

TWO-STEPS MIXED PENSION SYSTEM

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Abstract

The change in economic and sociodemographic reality, characterized by a continuous increase in longevity, the consequences of the economic crisis as well as the lack of adequate adjustments of the Social Security retirement pension systems everywhere, entail risks for workers and the Social Security itself. Many reforms of public pension systems have been carried out in recent years, based on modifying system parameters and structural changes. Some reforms aim at increase capitalization in the determination of the final pension through a life annuity to complement the public retirement pension as a second retirement income.

Against the background of the change of agents' behaviors throughout the life cycle and the presence of an adverse selection problem in the annuities market, we describe in this paper a 'two-steps mixed pension system' that tries to solve the pressure that increasing longevity is putting on pension schemes to provide adequate and sustainable pensions for all.

In our two-steps mixed system, when workers reach their ordinary retirement age they receive a 'term annuity' generated by their previous capitalized savings to be replaced by a Social Security defined contribution 'pure life annuity' when the so-called 'grand age' is reached. The analysis is carried out from an individual perspective, through the Internal Rate of Return that workers will receive since ordinary retirement in both schemes compared with the one they would get with the same contributions in the current situation. We also analyze some possible transition strategies to the new system.

Keywords: *individual analysis, grand age, two-steps mixed system, term annuity, JEL codes: G00; G22; G29*

1. Introduction

The objective of Social Security can be defined as protecting workers against old age and related risks, which they cannot cope with individually. The conventional mixed pension systems are based on a two-pillar structure with a first pillar being the conventional SS, pay-as-you-go scheme that provides a public retirement pension which is complemented with a life annuity, generated by a fully funded, employers' sponsored scheme. Both benefits are received during the entire retirement period in a simultaneous and complementary way. We will name this type of conventional complementary system "standard system" in our analysis.

Thus, many countries have included the complementarity of public and private pensions in their pension system, seeking that the income Social Security grants to pensioners, coming from the pay-as-you-go system, be supplemented with income generated by a private defined contribution funded system (Herce y Galdeano et al, 2017).

For income security reasons, the income which contemporarily complements the public system should be a life annuity similar in amount to Social Security benefits (Galdeano y Herce et al, 2018). This obliges the providers of the product and/or the benefit holders to be strongly exposed to the longevity risk, since as life expectancy increases regularly, the value of the periodic income will be reduced, given the premium paid ex ante by the beneficiary. It is therefore necessary to make a relevant saving effort, a careful planning of such effort and a sound risk assessment to adequately complement for life, the equally life-long provision of the Social Security, from retirement age to death.

Mitchell et al. (1999) show that life annuities in the United States are between 15% and 25% lower than those obtained when using overall population mortality tables. Finkelstein and Poterba (2000) also show that life annuities in the UK, which are taken out by 65-year-old men, are between 10% and 15% lower than those that would be obtained using ordinary overall population mortality tables. Note that the problem that occurs when adverse selection and over-weighted mortality combine in the case of life annuities is significantly reduced in the case of term annuities.

We can say that this "standard system" suffers several problems. Firstly, annuities entail a severe problem of adverse selection (see Blake, D; Dowd, K y Cairns, A., (2008), Domínguez, I; Herce, J. A. y del Olmo, F., (2018), Herce, J. A. y del Olmo, F., (2013), Whitehouse, E.R and Zaidi, A., (2008), Morales, M y Larraín, G. (2017)) and thus they become unduly expensive. On the other hand, the kind of longevity insurance, offered by the Social Security, is unsustainable

as the system has barely changed the retirement age since it was created, when life expectancy at birth was around 40 years, and around 10 years at age 65.

In this paper we present what we call a “two-step” mixed pension system in which contributions are paid as in the standard system but benefits are received by steps: a DC term annuity from retirement until grand age and a Social Security NDC life annuity afterwards.

This two-steps pension system solves two major problems: the adverse selection problem in private pensions and the pension adequacy problem in public pensions. Social Security, in particular, will continue to be PAYG but will have to pay better adjusted benefits for a shorter period thus managing more efficiently longevity risk.

As said, workers’ and employers’ contributions are assumed to be the same as in the standard model, while benefits since retirement are restricted to be at least as good as in the standard model. Our quest is thus for those conditions under which retirees improve under the two-steps system with respect to the standard system.

The pension model we have baptized as a ‘two-steps’ mixed pension system is based on the need to adapt pension schemes (public and private) to the longevity-induced behaviors of individuals, and responds to those financial needs that workers cannot cope with individually faced with ever increasing longevity.

In this paper, firstly, we define the two-steps model, in order to make a comparison in mathematical terms in the second section, from the individual point of view, of the standard system and the two-steps one. In the third section we perform a graphical comparison to illustrate the case and in the fourth section we analyze this empirical relationship concluding the paper with a summary discussion of the main results and some ideas about the transition from the standard mixed system to the two-steps mixed pension system.

2. Description of the ‘two steps mixed pension system’.

Pension systems are structured in two main flows: during the contribution phase individuals contribute a part of their salary until the retirement age x_j . After which, during the retirement stage, they perceive a total lifetime pension from the system, generated since the retirement moment until the death event. This general scheme runs everywhere, regardless of the system, either a pure public pay-as-you-go (PAYG) system, a funded system or a mixed system.

Taking into account the demographic risks and the fact that most of the reforms implemented seek to make the system sustainable, mainly through reduction in the amount of pensions, this implies a problem of adequacy of benefits and, therefore, a strong risk of impoverishment for retired people.

The two-steps system that we propose, considers a period of active life, from the beginning of the working life to the moment of retirement (chosen with sufficient flexibility); and a retirement period that is divided into two steps: one from ordinary retirement age to the so-called 'grand age' (the 'old age', that historical Social Security systems fixed at inception) and another one from that 'grand age' until the individual's death.

During the active stage, contributions will be made both to a DC funded scheme and to a reinvented NDC Social Security scheme. It is very important to indicate that, in our analysis, these levels of contributions will be the same in the two-steps system than in the standard system. Thus, annual contribution made by individuals is divided into two parts:

- A part of the contribution generates a term actuarial income that the worker will receive from the moment he or she retires to the so-called grand age.
- The other part of the contribution allows to finance pensions generated by a Social Security scheme based on NDC accounts, which consists of a life annuity which pays a monthly income from the mentioned grand age until the individual's death event.

Therefore, when an individual retires, in the two-step mixed system he or she will receive a term income, based on capitalization rules, from the retirement age he/she has freely chosen until the grand age, after which he or she will receive a retirement pension financed through a NDC account PAYG method, until the moment of death.

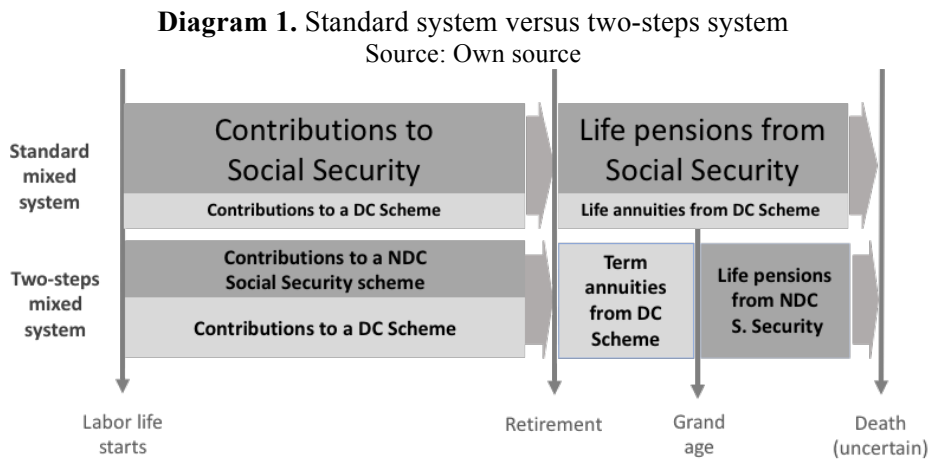
This ordering of contribution and benefit flows solves several crucial aspects of the pension arithmetics and workers' behavior, namely:

- Firstly, during the first retirement step, between the ordinary retirement age and the grand age, a term annuity or retirement income based on private capitalization is paid. Due to this reason, each worker, duly informed and according to his or her long-term savings achieved, will be in the best conditions, to choose the ordinary age in which he or she want to retire, or even making both statuses compatible as Social Security permits this often. Temporary incomes will ideally be insured through actuarial products and, due to the fact that they do not cover a lifetime period (but limited up to the grand age), longevity risk gets effectively capped and efficiently covered until grand age. The highest efficiency derives by the fact that term annuities are not designed with the aim of covering ordinary longevity risk, which increases enormously after grand age and which is, clearly, more expensive to hedge.
- Secondly, during the second step, which occurs between the grand age and the death event, retired people perceive a retirement pension from Social Security.

ty, which, by definition, is structured by an annuity calculated with strict actuarial criteria. As it is said above, this pension is financed through the pay-as-you-go method, so formally the system of this second step is equivalent to a public system with notional defined contribution individual accounts (an NDC scheme).

This way of presenting the sequence of contributions and benefits combined in time, is what allows us to talk about the ‘reinvention’ of Social Security (Herc and del Olmo, 2013).

The following diagram compares the standard model with the two-steps mixed system that we propose. Diagram 1 shows the contributions and benefits blocks for each scheme in a crude illustration of the actual numbers we have assumed for the standard system. It also shows the rearrangements that contributions and benefits must undergo in order to implement the two-steps idea without net losses for individuals.



Source: authors' elaboration

Fig. 1 shows the contributions that a typical individual would make to and the retirement income that he or she would receive both in the standard mixed model and in the two-steps mixed model. For this illustration a contribution rate of 15% of wages to the SS scheme (Notional Defined Contribution variety or NDC, in what follows) and a 5% rate to the funded scheme have been assumed. Initial working age is 25 years, the contributions career is not interrupted, the retirement age is 65 years and the grand age has been set at 75 years, with a maximum survival age of 100 years. PEM-2000 mortality tables have been used. A nominal interest rate of 3%, a notional rate of 1% and a wage revaluation of 1.5% per year have also been assumed for the computations.

This way of presenting the sequence of contributions and benefits combined in time, is what allows us to talk about the ‘reinvention’ of SS [4]. The two-steps

mixed system solves the vital algebra's inconsistency problem mentioned above since the pension funded by PAYG is paid from grand age until death; which means that the number of years the scheme must pay benefits to any retiree is far lower than under the current system. On the other hand, the fully funded scheme term annuities, payable since retirement until grand age, are marketed more efficiently than a life annuity, since the former are significantly cheaper on unit terms as they must not be insured against significant longevity but in a small part of this risk. This product is thus more interesting both for workers and for insurance companies, and could eventually lead to a significant expansion of annuities markets everywhere in a more natural way.

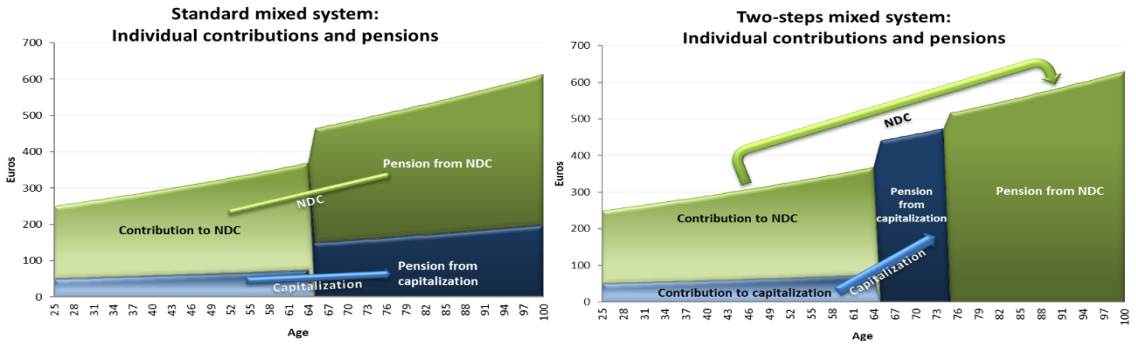


Fig. 1. (Illustrative figures). Source: Own computations

Fig. 2 shows the system's flows of income and expenses for the base year 2017 and for 2050. For each of the panels, on the left side, the standard mixed system's income and expenses are shown where those for the two-steps mixed system are shown in the right side of the panels. Income flows are the same for both systems as contribution rates are the same but, concerning expenditure flows, the two-steps system is cheaper to run, since retirees do not receive SS income until they are 75 years old (the grand age), although system's expenses from 75 years on are higher since the retirement income generated in the NDC scheme are higher than in the standard PAYG one.

The graph also shows how the financial problems of the standard PAYG scheme are exacerbated in 2050, as pensions expenses are higher and income is lower, due respectively, to increased longevity and a reduced workforce. The NDC scheme in the two-steps mixed system generates savings since inception year and increasingly so until 2050, that are accumulated over time.

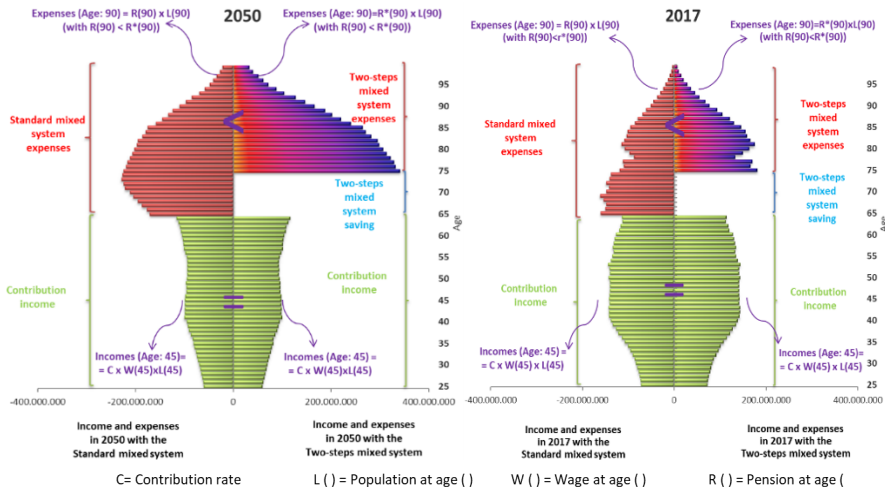


Fig. 2. Note: The fully funded scheme is not shown in this graph as for both systems it is sustainable. Source: Own computations

3. Comparative analysis of the standard system and the two-steps system in terms of individual financial solvency.

This section shows a comparative analysis, in terms of financial-actuarial solvency, between the standard system and the two-steps one. There are two basic periods: capital accumulation, which goes from X_0 (age of access to a first job) to X_r (retirement age) and the period of decumulation, which takes place from X_r to ω .

Some remarks are in order:

- 1) Funded capital $C_F(X_r)$ can be supplemented with other wealth sources, a house, for instance. The financial strategy is decided by individuals depending on their previous saving decisions.
- 2) In the two capital computations, life tables haven't been used, these will be used for computing benefits after retirement age X_r
- 3) An overlapping generations model with four periods, where steady state is defined as:

$$X_0 = X_{r-1} \quad , \quad X_g = X_{r+1} \quad \text{and} \quad w = X_{g+1}$$

3.1. Accumulation period:

Through a mixed system structured by a notional accounts pillar (NDC) and by a funded pillar (FDC), being the accumulation period the same both in the standard system and in the two-steps system. Both systems share the following information:

π_N = Contribution rate in NDC pillar.

π_F = Contribution rate in Funding pillar.

$C_F^{X_r}$ = Funded capital obtained through the NDC pillar at retirement age X_r

$C_N^{X_r}$ = Notional capital obtained through the FDC at retirement age X_r

These capitals are given by:

a.- Notional Part (NDC):

$$C_N^{X_r} = \sum_{X=X_0}^{X_r-1} \pi_N \cdot W(X) \cdot \left(\prod_{y=x}^{X_r-1} (1 + r_y) \right) \quad [1]$$

Where:

$W(X)$ = Salary at age X

r_y = Notional rate applied at age y (between y and $y + 1$)

For initial wage $W(X_0) = 1$ and being r the notional rate of the NDC we have:

$$C_N^{X_r} = \pi_N [(1 + r)^2 + (1 + \beta) \cdot (1 + k) \cdot (1 + r)] \quad [2]$$

b.- Funded part (FDC):

$$C_F^{X_r} = \sum_{X=X_0}^{X_r-1} \pi_F \cdot W(X) \cdot \left(\prod_{y=x}^{X_r-1} (1 + f_y) \right) \quad [3]$$

Where:

$W(X)$ = Salary at age X

f_y = Financial return at age y (between y and $y + 1$)

For the canonical choice (steady state) $(1 + r) = (1 + d) \cdot (1 + \beta)$, being i the technical interest rate of the insurer

$$C_F^{X_r} = \pi_F [(1 + i)^2 + (1 + \beta) \cdot (1 + k) \cdot (1 + i)] \quad [4]$$

3.1. Decumulation period

In the standard system, each of the two capital amounts is converted into a pure life annuity, following the NDC technique or the funding “insurance” technique. In the two-steps system, the funded capital is converted into a term annuity,

which will provide retirement income between the retirement age X_r and the grand age X_g being the notional capital deferred until age X_g and then converted into a life annuity, and a proper Social Security pension, however, between grand age X_g and death.

everal parameters have to be taken into account:

- d = rate of demographic increase;
- β = constant inflation rate;
- k = baremic increase of salary
- i = technical interest rate of the insurer

In relation with mortality, few other hypothesis are needed:

- 1) We have not assumed mortality before retirement age.
- 2) We have assumed that, after retirement, ${}_1p_{X_r}$ is the probability to survive until grand age being alive at retirement age.
- 3) We have assumed that the mortality rate used by the insurer is given by ${}_1p_{X_r}^* = {}_1p_{X_r} \cdot (1 + \alpha)$; with $\alpha > 0$ and α being the safety coefficient. Loading applied by the insurer (commission) on life annuity is g.

The price of the life annuity (indexed) is given by:

$$a_{X_r}^F = \frac{1}{1 - g} \cdot \left[1 + \frac{p(1 + \alpha)(1 + \beta)}{(1 + i)} \right]$$

3.2.1. In the standard system, total retirement income is composed by two different life pensions, one coming from funded scheme and the other coming from the NDC Social Security scheme.

3.2.1.a. Funded part in the standard system

First pension benefit at retirement age is:

$$R_F^{X_r} = \frac{C_F^{X_r}}{a_{X_r}^F}; \quad R_F^{X_r} = \pi_F \cdot \frac{[(1+i)^2 + (1+\beta) \cdot (1+k) \cdot (1+i)]}{\frac{1}{1-g} \cdot \left[1 + \frac{p(1+\alpha)(1+\beta)}{(1+i)} \right]} \quad [5]$$

The unfunded pension is computed with the following espression:

$$R_F^{X_r+1} = R_F^{X_r} \cdot (1 + \beta) \quad [6]$$

3.2.1.b. The notional part in the standard system

$$R_N^{X_r} = \pi_N \cdot \frac{[(1+r)^2 + (1+\beta) \cdot (1+k) \cdot (1+r)]}{1+p \frac{1+\beta}{1+r}} \quad [7]$$

The unfunded pension is now computed with the following expression:

$$R_N^{X_{r+1}} = R_N^{X_r} \cdot (1 + \beta) \quad [8]$$

3.2.2. In the two-steps system there are two also pension flows, one coming from the funded pillar and received from X_r to X_g and another one coming from the NDC pillar that is received from X_g to death.

3.2.2.a. Founded part (at retirement age) in the two-step system

$$R_F^{*X_r} = \pi_F \cdot \frac{[(1+i)^2 + (1+\beta) \cdot (1+k) \cdot (1+i)]}{\frac{1}{1-g}} \quad [9]$$

3.2.2.b. Notional part (at grand age) in the two-steps system where the notional capital at grand age becomes

$$\begin{aligned} C_N^{X_G} &= C_N^{X_r} \cdot \frac{1+r}{p} \\ &= \pi_N \cdot \frac{1+r}{p} \cdot [(1+r)^2 + (1+\beta) \cdot (1+k) \cdot (1+r)] \\ R_N^{*X_G} &= \pi_N \cdot \frac{1+r}{p} \cdot [(1+r)^2 + (1+\beta) \cdot (1+k) \cdot (1+r)] \\ R_F^{*X_G} &= 0 \end{aligned} \quad [10]$$

Table 1 shows the pension computations in each system.

Table 1. Pensions in each system in one overlapping generation model with four periods, and with steady state.

Age	Pensions in the classical complementary system	Pensions in the two steps system
X_r	$R_F^{X_r} + R_N^{X_r} = R^{X_r}$	$R_F^{*X_r}$
$X_G = X_{r+1}$	$(R_F^{X_r} + R_N^{X_r})(1+\beta) = R^{X_{r+1}}$	$R_N^{*X_G}$

Source: Own computations

We now compare both systems, from an individual point of view, though the Internal Rate Return (IRR).

In the standard system, we have:

$$C_N + C_F = R^{X_r} + \frac{P}{1+IRR} \cdot R^{X_{r+1}} \quad [11]$$

And in the two-step system we have:

$$C_N + C_F = R^{*X_r} + \frac{P}{1+IRR^*} \cdot R^{*X_{r+1}} \quad [12]$$

And the main question is how IRR compare in both systems. So we will try to find conditions such that it could be proved that two-steps system IRR* is igher than standard IRR.

Assuming capital amounts at retirement to be known, it implies that it is not necessary to work with the contribution rate and we can concentrate in what happens only after retirement provided capitals in both situations are the same.

Firstly, sensibility analysis is the best way to compare the interest rate i and the notional rate r . So we consider the following cases.

Case 1. Without management fees to be paid during funding and using the same as table to notional and the funding and with the interest rate i different to notional rate r .

$$IRR^* > IRR \quad \text{if and only if:} \quad \frac{p \cdot R_N^{*X_{r+1}}}{C - C_F} > \frac{p \cdot R^{X_r(1+\beta)}}{C - R^{X_r}}$$

$$\text{with } C = C_N + C_F; \quad R_N^{*X_{r+1}} (C - R^{X_r}) > R^{X_r} (1 + \beta) (C - C_F)$$

with

$$\text{a) } (C - C_F) = C_N$$

$$\text{b) } R_N^{*X_{r+1}} = C_N \cdot \frac{1+r}{p}$$

$$\text{c) } R^{X_r} = R_F^{X_r} + R_N^{X_r} = \frac{C_F}{(1+p) \frac{1+\beta}{1+i}} + \frac{C_N}{(1+p) \frac{1+\beta}{1+r}}$$

It is obtained:

$$C_N \frac{1+r}{p} \left[C_F + C_N - \frac{C_F}{1+p \frac{1+\beta}{1+i}} - \frac{C_N}{1+p \frac{1+\beta}{1+r}} \right] = \left(\frac{C_F}{1+p \frac{1+\beta}{1+i}} + \frac{C_N}{1+p \frac{1+\beta}{1+r}} \right) (1+p) C_N$$

Or,

$$\frac{1+r}{p} \left[C_F \frac{p \frac{1+\beta}{1+i}}{1+p \frac{1+\beta}{1+i}} + C_N \frac{p \frac{1+\beta}{1+r}}{1+p \frac{1+\beta}{1+r}} \right] = \left(\frac{C_F}{1+p \frac{1+\beta}{1+i}} + \frac{C_N}{1+p \frac{1+\beta}{1+r}} \right) (1 + \beta) \quad [13]$$

Finally

$$\frac{C_F \frac{1+r}{1+i}}{1+p \frac{1+\beta}{1+i}} + \frac{C_N}{1+p \frac{1+\beta}{1+i}} = \frac{C_F}{1+p \frac{1+\beta}{1+i}} + \frac{C_N}{1+p \frac{1+\beta}{1+r}} \quad [14]$$

Getting the final conventional condition that

$$\frac{1+r}{1+i} > 1 \quad [15]$$

for the NDC pension to be larger than the DC one.

Result 1. It can be appreciated that the relation between the IRR will be independent of the proportion C_F/C_N and then, without fees paid and using the same life table for notional and funding computations, in terms of IRR for both systems:

If $r > i$ then two-steps system is better in term of individual IRR

If $i > r$ then standard system is better in term of individual IRR

Case 2. Working with the general case with fees paid in the funding scheme and using the different like life table to notional and the funding and with the interest rate i different to notional rate r .

$$i \neq r ; p^* \neq p ; g \neq 0$$

It will be used the same relation that in Case 1 above $R_N^{*X_r+1} (C - R^{X_r}) > R^{X_r} (1 + \beta) (C - C_F)$ and the only difference now is the way to compute R^{X_r} :

$$R^{X_r} = R_F^{X_r} + R_N^{X_r} = \frac{C_F(1-g)}{1+p^*\frac{1+\beta}{1+i}} + \frac{C_N}{1+p\frac{1+\beta}{1+r}} \quad [16]$$

Getting now

$$C_N \frac{1+r}{p} \left[C_F + C_N - \frac{C_F}{1+p^*\frac{1+\beta}{1+i}} - \frac{C_N}{1+p\frac{1+\beta}{1+r}} \right] = \left(\frac{C_F}{1+p^*\frac{1+\beta}{1+i}} + \frac{C_N}{1+p\frac{1+\beta}{1+r}} \right) (1+p) C_N$$

$$\begin{aligned} & \frac{1+r}{p} \left[C_F \cdot \frac{p^* \frac{1+\beta}{1+i} + g}{1+p^*\frac{1+\beta}{1+i}} + C_N \cdot \frac{p \frac{1+\beta}{1+r}}{1+p\frac{1+\beta}{1+r}} \right] \\ &= \left(\frac{C_F(1+g)}{1+p^*\frac{1+\beta}{1+i}} + \frac{C_N}{1+p\frac{1+\beta}{1+r}} \right) (1+\beta) \end{aligned}$$

Or

$$\frac{\left(C_F \frac{p^*}{p} \cdot \frac{1+r}{1+i} (1+\beta) + g \cdot \frac{1+r}{p} \right)}{\left(1+p^*\frac{1+\beta}{1+i} \right)} = \frac{C_F (1+g)(1+\beta)}{1+p^*\frac{1+\beta}{1+i}}$$

Finally

$$\frac{p^*}{p} \cdot \frac{1+r}{1+i} (1 + \beta) + g \cdot \frac{1+r}{p} > (1 + g)(1 + \beta) \quad [17]$$

Or

$$\frac{p^*}{p} \cdot \frac{1+r}{1+i} > 1 - g - \frac{g}{p} \cdot \frac{1+r}{1+\beta} \quad [18]$$

If $\frac{1+r}{1+i} > \frac{p^*}{p} \left(1 - g - \frac{g}{p} \cdot \frac{1+r}{1+\beta}\right)$ then, the two-steps system is better.

But if $\frac{1+r}{1+i} < \frac{p^*}{p} \left(1 - g - \frac{g}{p} \cdot \frac{1+r}{1+\beta}\right)$ then, the standard system is better.

This can be written as a condition on the Rate of Return as follows:

$$\text{If } 1 + i > \frac{(1+r) \frac{p^*}{p}}{1 - g - \frac{g}{p} \cdot \frac{1+r}{1+\beta}}$$

Then, IRR in the standard system $>$ IRR in the two-steps one

In particular, if $p^* > p$ and $g > 0$ then $\partial_1 = \frac{p^*}{p} > 1$ and $\partial_2 = \frac{1}{1 - g - \frac{g}{p} \cdot \frac{1+r}{1+\beta}} > 1$

So if $1 + i > (1 + r) \cdot \partial_1 \cdot \partial_2$, then, IRR of the standard system $>$ IRR of the two-steps one

Result 2. The “ i ” must be very large to compensate for extra longevity in the life table (to cope with adverse selection) and for the fees paid and an important conclusion is that the condition is independent of the relative level of the two capitals under both systems, C_N and C_F .

On the other hand, and although the results on IRRs are independent of the relative level of the capitals, it is considered that it will be desirable that the two capitals at retirement (notional capital, C_N and funding capital C_F) are roughly equivalent in order to assure some continuity in the levels of retirement income in the transition from the funded income to the NDC income under the two-steps system.

The condition is that:

$$R^{*Xg} = R^{*Xg-1}(1 + \beta) \quad [19]$$

And in our 4 period model, $Xg = Xr + 1$

And $R^{*Xr} = C_F$; $R^{*Xr+1} = C_N \frac{1+r}{p}$; The condition $R^{*Xr+1} = R^{*Xr}(1 + \beta)$; becomes

$$C_N \frac{1+r}{p} = C_F(1 + \beta) \text{ or } \frac{C_F}{C_N} = \left(\frac{1+r}{1+\beta}\right) \left(\frac{1}{p}\right) \quad [21]$$

Result 3. In the two-steps system, if it is desirable to maintain continuity in pensions, it is necessary for the ratio set between the capital amount obtained by the funded pillar and the notional accounts pillar to be a function of the notional rate, the growth of wages and the probability of survival.

In the following section, through an empirical analysis, the results obtained in this section, from a theoretical perspective, are illustrated.

4. Empirical analysis.

In this section, the conclusions obtained above will be matched with two examples and the follows graphics show like IRR change in relation with the changes in another parameters. The basic parameters are shown in Table 2.

Table 2. Basic parameters considered

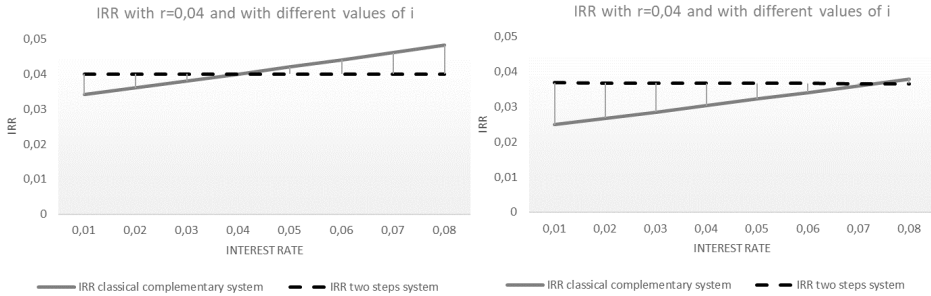
		W(Xo)	1		
Initial salary		B	0.02		
Constant inflation rate		R	0.01		
Increase of salary		C	1000		
Total Capital				Case 1	Case 2
Actuarial parameters	Increase of probability	Alfa	0	0.021	
	Probability company	p*	0.95	0.97	
	Demographic increase	D	0.08		
	Probability real	p	0.95		
NDC parameters	Contribution rate of NDC	μ_n	0.8		
	Notional rate	r	0.04		
DC parameters	Contribution rate of funding	μ_f	0.2		
	Fees for the life annuity	g	0	0.0125	

Source: Own

Case 1. Without commission in the funding and using the same like table to notional and the funding and with the interest rate i different to notional rate r .

Case 2. It is interesting to work with the general case with commission in the funding and using the different like table to notional and the funding and with the interest rate i different to notional rate r .

Case 1 is shown in left panel of Graph 1 below. We use different values of i to show the effect on IRR in the standard system and in the two-step system. The IRR of the standard system is higher than the IRR in the two-steps system only when the interest rate exceeds the value of the notional rate. This extreme case however is no realistic.



Graph 1. IRR with $r = 0.04$ and with different values of i

Source: Own

Case 2 is shown in right panel of Graph 1 above. Using different interest rate values, and then verifying that even in cases where the interest rate is higher than the notional rate, the IRR of the two-steps system can be higher than the IRR of the standard system, and only in cases in which the interest rate is more than twice the notional rate, the IRR of the standard system is higher.

From an individual perspective, thus, the IRR is better, under the assumptions adopted, for the two-steps system than for the standard system. This improvement is due to the superior profitability of a term annuity over that of the pure life annuity.

Now some other numerical examples will be presented, based on the the parameters assumed in Table 2 above, where capital amounts are such as to ensure continuity of benefits in the two-step system.

It is shown in left panel of Graph 2 below how the amounts of pensions are reduced as survival probability increases.

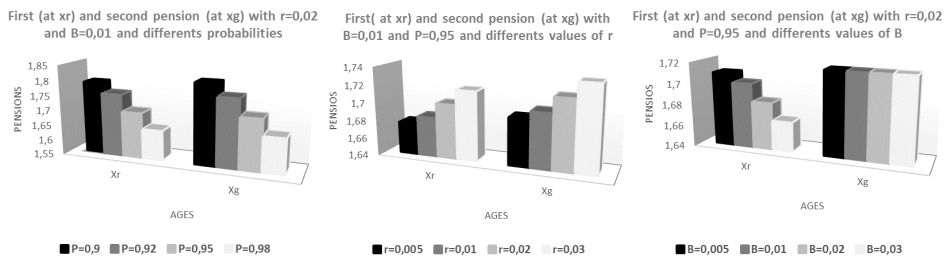


Fig. 3. Sensibility analysis

Source: Own

Central panel of Graph 3 above shows how the amounts of pensions increase as the notional rate increases, for given values of survival probability and inflation rate.

Finally, right panel of Graph 3 above shows how pensions are reduced in value as the inflation rate increases, with given survival probability and notional rate. To be noted in this graph how the amount of pension at x_g is constant.

5. Concluding remarks.

What we have termed the ‘standard mixed system’ or, in short, the ‘standard system’, has two pension schemes. A Social Security, PAYG albeit NDC, scheme and a fully funded (compulsory) DC scheme. In a way, this set up represents well many of advanced countries’ pension arrangements. Many other countries, where Social Security DB schemes are prevalent and DC pension arrangements are complementary and voluntary, are however marching towards that kind of total pension set ups through continuous reforms.

All these pension systems, however, are far away from having fully adapted to the increase of life expectancy that all nations have witnessed in last hundred years, just after Social Security was invented in continental Europe.

Our ‘two-steps mixed pension system’, or ‘two-steps system’, proposal tries to put pensions in line with the social, demographic and economic reality of the 21st Century. Its concept is simple, and amounts to a kind of reinvention of Social Security. We still consider ‘ordinary retirement’ around age 65, say, although workers should have large capacity to decide when to retire or whether to retire at all, as long as they are aware of the numbers concerning their savings till that age and how to handle them via term annuities until what we call their ‘grand age’ as we are not contemplating Social Security pensions until precisely that grand age. This is why we call this total pension scenario the ‘two-steps system’.

That is, workers, once retired at ordinary retirement age, count on a term annuity obtained out of their previous savings, real assets or contributions to a funded scheme, for their living, but cannot count on public pensions until they reach their grand age. When a renewed NDC Social Security, will grant them, against their previous social contributions, a life public pension till death.

Grand age, by the way, when defined, for instance (there are several metrics that are relevant here), as that age today at which life expectancy coincides with life expectancy at 65 in 1900, will easily lie, for many advanced countries, around 80! We are not that extreme in this paper, but what historical European Social Security systems did around 1900 was, exactly, this: to protect workers from their grand age until death.

In our view, this arrangement is more effective and more efficient than standard pension practice everywhere. As term annuities are cheaper than life annuities, they suffer from far less adverse selection, they don't need longevity adjustments

that are expensive, and, together with Social Security pensions, induce more productive behavior on workers.

Our numeric results show indeed that the two-steps system has a higher IRR than the standard system for workers and that only under rather exceptional conditions things would be the other way round. The economy would also profit from that because of the larger availability of long term saving and more active mature workers. This, of course, for identical saving efforts during labor life to both systems.

When considering transitions aspects, were we to depart from the standard system towards the two-steps system, it is important to consider that our simulations have been done for identical saving + contribution efforts, equivalent to current ones in advanced societies. So that transitional costs could be relatively small and easy to compensate with transitional benefits.

Other relevant transitional issue concerns who to let move from the standard system to the two-steps system. Many alternatives exist, but they share a dividing age line among current workers. However, a crucial element appears in this scenario. That is the fact that the NDC Social Security life annuity could well be higher than the DB Social Security it would replace.

Lastly, but not precisely least important, it is the issue of where to place the grand age in the time arrow. The grand age is the cornerstone of the two-steps system design and almost everything depends on its choice. To reassure the reader, we aren't saying that the grand age should be set at 80, or latter. In fact, in our computations, this age has been set at 75. Two other things are important about this variable. First, it should be set so that the aggregate balance of the NDC Social Security scheme, that continues to be of a PAYG nature, reaches and keeps a proper balance between its income and expenditure flows over time. Second, and closely related to the previous one, this age should be regularly reassessed to keep proper distance with life expectancy.

On the other hand, the first step DC scheme is sustainable by definition, even if it also should be rearranging some of its parameters with time if term annuities have to be kept adequate.

For the sake of easing transitional issues related to fairness, income adequacy and sustainability of the whole two-steps system, many things could be done at relatively low cost, given the superior efficiency and shorter term effectiveness of this system as compared, for instance, with a deep replacement of sole Social Security systems today with fully funded mixed systems tomorrow. The details concerning these crucial transitional issues are left for further research by the authors.

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