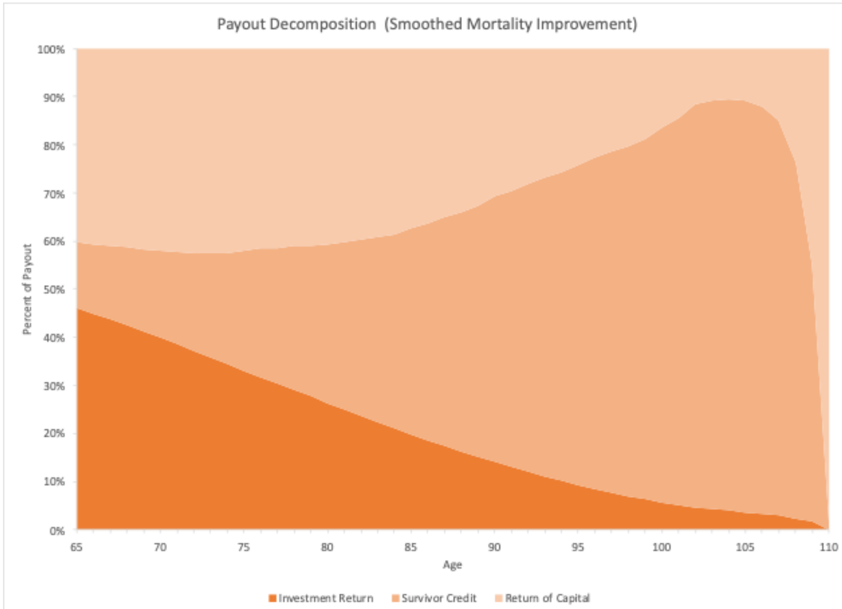
[36] The initial payout for both options varies across simulations because it depends on the annuity rate, which is a random variable. For any given simulation, the initial payout for the tontine will be the same as that of the programmed withdrawal. This is by design.

[37] The reason that the tontine payout falls on average is because the expected 3.50% net rate of return on a tontine investment in Fund D (the expected return of 3.65% less the 0.15% tontine administration fee) is less than the expected "programmed withdrawal rate" of 3.84% (the expected annuity rate of 3.24% plus 0.60%). We also modeled a tontine designed to have *flat* payouts on average, and in this case the expected payout in each year from age 65 to 110 was 6.81 UF. Note that if even higher initial payouts were the objective, we could have easily modeled a tontine with higher initial payouts and subsequent payouts that decline more rapidly.

[38] These figures do not consider the Solidarity Pillar, which is a mean-tested benefit for the 60% poorest of the population. For those pensioners entitled to get solidarity benefits, they received a minimum social pension once their programmed withdrawal gets exhausted.

3 shows the expected decomposition of the tontine payout for the 65-year-old male that was examined in Figure 2.[39]

Figure 3 Decomposition of tontine payout



Source: Authors estimates

In the early years of retirement, when the participant's account balance is still relatively high, investment returns represent the largest source of payout funding. However, its contribution gradually falls over time as the account balance is drawn down. Return of capital represents the next largest source of payout funding in the early years, but its contribution likewise falls over time as the account balance is drawn down. Conversely, the contribution of the survivor credits grows over time – it represents 14% of the payout at age 65 and rises to more than 80% of the payout by age 100.[40] These credits provide the power to sustain the payouts into advanced ages.

We can measure this on a cumulative basis as well. For every 100 UF invested in a tontine by the age 65 male cohort using Fund D, the cumulative payout made to someone who lives to the advanced age of 110 has the expected values shown in Table 6:

[39] Note that the CB-2014 and RV-2014 mortality tables quite unnaturally use mortality improvement factors that "jump" every five years. Such artificial jumps would result in survivor credits that likewise jump every five years. For this reason, we used a simple algorithm to smooth the factors such that the new improvement factor $\bar{i}_t = (i_{t-2} + i_{t-1} + i_t + i_{t+1} + i_{t+2})/5$. In other words, the new mortality improvement factor at each age is set to the average of the original factors over the five closest years.

[40] Notice also that if the participant survives to the ultimate year of the mortality table (age 110 in this case, although this could be extended), any remaining account balance is paid out as a return of capital.

Table 6 Decomposition of expected cumulative payout at age 110, per 100 UF invested

Source	Amount
Return of capital	100 UF
Investment returns	60 UF
Survivor credits	140 UF

Source: Authors estimates

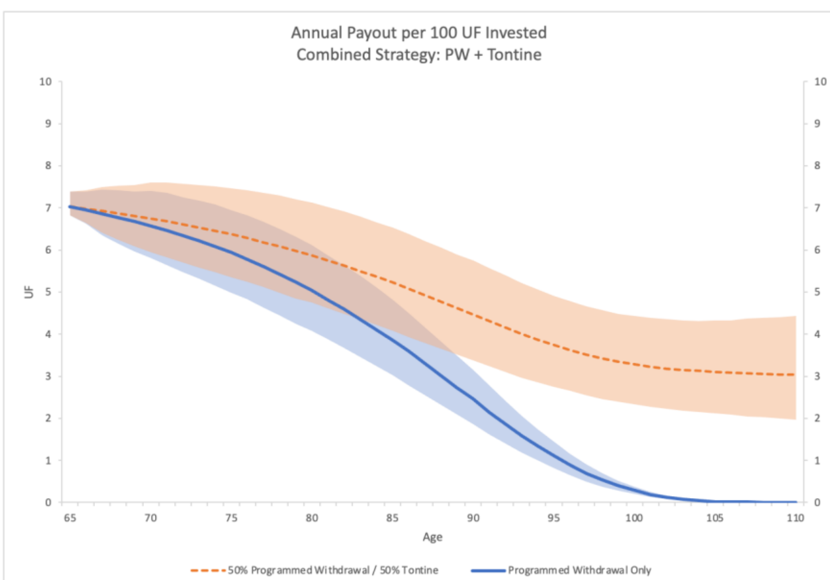
Because the source of the extra payouts offered by the tontine is the remaining balance of those who die, it provides no money for bequests. Those who wish to leave a bequest of some amount could do so by allocating their retirement account between programmed withdrawals and a tontine. This is examined in the next section.

5.2 Allocating between programmed withdrawal and tontine options

By allocating part of one's account to the programmed withdrawal option and part to the tontine option, participants may create payout streams that lie between the two outcomes shown in Figure 3. This "combined strategy" would deliver higher payouts and lower bequest amounts than if the participant had elected to take programmed withdrawals alone.

Figure 4 shows the resulting payout if the participant elects to allocate half of his Fund D investment to the tontine option and half to the programmed withdrawal option. The future payouts from the combined strategy are still boosted, but by a lesser amount than the "100% tontine" option from Figure 3 – in fact, since 50% of the account is invested in the tontine, the participant will receive 50% of the payout boost compared to the 100% tontine investment.

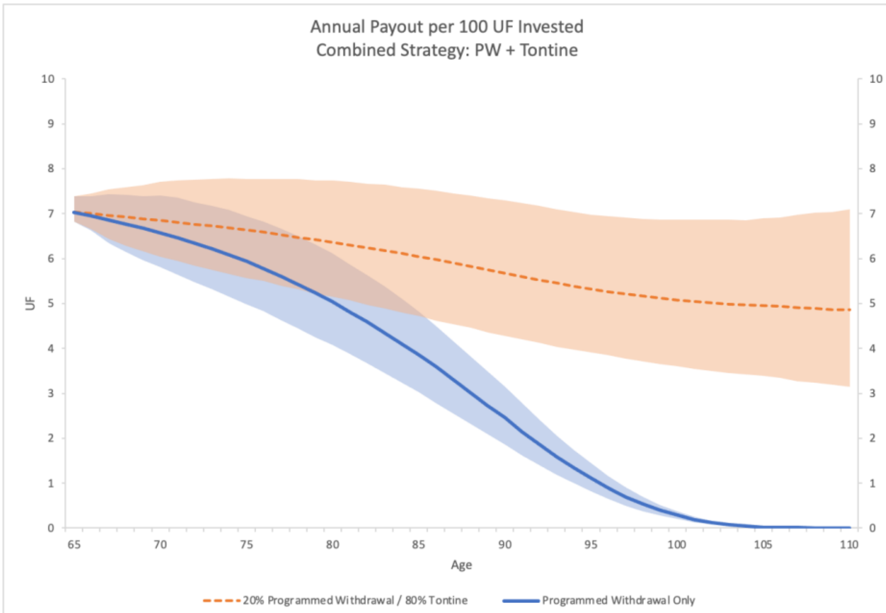
Figure 4 Payout of 50% Allocation each to Programmed Withdrawals and a Tontine



Source: Authors estimates

Figure 5 reveals the additional amount that can be received each year if the participant instead allocates his account balance at retirement with 20% to programmed withdrawals and 80% to the tontine.

Figure 5 Payout of 20% Allocation to Programmed Withdrawals and 80% to a Tontine

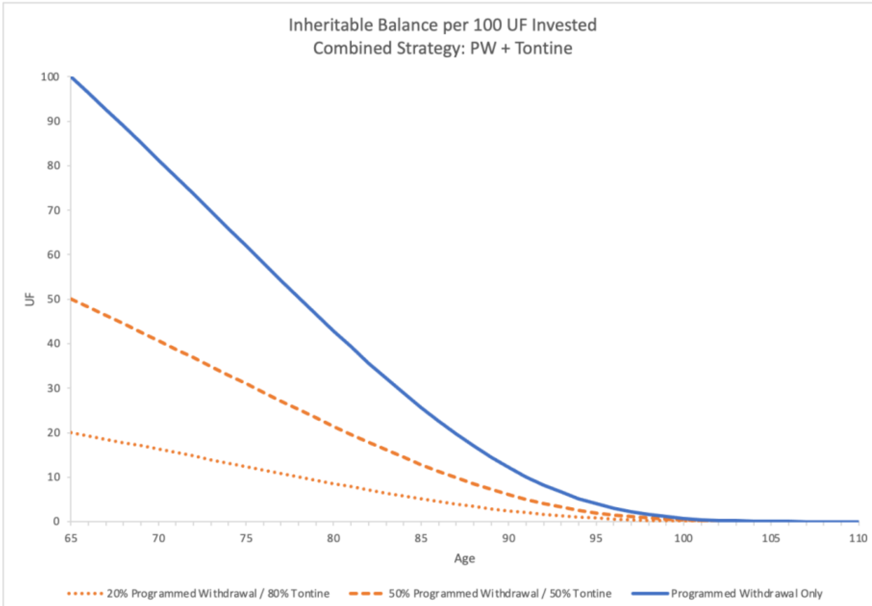


Source: Authors estimates

Figure 6 shows the inheritable balance of the two combined strategies shown in Figure 4 and Figure 5. Only that portion allocated to programmed withdrawals is inheritable by the participant's beneficiary.

It is easy to see that participants can easily increase or decrease their payouts and potential bequests simply by allocating more or less to the tontine option.

Figure 6 Inheritable Balance of 50% Allocation each to Programmed Withdrawals and a Tontine



Source: Authors' calculations

5.3 Using deferred tontines as longevity insurance

We also studied the strategy of combining temporal withdrawals with a deferred tontine. The former would provide income over a selected period (for example, the period covering the expected lifetime of the participant at the point of retirement), while the latter would act as longevity insurance if the participant lived longer than expected.

Temporal withdrawals refer to a withdrawal mechanism that pays out over a specific time horizon and completely depletes the investment with its last payout.[41] Thus, temporal withdrawals provide payments over a specific horizon only, with no longevity protection whatsoever.

A deferred tontine refers to payouts that start in the future rather than immediately. Thus, it acts very much like longevity insurance, paying if and only if the investor lives to receive the first payout, and then payouts will continue for as long as the investor is alive.[42]

Specifically, we modeled deferred tontine payouts that begin promptly after the temporal withdrawal period has ended. A portion of an investor's portfolio is allocated to the deferred tontine, while the remaining portion is allocated to temporal withdrawals. The

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[41] The formula for the temporal withdrawal is $TW_t = [S_t * i_t * (1 + i_t)^{N-1}]/[(1 + i_t)^N - 1]$, where N is the number of years of temporal withdrawals, S_t is the current account balance in year t that is dedicated to temporal withdrawals, and i_t is the current program withdrawal interest rate. The account balance evolves according to $S_{t+1} = (S_t - TW_t)(1 + r_t)$, where r_t is the rate of return for the investment portfolio in year t. Within each year, the monthly withdrawal amount equals the annual withdrawal amount divided by 12 – except in the final six months of the final year, the monthly withdrawal is recalculated each month to ensure that the last payment will fully exhaust the balance.

[42] Up to the assumed ultimate age of 110 (or whatever ultimate age is used in the tontine design)

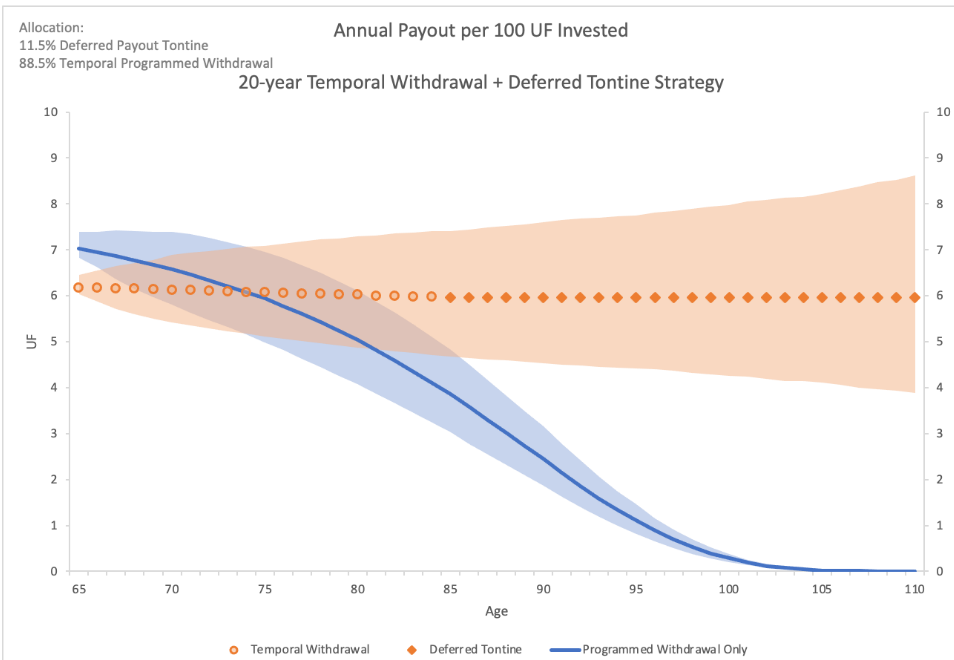
allocation was selected such that the expected value of the last temporal withdrawal payout was equal to the expected value of the first payout of the deferred tontine.

5.3.1 20-year temporal withdrawal horizon

In one example, we modeled a horizon of 20 years, roughly the life expectancy of an average 65-year-old male. In this case (a 65-year-old male investing in Fund D), the allocation was 11.5% to the deferred tontine and 88.5% to the temporal withdrawals.

Figure 7 compares this strategy to that of programmed withdrawals. Although the initial payout of the combined strategy is lower, the expected payout level is largely uniform. In the first 20 years, all payouts are from temporal withdrawals, while all payouts thereafter are from the deferred tontine. The expected payout is 6.18 UF in the first year, gradually declining each year until it reaches 5.97 UF in year 20 and 5.95 UF in year 21 and beyond.

Figure 7 Payout of 20-year Temporal Withdrawals + 20-year Deferred Tontine versus Programmed Withdrawal



Source: Authors estimates

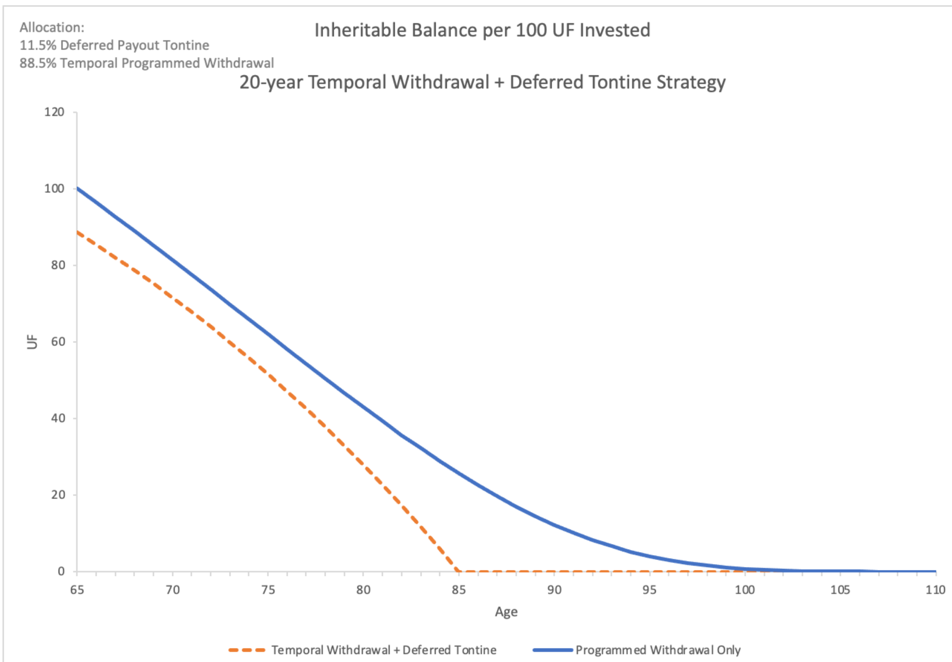
Relative to the programmed withdrawal strategy, investors in the combined strategy sacrifice some of their retirement income in the first few years for the benefit of higher income later in retirement.

Relative to the tontine-only strategy shown in Figure 2, pensioners also sacrifice some of their retirement income in the combined strategy. The sacrifice is about 12% of income in the first year, and then the difference between the two payouts steadily narrows each year thereafter. The combined strategy pays less than the tontine-only strategy because only a portion (11.5%) of the investors' account is allocated to longevity risk-sharing and the survivor credits that it provides.

Although the combined strategy pays less than the tontine-only strategy, it may be preferable to those with bequest motives. This is because the portion of the account allocated to temporal withdrawals may be bequeathed, whereas the tontine leaves no money as a bequest.

Figure 8 compares the inheritable balance of the programmed withdrawal strategy to the combined strategy. The inheritable balance of the combined strategy is 11.5% lower at the beginning due to 11.5% being invested in the deferred tontine. It declines over time for both strategies. For the combined strategy, it drops to zero at the end of the 20-year horizon.

Figure 8 Inheritable Balance of 20-Year Temporal Withdrawals + 20-Year Deferred Tontine versus Programmed Withdrawal



Source: Authors estimates

The combined strategy may be appealing to those who A) wish to protect against dramatically falling income later in life and B) want to retain some ability to leave assets as a bequest.

Note that an investor could increase (decrease) the payouts in the first 20 years and decrease (increase) the payouts in years 21 and beyond, if desired, simply by decreasing (increasing) the allocation to the deferred tontine. Changing the allocation would affect the inheritable balance as well – naturally, the more money allocated to longevity risk sharing, the less that will be available for bequest.

Another way to alter the income/bequest trade-off in the combined strategy is to use a longer or shorter temporal withdrawal horizon.[43]

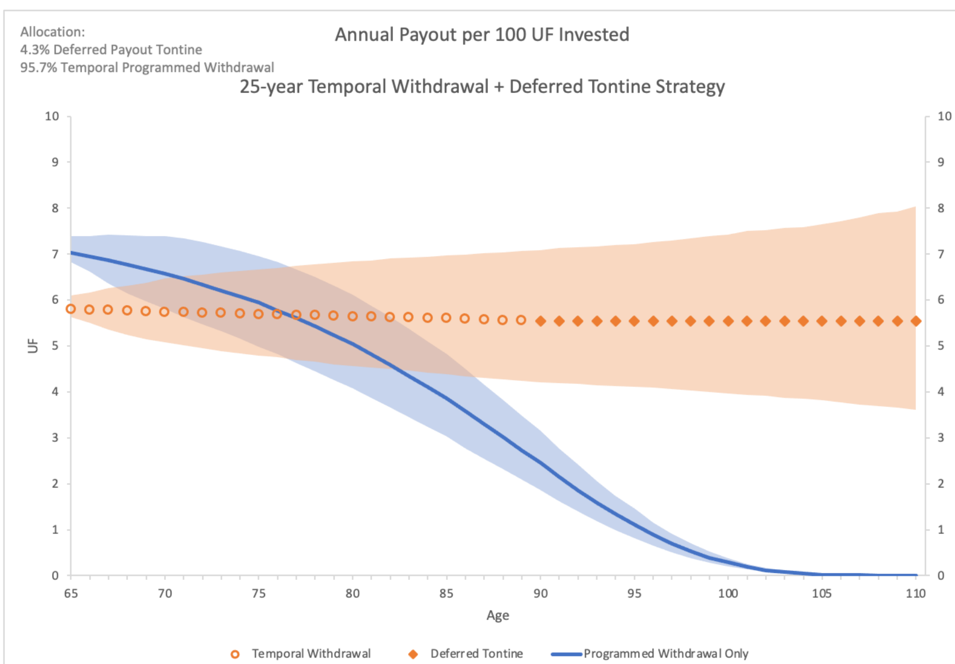
5.3.2 25-year temporal withdrawal horizon

To show the effect of the selected horizon, we extended the temporal withdrawal horizon to 25 years and likewise extended the deferral period for the tontine to 25 years so that it would again start paying after the temporal withdrawal payouts stop.

For a 65-year-old male investing in Fund D, the allocation is now approximately 4.3% to the deferred tontine and 95.7% to the temporal withdrawals. By making the deferral period longer, the longevity insurance provided by the deferred tontine becomes less expensive (4.3% of the investor's balance compared to 11.5%). However, because the temporal withdrawals must provide for an extra 5 years of payouts, the payout level each year is lower than with the 20-year strategy.

Figure 9 compares the 25-year combined strategy to that of programmed withdrawals. This time, all payouts in the first 25 years are from temporal withdrawals, while all payouts afterward are from the deferred tontine. The expected payout is now 5.80 UF in the first year, gradually declining each year until it reaches 5.55 UF in year 25 and then 5.54 UF in year 26 and beyond.

Figure 9 Payout of 25-year Temporal Withdrawals + 25-year Deferred Tontine versus Programmed Withdrawal

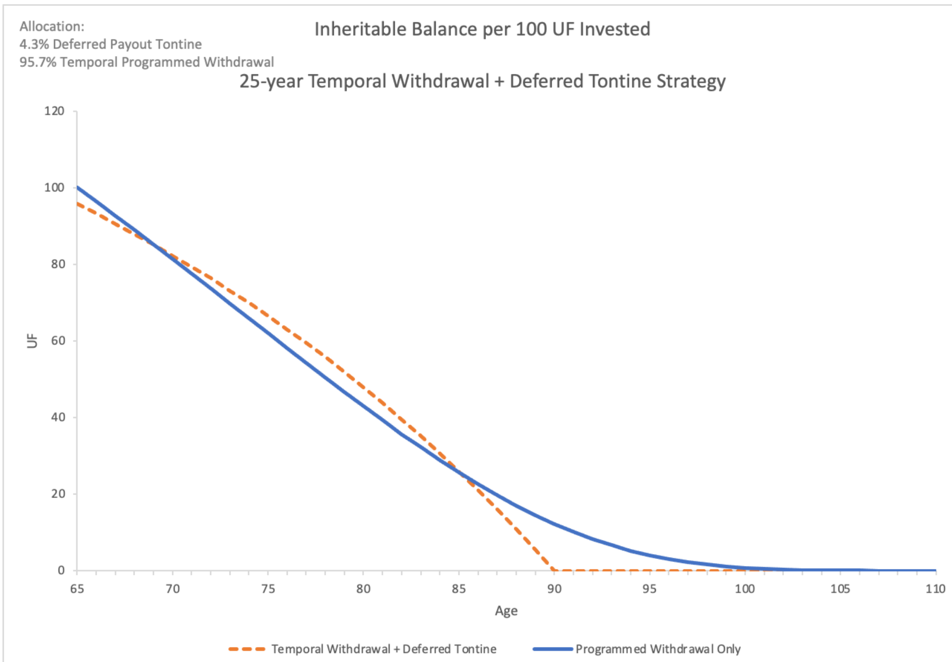


Source: Authors estimates

[43] For more on bequests and tontines see Weinert J.H. (2017b), Bernhardt and Donnelly (2019), and Chen and Rach (2020).

Figure 10 compares the inheritable balance of the programmed withdrawal strategy to the combined 25-year strategy. Interestingly, the inheritable balance between these two strategies tracks quite closely. The primary difference, then, lies in their differing payout trajectories.

Figure 10 Inheritable Balance of 25-Year Temporal Withdrawals + 25-Year Deferred Tontine versus Programmed Withdrawal



Source: Authors estimates

5.4 Comparison of tontines to annuities

Tontines and annuities are similar in many ways. Both offer longevity risk pooling, and as a result, both generally represent irrevocable decisions on the part of the participant buyer. A primary difference is that annuity buyers *transfer* longevity risk to an insurer in exchange for a guarantee, whereas tontine buyers *share* longevity risk among themselves.[44] Thus, tontine buyers neither receive a guarantee nor pay for one.

Guarantees are valuable, of course, and this is a primary benefit of annuities. The question for individuals is whether the value provided by the guarantee is worth its cost. For many, the answer will be yes. For others, the answer will be no.

The added cost of providing a guarantee dictates that annuity payout rates should be lower than tontine payout rates if the two products are invested similarly. The higher expected payout rate of tontines is a solid reason to consider them, especially in a world where many are undersaved, and the marginal benefit to them of higher income is significant.

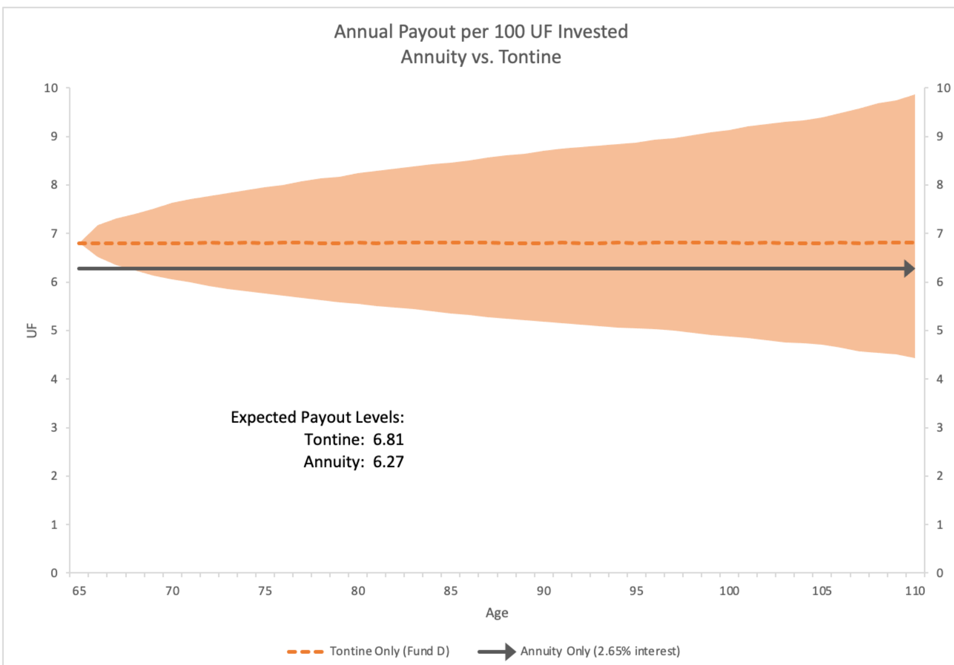
[44] Maurer et al. (2013) discuss participating annuities that do not transfer risk to the buyer, making them very tontine-like indeed.

A tontine's lack of a guarantee also brings other benefits. It is relatively simple to engineer a variety of different payout options using transparent formulas. Another advantage is that tontines do not necessarily require separate investment portfolios. They could be offered using the same investment fund options that already exist.

Our study did not attempt to explicitly quantify the difference in payout levels between tontines and an annuity because the answer is heavily dependent, among other things, on prevailing interest rates at the time of purchase.[45] For the sake of illustration, however, we modeled an annuity using recent pricing with a tontine that is designed to deliver a uniform UF payout in expected terms.[46] The annuity is priced using a 2.65% interest rate, which was the current annuity interest rate as of April 2021.

Figure 11 shows the payout range of a tontine compared to the annuity. The annuity pays the fixed rate of 6.27 UF for as long as the participant lives. The tontine has an initial payout of 6.81 UF, and this payout will vary over time – in some periods, it may be lower than the annuity payout, but in *expected* terms, the cumulative amount received is higher for the tontine.

Figure 11 Sample illustration of Annuity versus Tontine payout



Source: Authors' calculations

As with programmed withdrawals and tontines, it is also possible for participants to allocate their accounts between an annuity and a tontine. An illustration of this with a 50% allocation

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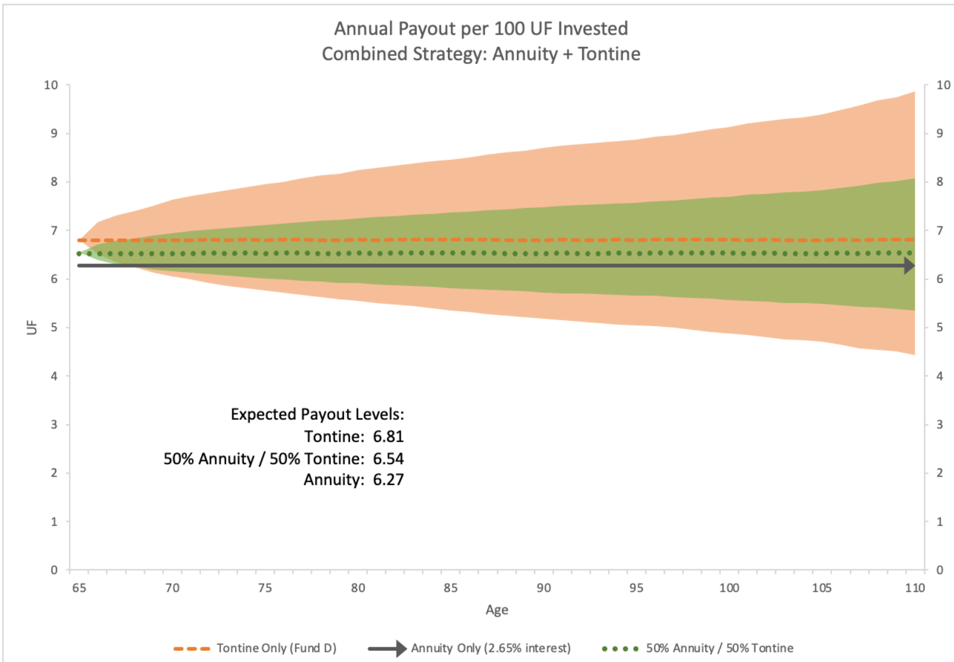
[45] Other studies have examined this question. See Milevsky (2015), Milevsky et al. (2018), Chen et al. (2020), and Fullmer and Sabin (2019a).

[46] To do so, we set the discount rate for the tontine payouts to the expected net rate of return on Fund D, which is 3.50% (the expected return of 3.65% less the 0.15% tontine administration fee).

to the annuity and a 50% allocation to the tontine is shown in Figure 12. With this combined strategy, the expected payout is now 6.54 UF, and the potential variability of the income stream is reduced relative to the tontine strategy alone. So, the annuity can be used to deliver greater certainty of the payout level. This is the purpose of insurance, and, naturally, participants who desire this will pay a premium (in the form of lower expected income relative to the tontine) to transfer risk to the insurer.

Figure 12 Sample illustration of combined Annuity + Tontine payout



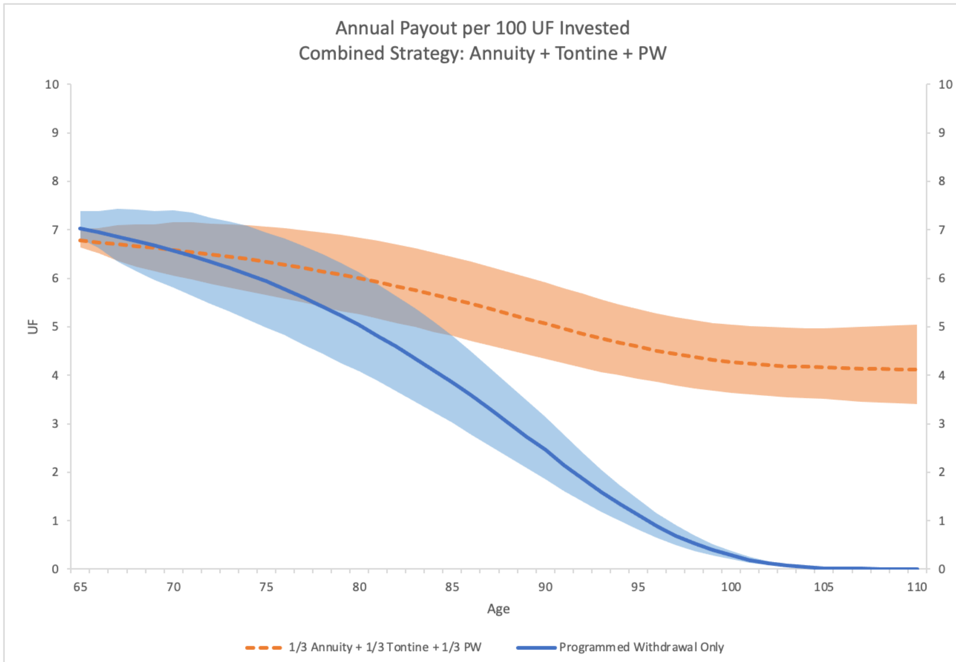
Source: Authors estimates

Again, we caution that the comparison to annuities is illustrative only because the payout rates are highly dependent on prevailing interest rates. The point is that providing tontines as an option gives participants greater flexibility to craft a pension that best meets their needs and preferences.

5.5 Three products combined

Naturally, participants could elect to combine any of the product options illustrated above. For example, if our 65-year-old participant selected to allocate his account in equal proportions between programmed withdrawals, a tontine, and an annuity, the result would appear as shown in Figure 13. Here, we again assume that the annuity interest rate is 2.65% and that the tontine uses the "programmed withdrawal rate" for computing its payouts, as discussed in section 5.1 and illustrated in Figure 2.

Figure 13 Sample illustration of combined Annuity + Tontine + Programmed Withdrawal payout



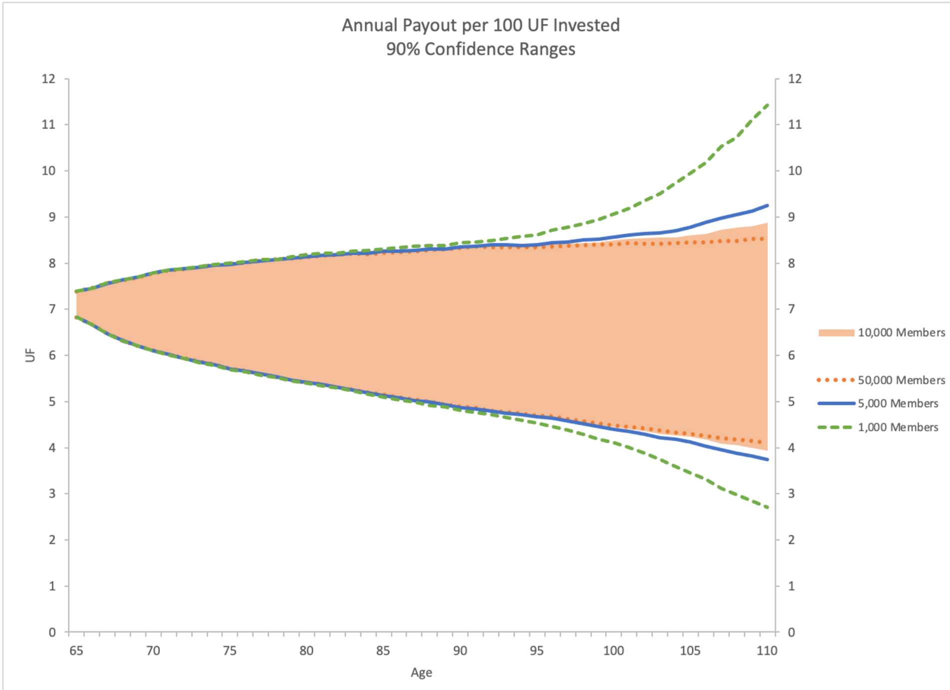
Source: Authors estimates

5.6 The effect of pool size

Up to this point, our analysis has assumed a tontine pool with a constant size of 10,000 members. The pool size matters because it affects the degree of longevity risk diversification enjoyed by the participants, which in turn affects the degree of volatility in the survivor credits that participants receive over time, affecting the variability of the payouts. To clarify: the pool size does not affect the expected *level* of the payouts, but it does affect the expected *variance* of the payouts.

Our previous analysis considered a tontine pool with 10,000 members. Figure 14 illustrates tontine payouts for a 65-year-old male who invests in Fund D at the start of 2021 and immediately begins taking payouts. It shows the effect of pool size on the potential range of the payout distribution by comparing the potential range of payouts at the 5th and 95th percentiles using four different pool sizes – 1,000, 5,000, 10,000, and 50,000 members.

Figure 14 Effect of membership pool size on payouts



Source: Authors estimates

The effect of pool size is minimal and virtually unnoticeable at ages below about 90. At higher ages, the effect becomes more noticeable. The reason for this is that the survival credits that a member receives are a function of mortality rates, which increase with age.

As expected by the law of large numbers, the confidence interval narrows as the membership size grows, and the marginal benefit of adding to the membership pool is a decreasing function. For all but the very long-lived, almost all the benefit is achieved with only 1,000 members. Including the very long-lived, most of the benefit is achieved with only 5,000 members. Bigger is better, of course, but a tontine with even as few as 1,000 members is enough to substantially diversify its members' longevity risks. Even as few as 500 members is enough to achieve a workable solution.

6. Conclusions and Policy Considerations

Many countries are promoting private pensions to take pressure off unsustainable first pillar schemes in response to increasing longevity. DB arrangements have been replaced by DC schemes or supplemented by DC features. These trends have shifted many complex decisions from the government and employers to individuals, such as choosing an investment strategy and choosing how to spend down one's savings in retirement.

Pension system challenges have gained significant public visibility. It is an international discussion with substantial media coverage. For example, an April 2021 publication by the Financial Times Editorial Board discussed the main challenges facing young generations in ensuring adequate pensions, recognizing longevity risk as one of the key threats affecting future retirement income.[47] Additional risks faced by pension members include market risk, labor/human capital risk, inflation risk, and annuitization risk. These risks are not independent; instead, they overlap, and their interaction is complex.

Evidence indicates that young individuals are not well-equipped at managing such risks and usually underestimate the need to save for old age. Behavioral biases and a lack of financial education may trigger poor decision-making, leading to inadequate income in retirement. It has been argued that there is a missing element in pension systems (ICPM, 2018a and 2018b) and that a "new approach" is needed for younger generations. From an international perspective and leading the way, countries such as the Netherlands, Singapore, and Sweden have incorporated risk-sharing components to their pension systems, where mutualistic and tontine-like arrangements can efficiently hedge investment and longevity risks. Other jurisdictions are evaluating or in the process of introducing these features, such as Australia, Canada, South Africa, and the United Kingdom. Unfortunately, the LATAM region and other jurisdictions are laggards on these developments.

To manage a tontine scheme, pension fund managers will need sufficient actuarial expertise in the discipline of fair tontine design, administrative systems for computing and processing survivor credits and payout amounts, tools to help individuals make asset allocation decisions (for example, how much to invest in a tontine versus other options), and upgraded reporting and auditing systems. Some might insource the work by hiring experts and upgrading the necessary systems. Others might elect to outsource the job by hiring external consultants and third-party administration providers who specialize in this. Either way, we believe that the capability to develop tontine solutions seems readily within the reach of pension product providers in Chile because not only has the literature on fair tontine design expanded significantly in recent years, but equivalent products have been successfully launched in several countries, most recently in Australia and Canada.

There is a need for pension systems to significantly improve pension payments' level, stability, and sustainability as pensioners age. The solution should be not limited to increasing the take-up rate of annuities – explicit guarantees are costly and are especially challenging in a low-interest rate environment, and the lock-in of savings may not be in line with members' preferences. We propose to develop new lifetime retirement income products in a more flexible and cost-efficient way.

Flexible products that are better suited to satisfying the needs and preferences of members are key for improving the pay-out phase. It cannot be overstated that pension adequacy is relevant at the time of retirement and in the long run. When retirement income declines with age, as it does with Chile's programmed withdrawals scheme, a retiree's financial situation becomes much worse at more advanced ages. This issue is especially crucial for women as they are likely to live longer and end life single.

We believe our variable lifetime income proposal has advantages over other proposals currently under consideration for the Chilean Pension System. Our proposal provides clear

[47] "A new deal for the young: ensuring fair pensions." The link is https://www.ft.com/content/14b9d858-7e57-4093-b2c6-b51fa3929ac5.

transparency and investment flexibility with higher expected income streams; it is easier to implement; it does not involve higher costs since there are no explicit guarantees; it does not distort the annuity market – on the contrary, it complements it; it provides a means to offer a form of longevity insurance even if insurers are unwilling to supply it, and it is in line with the transition of many countries to include tontine-like longevity-risk sharing in their defined-contribution designs.

As policy recommendations, we envision that the tontine could be administered countrywide, in which case every person who elects to purchase a tontine would become part of a single, countrywide longevity risk pool. The benefit of this approach is that it would promote economies of scale and maximize longevity risk diversification. Another advantage is that individuals could change pension fund manager providers at any time without affecting the longevity pool whatsoever. The implementation of this design may require setting up a clearinghouse basis risk exchange between providers, as was visualized by Edwards and Diaz (2009, 2011) and Edwards and Valdes (1998).

Alternatively, each pension fund manager offering tontines could manage their own longevity risk pool. Compared to the countrywide alternative, this would result in smaller pools and therefore reduced levels of risk diversification. However, if each pension fund manager's tontine can attain and maintain an adequate enrollment size (say, at least 500 members), the effect would be reasonably small. One question under this scenario is whether individuals that own a tontine would still be allowed to switch to a different provider, and again we believe that the effect of doing so would be small provided that each pension fund manager's tontine enrollment remains sufficiently large.[48]

Investment strategy is another potential area of innovation. For example, providers could develop dynamic cash-flow-driven investment strategies that seek to minimize payout variability beyond what is possible using the largely static asset allocation-based strategies offered today (Fullmer and Sabin, 2019b; Mantilla-García et al., 2022; Bernhardt et al., 2021; Mantilla-García, 2021; Martellini et al., 2018). Such dynamic strategies would be inspired in many ways by the liability-driven investment strategies used by defined-benefit pension plans and annuity providers.

Our proposal also creates new possibilities in terms of institutional design.[49] Although we assume that AFPs are a natural candidate to offer variable lifetime income products in the Chilean context, insurers, and other financial institutions could also be providers should they wish to enter this market. Henceforth, to the extent that increasing the number of retirement plans is a policy objective, keep in mind that any asset manager could offer variable lifetime income products. Our proposal enriches the institutional possibilities in the decumulation stage and enhances the welfare of individual participants. To this end, it is essential to emphasize a result in the literature comparing the fees that could be charged

[48] This question of transferability across different risk pools is a subject for further research. It is moot in the ideal case of a single countrywide risk pool, but important in the case of pools that are managed separately by pension fund managers.

[49] As Price et al. (2021), point out, these institutional considerations "are fundamental to ensuring that a good product innovation is not simply added into a defective basic delivery system."

under a tontine versus other products and the efficiency gains associated with low costs that can lead to such welfare improvement. Indeed, Chen et al. (2021a) show that tontine providers and participants can agree on a fee level, making tontines an attractive alternative to annuities for tontine participants and simultaneously allowing providers to be profitable.

Finally, policy recommendations also need to address the heterogeneity in pension systems across a number of relevant dimensions. One relevant source of heterogeneity in the payout phase refers to life-expectancy differentials among different population segments. In fact, empirical evidence for many countries is robust to indicate that life expectancy depends largely on the socioeconomic characteristics of individuals (OECD, 2018). One advantage of the tontine methodological framework is easily incorporating such heterogeneity into its design. For example, survivor credit and income payout benefits can be designed to fairly compensate members for the actual longevity risk they take within the pension plan. A fair distribution of survivor credits will grant a higher proportion to participants with higher expected mortality (Fullmer and Sabin, 2019a). Older members should receive a larger proportion because their expected mortality rate is higher. In the same way, members of lower socioeconomic groups who have higher expected mortality rates can likewise be granted a higher proportion of survivor credits commensurate with their higher mortality risk. In this way, the design of the longevity risk sharing within tontines can explicitly – and fairly – compensate those having lower life expectancies. This is a subject for further research in the context of tontine design.

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Appendix

This section shows data for selected age/gender cohorts and investment fund options.

Table A 1 Female Age 60 at Beginning of Year 2021, Investment Fund C

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	Age	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile
1/1/2021	60	5.63	5.63	5.63	5.30	5.50	5.87	5.30	5.50	5.87
1/1/2022	61	5.27	5.63	6.04	5.14	5.51	6.00	5.13	5.51	5.99
1/1/2023	62	5.03	5.63	6.21	4.91	5.52	6.16	4.89	5.51	6.14
1/1/2024	63	4.87	5.63	6.36	4.75	5.53	6.30	4.73	5.51	6.27
1/1/2025	64	4.75	5.63	6.49	4.65	5.54	6.44	4.62	5.51	6.40
1/1/2026	65	4.67	5.63	6.62	4.56	5.55	6.58	4.52	5.50	6.52
1/1/2027	66	4.59	5.63	6.73	4.50	5.56	6.71	4.44	5.50	6.62
1/1/2028	67	4.52	5.63	6.81	4.44	5.57	6.79	4.37	5.49	6.69
1/1/2029	68	4.45	5.63	6.89	4.38	5.58	6.87	4.30	5.48	6.74
1/1/2030	69	4.39	5.63	6.99	4.32	5.59	6.97	4.23	5.47	6.81
1/1/2031	70	4.33	5.63	7.03	4.26	5.60	7.04	4.15	5.45	6.84
1/1/2032	71	4.27	5.63	7.13	4.22	5.61	7.15	4.09	5.44	6.93
1/1/2033	72	4.21	5.62	7.21	4.16	5.63	7.25	4.01	5.42	6.98
1/1/2034	73	4.16	5.62	7.27	4.13	5.63	7.31	3.96	5.40	7.01
1/1/2035	74	4.12	5.63	7.36	4.09	5.65	7.41	3.89	5.37	7.05
1/1/2036	75	4.07	5.63	7.42	4.04	5.65	7.50	3.82	5.35	7.09
1/1/2037	76	4.01	5.62	7.48	4.01	5.67	7.56	3.75	5.31	7.09
1/1/2038	77	3.97	5.63	7.55	3.97	5.67	7.64	3.68	5.27	7.08
1/1/2039	78	3.93	5.62	7.64	3.93	5.69	7.77	3.61	5.22	7.13
1/1/2040	79	3.89	5.62	7.70	3.89	5.70	7.83	3.53	5.16	7.09
1/1/2041	80	3.85	5.62	7.75	3.86	5.71	7.94	3.46	5.10	7.08
1/1/2042	81	3.81	5.63	7.82	3.81	5.72	8.00	3.35	5.01	7.00
1/1/2043	82	3.77	5.62	7.89	3.79	5.73	8.09	3.26	4.92	6.95
1/1/2044	83	3.73	5.63	7.94	3.76	5.74	8.14	3.15	4.81	6.82
1/1/2045	84	3.69	5.62	8.01	3.73	5.75	8.24	3.05	4.69	6.73
1/1/2046	85	3.67	5.63	8.08	3.70	5.75	8.34	2.94	4.56	6.61
1/1/2047	86	3.63	5.62	8.13	3.68	5.77	8.39	2.81	4.40	6.40
1/1/2048	87	3.60	5.63	8.13	3.66	5.77	8.44	2.68	4.23	6.18
1/1/2049	88	3.57	5.63	8.19	3.63	5.79	8.51	2.54	4.05	5.96
1/1/2050	89	3.53	5.63	8.27	3.60	5.80	8.59	2.40	3.87	5.72
1/1/2051	90	3.50	5.62	8.31	3.58	5.81	8.65	2.27	3.67	5.47
1/1/2052	91	3.46	5.62	8.34	3.55	5.82	8.70	2.10	3.43	5.13
1/1/2053	92	3.42	5.63	8.38	3.51	5.83	8.75	1.92	3.19	4.78
1/1/2054	93	3.39	5.62	8.45	3.49	5.84	8.84	1.76	2.94	4.45
1/1/2055	94	3.35	5.63	8.50	3.44	5.84	8.92	1.59	2.69	4.09
1/1/2056	95	3.35	5.63	8.57	3.44	5.86	9.01	1.43	2.43	3.74
1/1/2057	96	3.33	5.63	8.62	3.43	5.87	9.07	1.24	2.13	3.28
1/1/2058	97	3.32	5.63	8.64	3.43	5.88	9.11	1.07	1.83	2.83
1/1/2059	98	3.28	5.62	8.70	3.40	5.89	9.20	0.90	1.56	2.42
1/1/2060	99	3.24	5.63	8.75	3.36	5.90	9.26	0.75	1.31	2.04
1/1/2061	100	3.21	5.62	8.79	3.34	5.91	9.30	0.61	1.08	1.68
1/1/2062	101	3.19	5.63	8.83	3.32	5.91	9.38	0.46	0.82	1.29
1/1/2063	102	3.17	5.62	8.93	3.31	5.92	9.51	0.34	0.61	0.97
1/1/2064	103	3.15	5.62	8.96	3.28	5.94	9.56	0.25	0.44	0.71
1/1/2065	104	3.12	5.62	8.97	3.26	5.95	9.59	0.17	0.31	0.49
1/1/2066	105	3.09	5.62	9.05	3.24	5.95	9.69	0.12	0.21	0.34
1/1/2067	106	3.06	5.63	9.07	3.21	5.96	9.74	0.075	0.147	0.220
1/1/2068	107	3.02	5.62	9.12	3.17	5.97	9.82	0.046	0.093	0.139
1/1/2069	108	3.00	5.62	9.15	3.14	5.98	9.88	0.028	0.056	0.084
1/1/2070	109	2.98	5.63	9.33	3.15	5.99	10.06	0.016	0.032	0.048
1/1/2071	110	2.94	5.63	9.33	3.11	6.02	10.10	0.009	0.017	0.026

Source: Author's calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.
"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 2 Female Aged 60 at Beginning of Year 2021, Investment Fund D

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	**Age**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**
1/1/2021	60	5.28	5.28	5.28	5.30	5.50	5.87	5.30	5.50	5.87
1/1/2022	61	5.05	5.28	5.56	5.19	5.49	5.88	5.19	5.48	5.87
1/1/2023	62	4.92	5.28	5.66	5.05	5.47	5.97	5.03	5.45	5.95
1/1/2024	63	4.83	5.28	5.74	4.93	5.45	6.00	4.91	5.43	5.97
1/1/2025	64	4.76	5.28	5.82	4.85	5.43	6.05	4.81	5.40	6.01
1/1/2026	65	4.70	5.28	5.91	4.77	5.42	6.12	4.73	5.37	6.07
1/1/2027	66	4.65	5.28	5.97	4.70	5.40	6.17	4.65	5.33	6.09
1/1/2028	67	4.59	5.28	6.03	4.64	5.38	6.18	4.57	5.30	6.09
1/1/2029	68	4.54	5.28	6.07	4.58	5.36	6.22	4.50	5.26	6.10
1/1/2030	69	4.51	5.28	6.11	4.53	5.35	6.25	4.43	5.23	6.10
1/1/2031	70	4.47	5.28	6.16	4.47	5.33	6.25	4.35	5.19	6.09
1/1/2032	71	4.44	5.28	6.21	4.43	5.31	6.28	4.29	5.15	6.09
1/1/2033	72	4.39	5.28	6.26	4.37	5.30	6.31	4.21	5.10	6.08
1/1/2034	73	4.36	5.28	6.30	4.33	5.28	6.33	4.15	5.06	6.07
1/1/2035	74	4.32	5.28	6.33	4.26	5.26	6.35	4.06	5.01	6.05
1/1/2036	75	4.30	5.28	6.39	4.24	5.24	6.37	4.01	4.96	6.03
1/1/2037	76	4.27	5.28	6.41	4.20	5.23	6.39	3.94	4.90	5.99
1/1/2038	77	4.24	5.28	6.45	4.15	5.21	6.40	3.85	4.84	5.94
1/1/2039	78	4.23	5.28	6.48	4.11	5.19	6.42	3.77	4.77	5.90
1/1/2040	79	4.19	5.28	6.52	4.06	5.18	6.43	3.68	4.69	5.82
1/1/2041	80	4.16	5.28	6.55	4.03	5.16	6.45	3.60	4.61	5.75
1/1/2042	81	4.12	5.28	6.58	3.98	5.14	6.46	3.49	4.51	5.67
1/1/2043	82	4.10	5.28	6.63	3.95	5.13	6.47	3.39	4.40	5.55
1/1/2044	83	4.08	5.28	6.66	3.91	5.11	6.48	3.28	4.29	5.42
1/1/2045	84	4.06	5.28	6.70	3.87	5.09	6.49	3.17	4.16	5.30
1/1/2046	85	4.02	5.28	6.73	3.83	5.08	6.52	3.04	4.03	5.17
1/1/2047	86	4.00	5.28	6.76	3.80	5.06	6.52	2.90	3.86	4.98
1/1/2048	87	3.99	5.28	6.80	3.77	5.05	6.53	2.76	3.70	4.78
1/1/2049	88	3.96	5.28	6.82	3.73	5.03	6.53	2.62	3.52	4.57
1/1/2050	89	3.95	5.28	6.84	3.71	5.01	6.53	2.48	3.35	4.35
1/1/2051	90	3.93	5.29	6.85	3.68	5.00	6.52	2.33	3.16	4.13
1/1/2052	91	3.91	5.28	6.88	3.66	4.98	6.53	2.16	2.94	3.86
1/1/2053	92	3.88	5.29	6.91	3.62	4.96	6.55	1.98	2.72	3.58
1/1/2054	93	3.86	5.29	6.96	3.58	4.95	6.56	1.81	2.49	3.30
1/1/2055	94	3.83	5.28	6.98	3.55	4.93	6.57	1.64	2.27	3.01
1/1/2056	95	3.81	5.29	7.01	3.51	4.92	6.57	1.46	2.04	2.73
1/1/2057	96	3.80	5.28	7.04	3.50	4.90	6.58	1.27	1.77	2.38
1/1/2058	97	3.77	5.28	7.06	3.46	4.88	6.57	1.08	1.52	2.04
1/1/2059	98	3.75	5.28	7.08	3.43	4.86	6.58	0.91	1.29	1.73
1/1/2060	99	3.73	5.29	7.11	3.39	4.85	6.59	0.75	1.07	1.46
1/1/2061	100	3.71	5.29	7.16	3.36	4.84	6.60	0.62	0.88	1.20
1/1/2062	101	3.69	5.29	7.18	3.34	4.82	6.61	0.47	0.69	0.92
1/1/2063	102	3.64	5.29	7.23	3.30	4.81	6.63	0.34	0.51	0.68
1/1/2064	103	3.64	5.29	7.27	3.27	4.79	6.65	0.25	0.37	0.49
1/1/2065	104	3.61	5.29	7.33	3.24	4.78	6.67	0.17	0.26	0.34
1/1/2066	105	3.60	5.29	7.35	3.21	4.76	6.68	0.11	0.17	0.23
1/1/2067	106	3.57	5.29	7.38	3.17	4.75	6.69	0.074	0.112	0.151
1/1/2068	107	3.54	5.29	7.43	3.14	4.73	6.71	0.046	0.070	0.094
1/1/2069	108	3.51	5.29	7.50	3.10	4.72	6.74	0.027	0.042	0.056
1/1/2070	109	3.47	5.30	7.53	3.06	4.71	6.74	0.016	0.024	0.032
1/1/2071	110	3.45	5.30	7.59	3.03	4.71	6.79	0.008	0.013	0.018

Source: Authors' calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.

"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 3 Female Age 60 at Beginning of Year 2021, Investment Fund E

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	Age	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile
1/1/2021	60	4.99	4.99	4.99	5.30	5.50	5.87	5.30	5.50	5.87
1/1/2022	61	4.82	4.99	5.20	5.20	5.46	5.83	5.19	5.46	5.82
1/1/2023	62	4.73	4.99	5.28	5.08	5.42	5.87	5.06	5.41	5.85
1/1/2024	63	4.68	4.99	5.34	4.98	5.38	5.85	4.96	5.36	5.82
1/1/2025	64	4.62	4.99	5.39	4.90	5.34	5.85	4.86	5.30	5.81
1/1/2026	65	4.58	4.99	5.45	4.81	5.30	5.86	4.77	5.25	5.81
1/1/2027	66	4.54	4.99	5.50	4.74	5.26	5.86	4.68	5.20	5.79
1/1/2028	67	4.50	4.99	5.53	4.66	5.22	5.84	4.59	5.14	5.76
1/1/2029	68	4.46	4.99	5.57	4.60	5.18	5.84	4.52	5.09	5.73
1/1/2030	69	4.44	4.99	5.60	4.54	5.14	5.83	4.44	5.03	5.70
1/1/2031	70	4.41	4.99	5.63	4.47	5.10	5.81	4.35	4.97	5.65
1/1/2032	71	4.38	4.99	5.67	4.42	5.07	5.80	4.28	4.91	5.62
1/1/2033	72	4.36	4.99	5.71	4.36	5.03	5.77	4.20	4.85	5.57
1/1/2034	73	4.33	4.99	5.73	4.29	4.99	5.76	4.12	4.78	5.52
1/1/2035	74	4.30	4.99	5.76	4.24	4.95	5.74	4.04	4.72	5.47
1/1/2036	75	4.28	4.98	5.79	4.19	4.91	5.74	3.96	4.65	5.43
1/1/2037	76	4.26	4.98	5.82	4.14	4.88	5.72	3.88	4.57	5.36
1/1/2038	77	4.23	4.99	5.83	4.08	4.84	5.69	3.79	4.49	5.28
1/1/2039	78	4.21	4.98	5.87	4.04	4.80	5.67	3.71	4.41	5.21
1/1/2040	79	4.20	4.99	5.89	3.98	4.77	5.65	3.61	4.32	5.11
1/1/2041	80	4.17	4.98	5.92	3.94	4.73	5.63	3.52	4.23	5.03
1/1/2042	81	4.15	4.98	5.92	3.89	4.70	5.61	3.41	4.12	4.92
1/1/2043	82	4.15	4.99	5.95	3.84	4.66	5.59	3.31	4.00	4.80
1/1/2044	83	4.12	4.99	5.96	3.80	4.63	5.56	3.19	3.88	4.67
1/1/2045	84	4.10	4.99	6.00	3.75	4.59	5.55	3.07	3.75	4.53
1/1/2046	85	4.09	4.99	6.02	3.71	4.56	5.55	2.94	3.61	4.40
1/1/2047	86	4.06	4.98	6.05	3.66	4.52	5.50	2.80	3.45	4.21
1/1/2048	87	4.05	4.99	6.07	3.63	4.49	5.48	2.66	3.29	4.02
1/1/2049	88	4.04	4.98	6.08	3.59	4.45	5.46	2.52	3.12	3.82
1/1/2050	89	4.03	4.99	6.10	3.54	4.42	5.44	2.37	2.95	3.63
1/1/2051	90	4.02	4.99	6.12	3.51	4.38	5.41	2.22	2.78	3.42
1/1/2052	91	3.99	4.98	6.13	3.47	4.35	5.38	2.05	2.57	3.18
1/1/2053	92	3.98	4.98	6.15	3.42	4.32	5.36	1.88	2.37	2.93
1/1/2054	93	3.95	4.99	6.18	3.38	4.29	5.34	1.71	2.16	2.69
1/1/2055	94	3.94	4.98	6.19	3.34	4.26	5.31	1.54	1.96	2.43
1/1/2056	95	3.92	4.98	6.22	3.30	4.22	5.29	1.37	1.76	2.19
1/1/2057	96	3.90	4.98	6.23	3.26	4.19	5.27	1.19	1.52	1.90
1/1/2058	97	3.88	4.98	6.25	3.22	4.16	5.25	1.01	1.30	1.63
1/1/2059	98	3.87	4.98	6.27	3.18	4.13	5.22	0.84	1.09	1.38
1/1/2060	99	3.85	4.98	6.30	3.14	4.10	5.21	0.70	0.91	1.15
1/1/2061	100	3.83	4.99	6.32	3.11	4.07	5.18	0.57	0.74	0.94
1/1/2062	101	3.82	4.98	6.34	3.07	4.03	5.16	0.43	0.56	0.71
1/1/2063	102	3.80	4.99	6.38	3.03	4.00	5.16	0.32	0.42	0.53
1/1/2064	103	3.77	4.98	6.43	2.99	3.97	5.15	0.23	0.29	0.38
1/1/2065	104	3.75	4.99	6.47	2.95	3.95	5.14	0.16	0.21	0.26
1/1/2066	105	3.72	4.99	6.49	2.90	3.92	5.13	0.10	0.14	0.18
1/1/2067	106	3.69	4.99	6.54	2.86	3.89	5.13	0.067	0.091	0.115
1/1/2068	107	3.68	4.98	6.58	2.83	3.86	5.12	0.042	0.057	0.072
1/1/2069	108	3.64	4.98	6.64	2.78	3.83	5.12	0.025	0.034	0.043
1/1/2070	109	3.63	4.99	6.67	2.74	3.81	5.12	0.014	0.019	0.024
1/1/2071	110	3.59	4.99	6.74	2.69	3.77	5.13	0.007	0.010	0.013

Source: Authors' calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.
"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 4 Male Aged 65 at Beginning of Year 2021, Investment Fund C

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	Age	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile	5th %tile	Mean	95th %tile
1/1/2021	65	7.15	7.15	7.15	6.83	7.03	7.39	6.83	7.03	7.39
1/1/2022	66	6.70	7.15	7.68	6.58	7.04	7.62	6.53	6.98	7.55
1/1/2023	67	6.39	7.15	7.90	6.28	7.06	7.84	6.18	6.93	7.70
1/1/2024	68	6.19	7.15	8.08	6.08	7.07	8.02	5.91	6.87	7.80
1/1/2025	69	6.04	7.15	8.24	5.94	7.08	8.21	5.72	6.81	7.89
1/1/2026	70	5.93	7.15	8.41	5.84	7.09	8.39	5.54	6.74	7.96
1/1/2027	71	5.83	7.15	8.55	5.75	7.11	8.54	5.39	6.65	7.99
1/1/2028	72	5.74	7.15	8.65	5.67	7.12	8.66	5.23	6.56	7.98
1/1/2029	73	5.65	7.15	8.75	5.60	7.13	8.76	5.08	6.47	7.94
1/1/2030	74	5.58	7.15	8.87	5.54	7.14	8.89	4.93	6.36	7.91
1/1/2031	75	5.49	7.15	8.92	5.46	7.16	8.99	4.76	6.24	7.82
1/1/2032	76	5.43	7.15	9.07	5.40	7.17	9.11	4.59	6.10	7.76
1/1/2033	77	5.35	7.15	9.16	5.32	7.18	9.24	4.41	5.95	7.66
1/1/2034	78	5.29	7.15	9.25	5.27	7.20	9.34	4.24	5.79	7.51
1/1/2035	79	5.24	7.15	9.36	5.23	7.21	9.46	4.07	5.61	7.37
1/1/2036	80	5.18	7.15	9.44	5.18	7.22	9.58	3.89	5.43	7.20
1/1/2037	81	5.10	7.15	9.51	5.11	7.23	9.66	3.69	5.22	6.97
1/1/2038	82	5.04	7.15	9.60	5.07	7.25	9.78	3.50	5.00	6.73
1/1/2039	83	5.00	7.15	9.71	5.02	7.26	9.91	3.30	4.77	6.50
1/1/2040	84	4.94	7.15	9.79	4.97	7.27	10.00	3.09	4.52	6.22
1/1/2041	85	4.89	7.15	9.88	4.93	7.28	10.15	2.89	4.27	5.93
1/1/2042	86	4.84	7.15	9.96	4.88	7.30	10.24	2.67	3.98	5.56
1/1/2043	87	4.80	7.15	10.04	4.84	7.31	10.32	2.44	3.69	5.21
1/1/2044	88	4.73	7.15	10.08	4.80	7.32	10.41	2.22	3.39	4.81
1/1/2045	89	4.69	7.15	10.17	4.77	7.34	10.53	2.01	3.09	4.43
1/1/2046	90	4.65	7.15	10.28	4.73	7.35	10.64	1.79	2.79	4.04
1/1/2047	91	4.61	7.16	10.33	4.69	7.37	10.73	1.57	2.45	3.57
1/1/2048	92	4.57	7.16	10.36	4.66	7.38	10.78	1.35	2.14	3.11
1/1/2049	93	4.53	7.15	10.43	4.62	7.39	10.88	1.15	1.83	2.69
1/1/2050	94	4.47	7.16	10.54	4.58	7.41	10.99	0.96	1.55	2.29
1/1/2051	95	4.44	7.16	10.59	4.55	7.42	11.08	0.80	1.29	1.92
1/1/2052	96	4.38	7.15	10.60	4.50	7.43	11.14	0.63	1.03	1.54
1/1/2053	97	4.34	7.15	10.68	4.46	7.44	11.23	0.49	0.81	1.21
1/1/2054	98	4.28	7.16	10.79	4.42	7.46	11.33	0.37	0.62	0.93
1/1/2055	99	4.25	7.15	10.85	4.38	7.48	11.45	0.27	0.47	0.71
1/1/2056	100	4.22	7.16	10.93	4.37	7.49	11.60	0.20	0.34	0.52
1/1/2057	101	4.21	7.16	11.03	4.35	7.50	11.70	0.13	0.23	0.35
1/1/2058	102	4.17	7.15	11.10	4.32	7.52	11.79	0.09	0.14	0.23
1/1/2059	103	4.10	7.16	11.19	4.27	7.54	11.90	0.05	0.09	0.14
1/1/2060	104	4.04	7.15	11.23	4.22	7.54	11.97	0.03	0.06	0.09
1/1/2061	105	4.02	7.16	11.31	4.19	7.56	12.04	0.02	0.03	0.05
1/1/2062	106	3.97	7.16	11.38	4.15	7.56	12.16	0.010	0.019	0.028
1/1/2063	107	3.93	7.17	11.50	4.11	7.57	12.32	0.005	0.010	0.015
1/1/2064	108	3.90	7.17	11.58	4.08	7.58	12.46	0.003	0.005	0.008
1/1/2065	109	3.88	7.15	11.68	4.07	7.60	12.55	0.001	0.003	0.004
1/1/2066	110	3.79	7.16	11.84	4.00	7.64	12.78	0.001	0.001	0.002

Source: Authors' calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.

"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 5 Male Age 65 at Beginning of Year 2021, Investment Fund D

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	**Age**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**
1/1/2021	65	6.81	6.81	6.81	6.83	7.03	7.39	6.83	7.03	7.39
1/1/2022	66	6.51	6.81	7.17	6.67	7.01	7.45	6.61	6.95	7.39
1/1/2023	67	6.35	6.81	7.30	6.47	6.99	7.57	6.35	6.86	7.43
1/1/2024	68	6.23	6.81	7.41	6.32	6.96	7.63	6.15	6.77	7.42
1/1/2025	69	6.13	6.81	7.51	6.20	6.94	7.69	5.97	6.67	7.39
1/1/2026	70	6.06	6.81	7.63	6.11	6.92	7.79	5.80	6.57	7.40
1/1/2027	71	6.00	6.81	7.71	6.02	6.90	7.84	5.64	6.46	7.35
1/1/2028	72	5.92	6.82	7.77	5.94	6.87	7.88	5.48	6.34	7.26
1/1/2029	73	5.86	6.81	7.82	5.85	6.85	7.91	5.32	6.21	7.17
1/1/2030	74	5.81	6.82	7.90	5.79	6.83	7.96	5.16	6.08	7.07
1/1/2031	75	5.76	6.81	7.95	5.71	6.81	7.97	4.99	5.94	6.95
1/1/2032	76	5.73	6.82	8.01	5.67	6.79	8.01	4.82	5.77	6.82
1/1/2033	77	5.67	6.82	8.07	5.59	6.76	8.05	4.64	5.60	6.67
1/1/2034	78	5.63	6.81	8.13	5.53	6.74	8.08	4.45	5.42	6.50
1/1/2035	79	5.58	6.81	8.17	5.46	6.72	8.10	4.25	5.23	6.31
1/1/2036	80	5.55	6.82	8.24	5.42	6.70	8.14	4.07	5.04	6.12
1/1/2037	81	5.51	6.81	8.28	5.37	6.68	8.16	3.87	4.81	5.88
1/1/2038	82	5.47	6.82	8.33	5.30	6.65	8.18	3.66	4.59	5.63
1/1/2039	83	5.44	6.82	8.38	5.25	6.63	8.21	3.45	4.35	5.38
1/1/2040	84	5.40	6.82	8.42	5.19	6.61	8.21	3.23	4.11	5.10
1/1/2041	85	5.35	6.82	8.47	5.14	6.59	8.24	3.02	3.86	4.82
1/1/2042	86	5.32	6.82	8.51	5.07	6.57	8.25	2.78	3.59	4.50
1/1/2043	87	5.28	6.82	8.57	5.03	6.55	8.27	2.55	3.30	4.16
1/1/2044	88	5.25	6.81	8.61	5.00	6.53	8.29	2.31	3.02	3.82
1/1/2045	89	5.22	6.81	8.64	4.93	6.50	8.30	2.08	2.73	3.49
1/1/2046	90	5.18	6.81	8.70	4.88	6.48	8.33	1.86	2.46	3.15
1/1/2047	91	5.15	6.82	8.76	4.84	6.46	8.35	1.62	2.15	2.77
1/1/2048	92	5.12	6.81	8.79	4.80	6.44	8.37	1.39	1.86	2.41
1/1/2049	93	5.09	6.81	8.81	4.75	6.42	8.36	1.18	1.59	2.06
1/1/2050	94	5.06	6.82	8.85	4.72	6.40	8.36	0.99	1.34	1.74
1/1/2051	95	5.05	6.81	8.87	4.68	6.38	8.36	0.82	1.11	1.45
1/1/2052	96	5.03	6.82	8.93	4.66	6.36	8.40	0.65	0.88	1.16
1/1/2053	97	4.99	6.82	8.96	4.60	6.34	8.41	0.50	0.69	0.91
1/1/2054	98	4.95	6.82	9.03	4.55	6.32	8.43	0.38	0.53	0.69
1/1/2055	99	4.91	6.82	9.08	4.50	6.30	8.45	0.28	0.39	0.52
1/1/2056	100	4.87	6.82	9.13	4.45	6.27	8.50	0.20	0.29	0.38
1/1/2057	101	4.84	6.81	9.21	4.40	6.26	8.52	0.14	0.19	0.25
1/1/2058	102	4.80	6.82	9.25	4.36	6.23	8.54	0.09	0.12	0.16
1/1/2059	103	4.75	6.81	9.30	4.30	6.21	8.56	0.05	0.08	0.10
1/1/2060	104	4.74	6.81	9.33	4.27	6.20	8.55	0.03	0.05	0.06
1/1/2061	105	4.70	6.81	9.40	4.23	6.18	8.60	0.02	0.02	0.04
1/1/2062	106	4.64	6.82	9.48	4.17	6.16	8.64	0.010	0.015	0.020
1/1/2063	107	4.57	6.81	9.58	4.09	6.13	8.72	0.005	0.008	0.011
1/1/2064	108	4.54	6.82	9.69	4.05	6.11	8.76	0.003	0.004	0.005
1/1/2065	109	4.50	6.82	9.75	3.99	6.08	8.80	0.001	0.002	0.003
1/1/2066	110	4.44	6.82	9.87	3.93	6.08	8.87	0.001	0.001	0.001

Source: Authors' calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.

"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 6 Male Age 65 at Beginning of Year 2021, Investment Fund E

		Tontine (Flat Rate)			Tontine - (PW Rate)			Programmed Withdrawal		
Date	**Age**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**
1/1/2021	65	6.53	6.53	6.53	6.83	7.03	7.39	6.83	7.03	7.39
1/1/2022	66	6.30	6.53	6.80	6.68	6.98	7.38	6.62	6.92	7.31
1/1/2023	67	6.19	6.52	6.90	6.52	6.93	7.42	6.40	6.80	7.29
1/1/2024	68	6.12	6.53	6.98	6.39	6.87	7.42	6.22	6.68	7.21
1/1/2025	69	6.04	6.52	7.05	6.28	6.82	7.42	6.04	6.56	7.14
1/1/2026	70	5.99	6.53	7.13	6.16	6.77	7.45	5.85	6.43	7.07
1/1/2027	71	5.94	6.53	7.19	6.07	6.72	7.45	5.69	6.29	6.98
1/1/2028	72	5.88	6.52	7.24	5.97	6.67	7.43	5.51	6.15	6.86
1/1/2029	73	5.84	6.52	7.29	5.89	6.62	7.43	5.34	6.00	6.74
1/1/2030	74	5.81	6.53	7.33	5.81	6.57	7.42	5.17	5.85	6.60
1/1/2031	75	5.76	6.52	7.37	5.72	6.52	7.40	4.99	5.69	6.45
1/1/2032	76	5.73	6.52	7.42	5.66	6.47	7.38	4.82	5.51	6.28
1/1/2033	77	5.70	6.52	7.47	5.58	6.42	7.37	4.63	5.32	6.10
1/1/2034	78	5.66	6.53	7.49	5.50	6.37	7.35	4.42	5.13	5.91
1/1/2035	79	5.62	6.52	7.54	5.42	6.32	7.32	4.22	4.92	5.70
1/1/2036	80	5.60	6.53	7.58	5.36	6.28	7.32	4.03	4.72	5.50
1/1/2037	81	5.57	6.52	7.62	5.29	6.23	7.30	3.82	4.49	5.26
1/1/2038	82	5.54	6.52	7.64	5.22	6.18	7.27	3.60	4.26	5.00
1/1/2039	83	5.51	6.52	7.68	5.16	6.14	7.25	3.39	4.03	4.75
1/1/2040	84	5.48	6.52	7.71	5.08	6.09	7.21	3.17	3.79	4.48
1/1/2041	85	5.45	6.52	7.75	5.03	6.04	7.21	2.96	3.55	4.21
1/1/2042	86	5.42	6.53	7.76	4.96	6.00	7.17	2.71	3.27	3.90
1/1/2043	87	5.41	6.52	7.80	4.91	5.95	7.14	2.48	3.01	3.60
1/1/2044	88	5.38	6.53	7.82	4.85	5.91	7.10	2.25	2.73	3.28
1/1/2045	89	5.35	6.53	7.86	4.79	5.86	7.10	2.02	2.47	2.98
1/1/2046	90	5.33	6.53	7.91	4.73	5.82	7.10	1.80	2.21	2.68
1/1/2047	91	5.30	6.53	7.92	4.67	5.77	7.04	1.56	1.92	2.34
1/1/2048	92	5.29	6.53	7.96	4.63	5.73	7.02	1.34	1.66	2.02
1/1/2049	93	5.27	6.52	7.99	4.57	5.69	7.01	1.14	1.41	1.72
1/1/2050	94	5.25	6.53	8.01	4.51	5.64	6.97	0.95	1.18	1.45
1/1/2051	95	5.23	6.52	8.04	4.45	5.60	6.94	0.78	0.98	1.20
1/1/2052	96	5.20	6.52	8.07	4.41	5.56	6.92	0.62	0.77	0.96
1/1/2053	97	5.16	6.52	8.11	4.33	5.52	6.89	0.47	0.60	0.74
1/1/2054	98	5.13	6.53	8.14	4.28	5.48	6.87	0.36	0.45	0.56
1/1/2055	99	5.10	6.52	8.18	4.22	5.44	6.85	0.27	0.34	0.42
1/1/2056	100	5.06	6.53	8.20	4.16	5.39	6.83	0.19	0.24	0.31
1/1/2057	101	5.03	6.52	8.26	4.11	5.35	6.81	0.13	0.16	0.20
1/1/2058	102	4.99	6.53	8.31	4.04	5.31	6.81	0.08	0.11	0.13
1/1/2059	103	4.95	6.53	8.34	3.99	5.27	6.80	0.05	0.06	0.08
1/1/2060	104	4.90	6.53	8.39	3.92	5.24	6.77	0.03	0.04	0.05
1/1/2061	105	4.86	6.52	8.46	3.86	5.19	6.77	0.02	0.02	0.03
1/1/2062	106	4.83	6.52	8.55	3.80	5.15	6.79	0.009	0.013	0.016
1/1/2063	107	4.78	6.53	8.64	3.73	5.12	6.81	0.005	0.007	0.008
1/1/2064	108	4.73	6.52	8.75	3.67	5.08	6.83	0.003	0.003	0.004
1/1/2065	109	4.68	6.54	8.83	3.60	5.05	6.85	0.001	0.002	0.002
1/1/2066	110	4.61	6.54	8.86	3.53	5.00	6.83	0.001	0.001	0.001

Source: Authors' calculations

Notes: "Tontine (Flat Rate)" refers to a tontine designed to deliver uniform UF payouts, using the net investment rate of return as the discount rate in its payout formula.

"Tontine (PW Rate)" refers to a tontine that uses the "programmed withdrawal rate" as its discount rate.

Table A 7 Male Aged 65 at Beginning of Year 2021, Investment Fund D, Combined Strategy with allocation to both Programmed Withdrawals and a Tontine

		50% Programmed Withdrawal / 50% Tontine			20% Programmed Withdrawal / 80% Tontine		
Date	**Age**	**5th %tile**	**Mean**	**95th %tile**	**5th %tile**	**Mean**	**95th %tile**
1/1/2021	65	6.83	7.03	7.39	6.83	7.03	7.39
1/1/2022	66	6.64	6.98	7.42	6.66	7.00	7.44
1/1/2023	67	6.41	6.93	7.50	6.44	6.96	7.54
1/1/2024	68	6.23	6.87	7.52	6.29	6.92	7.59
1/1/2025	69	6.09	6.81	7.54	6.16	6.89	7.63
1/1/2026	70	5.96	6.75	7.59	6.05	6.85	7.71
1/1/2027	71	5.83	6.68	7.60	5.94	6.81	7.74
1/1/2028	72	5.71	6.61	7.57	5.84	6.76	7.75
1/1/2029	73	5.58	6.53	7.54	5.75	6.72	7.77
1/1/2030	74	5.47	6.46	7.52	5.66	6.68	7.78
1/1/2031	75	5.35	6.38	7.46	5.57	6.64	7.76
1/1/2032	76	5.24	6.28	7.41	5.50	6.59	7.77
1/1/2033	77	5.11	6.18	7.36	5.40	6.53	7.77
1/1/2034	78	4.99	6.08	7.29	5.31	6.48	7.76
1/1/2035	79	4.85	5.98	7.20	5.21	6.42	7.74
1/1/2036	80	4.75	5.87	7.13	5.15	6.37	7.73
1/1/2037	81	4.62	5.75	7.02	5.07	6.31	7.71
1/1/2038	82	4.48	5.62	6.91	4.97	6.24	7.67
1/1/2039	83	4.35	5.49	6.79	4.89	6.17	7.64
1/1/2040	84	4.21	5.36	6.65	4.80	6.11	7.59
1/1/2041	85	4.08	5.23	6.53	4.72	6.04	7.56
1/1/2042	86	3.93	5.08	6.38	4.62	5.97	7.50
1/1/2043	87	3.79	4.93	6.22	4.53	5.90	7.45
1/1/2044	88	3.66	4.78	6.06	4.46	5.83	7.40
1/1/2045	89	3.51	4.62	5.89	4.36	5.75	7.34
1/1/2046	90	3.37	4.47	5.74	4.27	5.68	7.30
1/1/2047	91	3.23	4.31	5.56	4.20	5.60	7.24
1/1/2048	92	3.10	4.15	5.39	4.12	5.52	7.18
1/1/2049	93	2.97	4.01	5.21	4.04	5.45	7.10
1/1/2050	94	2.86	3.87	5.05	3.98	5.39	7.04
1/1/2051	95	2.75	3.75	4.91	3.91	5.33	6.98
1/1/2052	96	2.65	3.62	4.78	3.86	5.26	6.95
1/1/2053	97	2.55	3.52	4.66	3.78	5.21	6.91
1/1/2054	98	2.46	3.43	4.56	3.71	5.16	6.89
1/1/2055	99	2.39	3.35	4.48	3.65	5.12	6.86
1/1/2056	100	2.33	3.28	4.44	3.60	5.07	6.87
1/1/2057	101	2.27	3.23	4.39	3.55	5.05	6.87
1/1/2058	102	2.22	3.18	4.35	3.51	5.01	6.86
1/1/2059	103	2.18	3.15	4.33	3.45	4.98	6.87
1/1/2060	104	2.15	3.13	4.31	3.42	4.97	6.86
1/1/2061	105	2.12	3.10	4.32	3.39	4.95	6.89
1/1/2062	106	2.09	3.09	4.33	3.34	4.93	6.92
1/1/2063	107	2.05	3.07	4.36	3.27	4.91	6.97
1/1/2064	108	2.02	3.06	4.38	3.24	4.89	7.01
1/1/2065	109	2.00	3.04	4.40	3.20	4.86	7.04
1/1/2066	110	1.97	3.04	4.44	3.15	4.86	7.10

Source: Authors' calculations