Optimal Social Insurance and Universal Day Care

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Abstract

We investigate whether the possibility of engaging in household child care activities may incentivize parents and grandparents to falsely claim disability benefits. Within a dynamic Mirrleesian framework, where the government seeks to provide social insurance while actual disability shocks are private information to households, we show that the optimal scheme may be implemented via universal day care, combined with non-linear income taxation and asset limits. We calibrate a multi-generational family model, with persistence in privately observed shocks, to key features of the U.S. labor and child care markets, and find that the optimal scheme may lead to sizeable cost savings.

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1 Introduction

Disability insurance and child related tax benefits are important components of government policy in the United States. Social Security disability benefit payments totalled $135 billion and made up 17.5% of Social Security benefits in 2012 (Budget of the U.S. Government, FY2014, Table 13.1). There were 10.1 million recipients of Social Security Disability Insurance (SSDI) benefits and 9.2 million recipients of Social Security Income (SSI) benefits (Social Security Administration, 2013). Child related tax benefits, such as the Earned Income Tax Credit (EITC) and dependent tax exemptions, are also relatively large in scope and magnitude. The Tax Policy Center estimated that families' tax savings from those two programs amounted to $104 billion in 2013 (Maag, 2013). The EITC benefited 26 million working families in 2012 and made an estimated 13.1 million children less poor in 2013 (Center on Budget and Policy Priorities, 2014).

In contrast, U.S. price related child care subsidy programs that are linked to formal child care costs, are relatively small. Families' tax savings from the Child and Dependent Care Tax Credit (CDCTC) totalled $4 billion in 2013 (Maag, 2013) while the budget request for the Child Care and Development Fund (CCDF) was $6 billion to support 1.4 million children in 2015 (Department of Health and Human Services, 2014). Meanwhile, child care subsidies tend to be relatively large in several other developed countries such as Scandinavia (Guner *et al.* , 2014; Havnes & Mogstad, 2011), whereas recent policy debates in Europe expressed a desire to move towards universal day care. In 2002, the European Union stated as policy goal "to provide childcare by 2010 to at least 90% of children between 3 years old and the mandatory school age and at least 33% of children under 3 years of age" so as to encourage labor force participation of mothers (European Council, 2002).

In this study, we present a case for universal day care, that is, a full price subsidy on formal child care costs for all families. Our context is one of optimal social insurance in the spirit of Diamond & Mirrlees (1978) and Golosov & Tsyvinski (2006). In a standard optimal social insurance framework, where disability shocks of an agent are private information, disability benefits cannot be too generous or else, healthy individuals may be tempted to mimic the disabled by not working, and claim the benefits.[1] We add to the standard framework in several ways. First, we consider multi-member households which may have several adult members, such as parents and grandparents, subject to privately observed disability shocks. Second, we consider a multi-choice framework where healthy members may allocate their time (effort) between working on the market and household child care activities. Third, we calibrate our multi-generational family model to match key features of the U.S. labor and child care markets, and quantify the potential cost savings from the optimal policy.

In spite of the multi-dimensional nature of our framework, the model remains very tractable with clear policy implications.[2] We use a recursive formulation with history dependence in privately observed shocks and adapt the threat keeping constraint theoretically proposed by (Fernandes & Phelan, 2000) to account for the absorbing nature of disability shocks[3]. We show that universal day care, in addition to non-linear income taxation and asset limits, implement the constrained optimum, where it is *as though* the government may monitor household child care. The intuition behind our result is as follows: healthy household members who mimic the disabled by not working, may not only claim disability benefits, but also save on formal child care costs by looking after the children themselves. The possibility of engaging in privately observed household child care therefore exacerbates the incentives of healthy members to mimic the disabled. The use of universal day care helps to directly counterbalance private household child care incentives by making such activities unattractive relative to formal child care.

[1] In 1984, congressional reforms redefined disability as the inability to engage in substantial gainful activity due to a physical or psychological impairment. Those include back pain and mental illness which are hard to verify. Among SSDI beneficiaries, 27% received the benefits on the grounds of musculoskeletal diseases and 35% on the grounds of mental disorders in December 2012 (Social Security Administration, 2012). The controlling evidentiary weight is also placed on a disability applicant's own health practitioner, thereby making the disability screening process easier for applicants (Autor & Duggan, 2006; Hu *et al.*, 2001). See Autor *et al.* (2014) for recent evidence on moral hazard in long term disability insurance policies. See also Gruber (2000), Haveman *et al.* (1991), and Maestas *et al.* (2013).

[2] This is partly thanks to the discrete nature of disability (Armstrong & Rochet, 1999) and to the fact that the disabled cannot mimic the healthy.

[3] Social Security pays disability benefits only for long-term total disability. Less than 0.5% of SSDI and SSI beneficiaries leave the disability rolls and return to work. *Source:* 42 U.S.C. 1320b-19, The Public Health and Welfare.

We make our case for universal day care within the context of multi-generational households. The increased prevalence of such households makes it important for policy to take intergenerational linkages into consideration.[4] According to the U.S. Census, approximately 7.5 million (10%) children under age 18 lived with a grandparent in 2010. In 2011, 21.1% of children of pre-primary school age with a working mother benefited from grandparent-provided child care as their primary source of day care, averaging 23 hours per week (Laughlin, 2013). Grandparent provided child care has been found to increase labor supply of mothers (Cardia & Ng, 2003; Compton, 2013; Compton & Pollak, 2013; Maurer-Fazio *et al.* , 2011) while grandchild care needs have been found to influence labor supply of grandparents (Ho, 2013; Marcotte & Wang, 2007). It has also been found in the literature that child care subsidies lead to a substitution from relative care to formal child care (Havnes & Mogstad, 2011; Tekin, 2007), and a possible increase in the labor supply of grandmothers who live with their grandchildren (Ho, 2015). This suggests that day care subsidies may not only lead to welfare gains by increasing the labor supply of parents, but also by increasing the labor supply of grandparents.

In our quantitative exercise, we extend the theoretical model to incorporate demographic heterogeneity across households in terms of presence of parents and grandparents, marital status, gender, and education, and number and age of children. We calibrate our multi-generational family model taking into account the U.S. tax and benefit system and match key features of the U.S. labor and child care markets. We find that our proposed implementation with universal day care may lead to average costs savings of 0.05% to 3.31%, relative to the case that would yield the same level of welfare but where the government may not use universal day care. We find higher cost savings for single parent households and for multi-generational households with both a single parent and a single grandparent present. Average cost savings ranged between 0.23% and 2.2% for single mothers with higher cost savings among the less educated mothers. Cost savings for single fathers ranged between 0.1% and 1.23%, for two parent households between 0.05% and 0.8%, and

[4] A recent newspaper article headlines: "Grandmother on disability benefits caught running a mile (while pushing a pram)", which provides an anecdotal story that some disability benefits cheats may be babysitting their grandchildren. *Source*: Daily Mail, 2009.

for households with a grandparent present between 0.2% and 3.31%.

Our study makes a contribution to the child care subsidies literature. There is general agreement that child care subsidies are positively associated with higher labor supply of mothers (Blau, 2003; Guner *et al.* , 2014; Havnes & Mogstad, 2014; Tekin, 2007). Proponents have argued for such subsidies on the grounds of encouraging mothers to become self-sufficient (Blau, 2003), in order to counteract the disincentive effects of income taxation (Barnett, 1993), and to promote higher quality child care (Currie, 2001; Heckman & Cunha, 2010).

Our work is also related to the optimal taxation literature with home production. Domeij & Klein (2013) find that child care subsidies can lead to welfare gains within a Ramsey optimal taxation framework. In their context, there would be no need to subsidize day care if tax rates on income were zero, that is, the role of day care subsidies is to help counterbalance the effects of distortionary income taxation. Our argument is somewhat reversed: In an optimal social insurance context, with asymmetric information between the government and households, day care subsidies, together with non-linear income taxation and asset limits, help counterbalance the existing incentive issues of household members.[5]

Universal day care allows for a simple child care subsidy scheme that helps circumvent a complicated system with multiple targeted subsidy rates. With multi-member households, household members would have different child care margins, that is, different incentives to engage in household child care activities. Implementation of targeted child care subsidies would therefore require policy makers to provide different subsidy rates to different household members across different households. In addition, for such targeted subsidies to be feasibly implemented, household child care would need to be monitored. In our context, with multi-member households and privately observed household child care activities, universal day care not only help counter the incentive issues of mothers but also the incentive issues of related family members.

[5]In our context, the optimal scheme is designed so as to prevent healthy members from mimicking the disabled. Ho & Pavoni (2014) make a somewhat similar argument for targeted sliding scale child care subsidies within a Mirrlees optimal income taxation problem with verifyable household child care. They focus on a static model with single mothers who have heterogeneous market productivities.

In Section 2, we describe our theoretical framework and present the recursive formulation of the government problem. In Section 3, we make a case for universal day care and discuss implementation of the constrained optimum. In Section 4, we describe our quantitative exercise and present numerical results from simulations of the optimal policy as well as from counterfactual policies without universal day care. Finally, we conclude in Section 5.

2 Model

2.1 Framework

2.1.1 Agents

Agents in our model are multi-generational family households that may consist of grandparents, parents, and children. We use the terms family and household interchangeably throughout the text. Let I be the total number of adult members in the household and denote the adult members of the family by the index $i \in \{1, 2, .., I\}$.[6] We consider a finite horizon time frame with T discrete periods, $t = 0, 1, ..., T$, during which the household's adult composition is fixed and child care needs may be relevant to the family. The family household is a decision unit in our model.

Time allocation In each period, households have child care needs $n \geq 0$. Healthy adults can devote effort (time) to the labor market $l \geq 0$ or to household child care $h \geq 0$. The remaining child care needs not covered by household child care, $n - \sum_i h^i$, is purchased from the child care market at price $p > 0$ per unit. When a healthy adult i works on the labor market, the latter earns $y^i = w^i l^i$ where w^i is wage per unit of labor. We assume that $\forall i$, $w^i > p$, which is in line with the fact that average cost of formal child care in the U.S. tends to be below the minimum wage.[7] n, p and w^i evolve deterministically over time.

[6]For example, $i = 1$ could represent the father, $i = 2$ the mother and so on.

[7]The federal minimum wage is \$7.25 per hour while cost of formal child care for an infant averaged \$3.88 per hour. See Section 4 for details on the calibration process. Our model is extendable to incorporate household members with $w^i < p$. See the Appendix for a discussion of this extension.

Preferences Family preferences are separable over time and the future is discounted at rate $\beta \in (0,1)$. Within period family preferences are separable in family consumption $c > 0$ and effort of each adult member $e^i = l^i + h^i$. Intra period family preferences are given by

$$u(c) - \sum_i v_i(e^i).$$

u is a concave function with $u' > 0$, $u'' < 0$. For all i, v_i is a convex function with $v_i' \geq 0$, $v_i'' \geq 0$ and $v_i(0) = v_i'(0) = 0$.

Disability states Adults are subject to absorbing disability shocks, that is, once an adult is disabled, the disability is permanent. Disabled adults can neither work on the market nor at home so that $e = 0$. In any period, a family may have from zero to all members disabled. Let S denote the set of relevant disability states that a family may be subject to and denote the state of family disability by the index $s \in S$, which captures the number and the identity of disabled members. Note that by the absorbing nature of disability shocks, the relevant state set in the current period, S, would be a subset of the the previous period's state set, S_{-1}. For example, if one family member is disabled in the previous period, then the same member cannot be healthy in the current period. Let $\pi(s, s_{-1})$ be the conditional probability that the family is in state s in the current period, given state s_{-1} at the start of the period, and normalize $\sum_{s \in S} \pi(s, s_{-1}) = 1$.

2.1.2 Government

The government is risk-neutral and provides disability insurance to families in the least costly way possible, while guaranteeing the family an expected utility level of at least V. The initial promised utility, V, can be interpreted as an exogenously determined parameter capturing the generosity level of the welfare system. We consider a setting where the government discounts future costs at rate $\frac{1}{1+r}$, where r is the interest rate, and assume that $\beta = \frac{1}{1+r}$.

Information structure The government knows the distribution of disability shocks but actual disability shocks are private information to the household. y and l are verifiable for those employed.

Thus, if $l^i > 0$, the government can infer that household member i is healthy, but if $l^i = 0$, the government cannot infer whether i is disabled or healthy but exerting no effort on the labor market. We assume that the government may also monitor household assets.

2.2 Government Problem

We are interested in modeling the constrained optimum where it is *as though* the government could perfectly monitor household child care activities. We will therefore write the government problem as though h were public information. We will show in Section 3 that the implementation of this optimum is possible via the use of universal day care (which precludes the need to monitor h), in addition to non-linear income taxation and asset limits.

2.2.1 Recursive formulation with history dependence

We model the government problem recursively. By the revelation principle, we can focus on direct mechanism where in each period, households declare their disability state $s \in S$. The government then specifies allocations according to the declared state.

Standard recursive formulation In a standard recursive framework, the optimal allocations may be solved recursively as a government minimizing costs subject to promise keeping constraint and incentive compatibility constraints. Within our context, in each period, for a declaration of state s, the government specifies for all i, earnings $y^i(s)$ and household child care $h^i(s)$, and allocates consumption $c(s)$, formal child care costs $p\left(n - \sum_i h^i(s)\right)$, and continuation utility $V(s)$ to households. The continuation utility allocated in the current period then becomes the expected discounted utility that the government needs to deliver to agents via the following period's promise keeping constraint.

The promise keeping constraint in our model is given by:

$$\sum_{s \in S} \pi(s, s_{-1}) \left[u(c(s)) - \sum_i v_i \left(h^i(s) + \frac{y^i(s)}{w^i} \right) + \beta V(s) \right] = V(s_{-1}). \tag{PK}$$

$V(s_{-1})$ is the promised continuation utility for agents who declared to be in state s_{-1} in the previous period. $V(s_{-1}) = V$ in the first period and $V(s) = 0$ in the last period.

If disability shocks were independent over time, we would have unconditional probabilities $\pi(s, s_{-1}) = \widehat{\pi}(s)$. In a framework where privately observed shocks are assumed to be independent over time, the promised continuation utility would be the only state variable that we need to keep track of (Albanesi & Sleet, 2006; Atkeson & Lucas, 1992).[8] With time-independent shocks, expected utility of agents would be common knowledge in every period. In other words, expected utility in the current period would be the same for all agents who declared s_{-1} in the previous period, irrespective of whether the agents were truthful or not. This arises because agents would have the same (non-history dependent) distribution of privately observed shocks in the current period, irrespective of the actual state in the previous period. The promise keeping and incentive compatibility constraints would therefore follow a straightforward recursive formulation, based on the commonly known preferences.

History dependence in privately observed shocks Now consider the case where shocks are history dependent, such as with absorbing disability shocks. In this case, expected utility of agents would not be common knowledge every period. To see this, consider two agents who declared to be in state s_{-1} in the previous period. The first agent was truthful while the second agent was untruthful and was actually in state $\widetilde{s}_{-1} \neq s_{-1}$. Then, in the current period, the expected utility based on the distribution of disability shocks would be conditional on s_{-1} for the first agent and conditional on $\widetilde{s}_{-1}$ for the second agent. Even though both agents declared to be in state s_{-1} in the previous period, they would have different expected utilities in the current period. Modeling the promise keeping and incentive compatibility constraints recursively, therefore, requires us to take into account the history dependence in privately observed shocks.

In addition to keeping track of the promised continuation utility, $V(s)$, for truthful agents, we

[8] Even with time-independent shocks, allocations would still be history dependent. In such a context, the promised continuation utility would be a state variable that captures all relevant information on those past histories.

therefore need to keep track of the history of disability shocks, and of additional state variables, $\widetilde{V}(s,\widetilde{s})$ for all $\widetilde{s}$ pretending to be s. $\widetilde{V}(s,\widetilde{s})$ can be interpreted as the *threatened continuation utility* of agents in state $\widetilde{s}$ pretending to be in state s (Fernandes & Phelan, 2000). The threatened continuation utilities allocated in the current period then becomes the expected discounted utilities that the government needs to deliver to previously untruthful agents via the following period's *threat keeping constraints*.

The threat keeping constraints in our model are given by:

$$\forall \widetilde{s}_{-1} \in S_{-1}, \sum_{\widetilde{s} \in S} \pi(\widetilde{s}, \widetilde{s}_{-1}) \underset{s}{max} \left[u\left(c(s)\right) - \sum_i v_i \left(h^i(s) + \frac{y^i(s)}{w^i} \right) + \beta \widetilde{V}(s,\widetilde{s}) \right] = \widetilde{V}(s_{-1}, \widetilde{s}_{-1}). \tag{TK}$$

$\widetilde{V}(s_{-1}, \widetilde{s}_{-1})$ is the threatened continuation utility of agents who were in state $\widetilde{s}_{-1}$ in the previous period but declared to be in state s_{-1}. Agents who were previously in state $\widetilde{s}_{-1}$ may still privately choose a state declaration s that would maximize their current expected discounted utility, even if they are in a different state $\widetilde{s}$ in the current period.[9] The threat keeping constraint therefore captures the different probability distribution of mimicking agents, as well as their private optimizing behavior. It keeps track of potential multi-period deviations.

The government also needs incentivize agents to truthfully declare their state. In other words, the expected discounted utility from truth-telling needs to be at least greater than or equal to the expected discounted utility from mimicking. The incentive compatibility constraints in our model are given by:

$$\forall s, \widetilde{s} \in S,\ u\left(c(s)\right) - \sum_i v_i \left(h^i(s) + \frac{y^i(s)}{w^i} \right) + \beta V(s) \geq u\left(c(\widetilde{s})\right) - \sum_i v_i \left(h^i(\widetilde{s}) + \frac{y^i(\widetilde{s})}{w^i} \right) + \beta \widetilde{V}(\widetilde{s}, s). \tag{IC}$$

Note that by the nature of disability, only abled family members may mimic the disabled and not vice versa so that only a subset of states are relevant for mimicking purposes. For the sake of notational simplicity, we keep the notation as it is in the main text and we will loosely use $\widetilde{s} < s$

[9]Note that $\widetilde{V}(s,s) = V(s)$ when an agent is in state s and chooses to truthfully report s.

in the Appendix to indicate that agents in state s may mimic agents in state $\widetilde{s}$.

2.2.2 Constrained optimization problem

Let $\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})$ be a vector of threatened continuation utilities capturing $\widetilde{V}(s,\widetilde{s})$ for all $\widetilde{s} \in S$. In every period, for agents who declared to be in state $s_{-1} \in S_{-1}$ in the previous period, the government chooses allocations so as to minimize the expected discounted value of costs:

$$G\left(V(s_{-1}), \widetilde{\mathbf{V}}(\mathbf{s_{-1}},\widetilde{\mathbf{s}}_{\mathbf{-1}})\right) =$$

$$\underset{c,y,h,V,\widetilde{V}}{Min} \sum_{s \in S} \pi(s, s_{-1}) \left[c(s) + p\left(n - \sum_i h^i(s)\right) - \sum_i y^i(s) + \beta G\left(V(s), \widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right)\right], \qquad \text{(G)}$$

s.t. (PK), (TK), and (IC).

In the initial period, $V(s_{-1}) = V$, the initial promised utility, and $\widetilde{V}(s_{-1}, \widetilde{s}_{-1}) = 0$, $\forall \widetilde{s}_{-1} \in S_{-1}$. In the last period, $G(V(s), \widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})) = 0$, $V(s) = 0$, and $\widetilde{V}(s,\widetilde{s}) = 0$, $\forall s, \widetilde{s} \in S$.

3 Policy Implications

In this Section, we characterize the optimal allocations and propose an implementation of the constrained optimum.

3.1 Full Information Benchmark

With perfect information on disability shocks, the government can choose allocations so as to minimize expected costs subject to promise keeping constraint only. In this case, we have full insurance with constant consumption, $\overline{c}$, across all states and across periods. Moreover, since $\forall i$, $w^i > p$, it is optimal for healthy household members to devote all of their effort to the labor market. The consumption-labor margin is also not distorted:

$$u'\left(\overline{c}\right) w^i = v_i'\left(\frac{y^i}{w^i}\right).$$

3.2 Constrained Optimal Allocations

It is straightforward to see that when actual disability shocks are private information, healthy agents would have an incentive to mimic the disabled if consumption were constant. There is therefore a trade-off between providing insurance and preserving work incentives of healthy agents.

Proposition 1. Let (c^*, y^*, h^*) solve the constrained optimization problem (G).

(i) For each period and state, it is optimal for healthy household members i to devote all of their effort to the labor market:

$$y^{i*}(s) > 0 \text{ and } h^{i*}(s) = 0.$$

In addition, the consumption-labor margin of healthy household members are not distorted:

$$u'\left(c^*(s)\right) w^i = v_i'\left(\frac{y^{i*}(s)}{w^i}\right).$$

(ii) For each period $t < T$, the inverse Euler equation holds:

$$\frac{1}{u'(c^*(s))} = \sum_{s_{+1} \in S_{+1}} \frac{\pi(s_{+1}, s)}{u'(c^*(s_{+1}))},$$

where $s_{+1} \in S_{+1}$ are the possible states in the following period given state s in the current period. There is an inter-temporal wedge between current and future marginal utilities of consumption:

$$u'\left(c^*(s)\right) < \sum_{s_{+1} \in S_{+1}} \pi(s_{+1}, s) u'\left(c^*(s_{+1})\right).$$

The proof is derived from the first order conditions of the constrained optimization problem and is outlined in the Appendix.

3.3 Private Deviation Incentives

In this Section, we discuss the private deviation incentives of agents.

Child care Suppose that household child care is private information to the household. Households take as given the allocations of consumption c^*, formal child care costs pn, and earnings y^*, specified by the government from the constrained optimal problem (G). Given such allocations, the household then privately chooses h, in order to maximize expected utility.

Consider the private problem of a household in which one member always mimics disability while the remaining members remain truthful about their health status. Given true state s, denote $\widetilde{s}$ as the declared state in which only member j mimics the disabled. The private problem of the household is given by:

$$\widetilde{U}(\widetilde{s}_{-1}, s_{-1}) = \underset{h}{Max} \sum_{s \in S} \pi(s, s_{-1}) \left[u(\underbrace{c^*(\widetilde{s}) + ph^j(\widetilde{s}, s)}_{\tilde{c}(\widetilde{s}, s)}) - v_j(h^j(\widetilde{s}, s)) - \sum_{i \neq j} v_i \left(\frac{y^{i*}(\widetilde{s})}{w^i} \right) + \beta \widetilde{U}(\widetilde{s}, s) \right].$$

By engaging in privately observed household child care, the household can save on formal child care costs and increase consumption. $\tilde{c}(\widetilde{s}, s)$ is consumption of the mimicker household. $\widetilde{U}(\widetilde{s}, s)$ is the expected private continuation utility for a household in state s but where member j is mimicking the disabled. $\widetilde{U}(\widetilde{s}, s) = 0$ in the last period.

The private consumption-child care margin when member j is actually healthy is given by:

$$u'(\tilde{c}(\widetilde{s}, s))p = v_j'(\tilde{h}^j(\widetilde{s}, s)).$$

Household member j would therefore engage in household child care activities when mimicking the disabled. Thus, the maximized utility of a mimicker household is greater with the possibility of engaging in privately observed household child care compared to the case where the mimicker household may not engage in such activities. The incentives of household members to mimic the disabled are therefore exacerbated.

Savings Suppose that instead, assets are private information to the household. The household then privately chooses assets $A \in \mathbb{R}$, in order to maximize expected utility. Consider the private problem of a household which is always truthful about its state s:

$$U(s_{-1}) = \underset{A}{Max} \sum_{s \in S} \pi(s, s_{-1}) \left[u(c(s)) - \sum_i v_i \left(\frac{y^{i*}(s)}{w^i} \right) + \beta U(s) \right],$$

s.t.

$$c(s) = c^*(s) - A(s) + (1+r)A(s_{-1}).$$

The possibility of having hidden assets enables households to privately smooth consumption across periods. $A(s_{-1})$ are assets carried over in the previous period and $A(s)$ are assets to be chosen in the current period. $A(s_{-1}) = 0$ in the first period, and $U(s) = 0$ in the last period.

From the household's first order conditions, the private Euler equation holds:

$$u'(c(s)) = \sum_{s_{+1} \in S_{+1}} \pi(s_{+1}, s) u'(c(s_{+1})).$$

The household privately equates expected marginal utility of consumption across periods. This is in contrast to the inter-temporal wedge in Proposition 1(ii). Thus, the agent may have private incentives to underconsume and oversave in the earlier periods, relative to the optimal allocation. The maximized utility of the household is therefore greater with the possibility of saving privately.

Triple deviation incentives Should both household child care and savings be private information to households, it may therefore be privately optimal for households to (i) be untruthful about their state, (ii) engage in non socially optimal household child care activities, and (iii) accumulate non socially optimal assets. The way that (ii) interacts with (iii) depends on the relative profile of child care needs and formal child care costs over the life-cycle. On one hand, front-loaded household child care enables households to increase consumption in earlier periods and therefore exacerbates the incentive to oversave for the future. On the other hand, back-loaded household child care enables households to increase consumption in later periods and therefore smoothens the incentive to oversave in earlier periods. In either case, such possibilities exacerbate the incentive of household members to mimic the disabled.

3.4 Implementation

We show that a combination of universal day care, non-linear income taxation and asset limits implement the constrained optimum *as though* the government could monitor household child care.

Proposition 2. The following scheme implements the constrained optimal allocations (c^*, y^*, h^*).

(i) *Universal day care.* Every period, the government subsidizes formal child care price at a rate of $s = 1$, such that the subsidized price of child care is $(1 - s)p = 0$.

(ii) *Lump sum net taxes and asset limits.* Every period, the government imposes net taxes such that: $\forall s$, $T(s) = \sum_i y^{i*}(s) - c^*(s)$ if $A(s_{-1}) \leq 0$ and $T(s) = \sum_i y^{i*}(s)$ if $A(s_{-1}) > 0$.

The proof of Proposition 2 is presented in the Appendix.

The universal day care may be implemented in the form of free public day care or of direct reimbursement to day care centers. With universal day care, households face an effective child care cost of zero. It follows, that household child care would not help households save on formal child care costs while it would still be costly in terms of effort. Households would therefore be discouraged from engaging in non optimal household child care activities.

By construction, non-linear income taxation and asset limits, ensure that household choices would then coincide with the constrained optimal allocations. It follows from incentive compatibility constraints (IC), that households would be truthful about their state and receive the same expected discounted utility as in the constrained optimum. By design, the government would also face the same expected costs as in the constrained optimum.[10]

[10] Our time-varying taxes are analogous to age-dependent taxation in the case of one person households (Weinzierl, 2011). We note that given universal day care, various combinations of time varying asset limits and non-linear income taxes could, in fact, also implement the constrained optimum. Such combinations may be defined recursively, from households' private budget constraints:

$$c^*(s) + \overline{A}(s) = \sum_i y^{i*}(s) - T(s) + (1 + r)\overline{A}(s_{-1}),$$

with assets limit $\overline{A}(s_{-1}) = 0$ in the first period and $\overline{A}(s) = 0$ in the last period. In addition, the present value of lifetime net taxes need to be equal to the government's expected costs at the constrained optimal allocations:

$$\sum_t \sum_s \widehat{\pi}_t(s) T_t(s) = \sum_t \sum_s \widehat{\pi}_t(s) \left[\sum_i y_t^{i*}(s) - c_t^*(s) \right],$$

where $\widehat{\pi}_t(s)$ is the unconditional probability of being in state s in period t. The time-varying asset limits would be similar in spirit to the asset-tested disability benefits system proposed by Golosov & Tsyvinski (2006).

4 Quantitative Analysis

4.1 Calibration

In the quantitative analysis, we allow for observed exogenous demographic heterogeneity across households in terms of structure (presence of parents and grandparents), adult characteristics (marital status, gender, and education) and child characteristics (number and age of children). We denote a particular family household type by the index k which conveys all the relevant information on household composition. The index i still denotes a particular adult family member. Each household type may face different life cycle profiles of child care needs, while the adult members may have different effort cost parameters, life cycle probabilities of being disabled, and wage profiles. The qualitative results presented in Section 3 are applicable to any family type.

Family preferences are given by

$$\ln(c_{kt}) - \sum_i \alpha_k^i \frac{(e_{kt}^i)^{1+\gamma}}{1+\gamma}$$

The felicity of consumption is logarithmic.[11] α_k^i is an effort cost parameter and γ is the reciprocal of the Frisch elasticity of labor supply.

Parameters to be calibrated are the discount factor β, preference parameters $\{\gamma, \alpha_k^i\}$, the life cycle probabilities of being disabled $\widehat{\pi}_{kt}^i$, the life cycle profiles of wages w_{kt}^i, child care needs n_{kt} and price of formal child care p_{kt}. We also need to calibrate initial promised utility V_k for each family type, and the U.S. tax and benefit system. In particular, Social Security and Federal Taxes, Earned Income Tax Credit (EITC), Child and Dependent Care Tax Credit (CDCTC), Child Care and Development Fund (CCDF), Social Security Disability Insurance (SSDI), and Supplemental Security Income (SSI).

[11] In the numerical analysis, we normalize the logarithmic function by adding 1 to its argument so that $\lim_{c\to 0} u(c) = 0$.

4.1.1 Demographics

Household adult structure The population size and adult structure of each household is assumed to be constant over the time frame $t = 0, ..., T$, which corresponds to the finite life cycle of a multi-generational household during which child care needs may be relevant. The demographic composition of households is designed to match the composition of U.S. households. We use data from the Current Population Survey (CPS) which is a nationally representative dataset of U.S. households. In particular, we use the March Annual Social and Economic Supplement of the CPS for years 2005 to 2014 as a series of repeated cross sections.

We define an adult to be part of the parent or grandparent generation based on age and irrespective of the presence of children. Those aged 25 to 49 are part of the parent generation and those aged 50 to 74 are part of the grandparent generation. To keep the terminology simple, we refer to an adult in the parent generation as "father" or "mother". Similarly, an adult in the grandparent generation is referred to as "grandfather" or "grandmother". We will describe households' child characteristics further on within this Section.

In terms of the living arrangements of related adults in a household[12], we consider parent households with adults aged only 25 to 49, grandparent households with adults aged only 50 to 74, as well as intergenerational households with adults aged 25 to 74. The adult members of the grandparent and parent generations may be single or married.

We consider a 5 period model where each period t corresponds to a 5 year time interval. We base the life cycle of a multi-generational household on the age of the mother when she is present in the household so that $t = 0, 1, 2, 3, 4$ correspond to a household with a mother aged respectively $25 - 29, 30 - 34, 35 - 39, 40 - 44, 45 - 49$. When there is no mother present but a grandmother is present, we base the life cycle of the household on the age of the grandmother so that $t = 0, 1, 2, 3, 4$ correspond to a household with a grandmother aged respectively $50 - 54, 55 - 59, 60 - 64, 65 -$

[12] In the CPS, a household is identified by the household number and current address of residence. Family members within a household are identified using the family identification number.

$69, 70-74$. Finally, when there are neither mother nor grandmother in the household, we use the age of the father.

We do not consider households with adult structures that make up less than 2% of the sample. This gives us 7 possible structures: 3 parent households (single mother, single father, two parents), 2 grandparent households (single grandmother, two grandparents), and 2 intergenerational households (single grandmother and single father, single grandfather and single mother). The different household structures and their proportions in the CPS are given in Table 1. Parent households make up 61% of our sample while grandparent households make up 28% of the sample, and 11% of households are intergenerational.

Household child characteristics Child arrival rates are exogenously given in our model. Our interpretation of child arrival is inclusive of births of own children and arrival of grandchildren into the household. We assume that children may arrive only in the first three periods, corresponding to when adults in the parent generation are aged 25 to 39. The arrival rates of children vary by household structure and time period, and is calibrated according to the proportion of households with children aged below 5 in the CPS.

For parent households, we limit the total number of children in a household to 3 and the maximum number of children arriving in one period is limited to 2.[13] This gives us a maximum of 17 profiles of child arrivals for each parent household. For grandparent households and intergenerational households with a single father, we limit the total number of children in a household to 1.[14] This gives us a maximum of 4 profiles of child arrivals for those households. For intergenerational households with a single mother, we limit the total number of children to 2 and the maximum number of children arriving in one period is also 2.[15] This gives us a maximum of 9 profiles of child arrivals for such intergenerational households.

[13]Less than 6% of such households had more than 3 children and less than 5% had more than two 5 year old children at any given point in time.

[14]Less than 6% of such households had more than 1 child.

[15]Less than 6% of such households had more than 2 children.

We report the proportion of households with children aged below 5 in Table 2. As can be seen from the Table, parent households and intergenerational households with mothers have the highest child arrival rates. On the other hand, grandparent households and intergenerational households with single fathers had the lowest child arrival rates. We compute the proportion of households facing each child arrival profile using the information in Table 2. For example, the proportion of single mothers who have one child in every period is computed as $a_0 \times a_1 \times a_2$ where a_t is the proportion of single mothers with one child aged below 5 in period t. We normalize the sum of the proportions to 1 for each household structure.

Child care needs and price of formal child care We define child care needs n_{kt} as the portion of the working week during which child care is required. A normal working week is 40 hours which we normalize to 1 unit of time. We assume that children require full time child care of 1 unit only in the first period of their life. Since an adult can look after several children at the same time, household child care needs are based on age of the youngest child and is 1 unit of time if the latter is a newborn.

Price of formal child care p_{kt} depends on the number of newborns in the household. We calibrate the cost of formal child care according to data from Child Care Aware of America (2014) fact sheet, which is an annual report on child care costs based on statistics from state Child Care Resource and Referral agencies and from the latest market rate surveys. Among families that use formal child care, infants and toddlers aged 4 were in either center-based care or family child care homes.[16] We observe the average annual child care costs for infants and for toddlers aged 4 in full time center-based care and in family child care home across states. We also observe the number of children aged less than 4 in each state as well as the proportion of space available in center-based care and family child care homes.

The calibration of hourly cost of formal child care for a child aged $0-5$ is done as follows.

[16] Family care homes are typically licensed facilities that provide paid formal child care to a small group of children. We classify this arrangement as formal child care in order to avoid confusion with the informal household child care provided by parents and grandparents in our model.

For each state, we first compute the average annual costs that a child would incur in center-based care and in family child care home by assuming that the child faces the infant cost for 2 years and the toddler cost for 3 years. We then pro-rate the center-based and family child care home costs according to the proportion of space attributed to each facility type. In the next step, given 50 working weeks of 40 hours each a year, we compute the hourly cost of child care in each state. Finally, we pro-rate this cost by the proportion of children aged less than 4 in each state and convert to 2010 dollars using the Bureau of Labor Statistics (BLS) Consumer Price Index (CPI) calculator. This gives us an hourly child care cost of \$3.88 per child.

Disability rates The life cycle probabilities of being disabled $\widehat{\pi}^i_{kt}$ vary according to individual gender, marital status and age group, as well as household structure. We assume that the probability of being disabled is independent across household members and across households. The probability of being disabled is calibrated according to the CPS question "Does ... have a health problem or a disability which prevents work or which limits the kind or amount of work?". Since disability is absorbing in our model, we also posit that the proportion of disabled individuals cannot decrease over time.[17]

We report the proportion of disabled individuals by household adult composition in Table 3. As can be seen from the Table, older individuals have higher probabilities of being disabled compared to younger individuals of the same gender and marital status. On the other hand, single parents and grandparents were more likely to be disabled compared their married counterparts of the same gender and age group. We determine the transition probabilities of households being in a given state π_{kt} based on the proportions in Table 3. For example, a single mother has probability $(1-\widehat{\pi}^i_{k0})$ of being healthy in period 0. For subsequent periods, she has conditional probability $\frac{(1-\widehat{\pi}^i_{kt})}{(1-\widehat{\pi}^i_{kt-1})}$ of being healthy in period t given that she is healthy in period $t-1$.

[17] In particular, if the proportion disabled is lower for a older age group compared to a younger age group, we assume that the proportion disabled is the same as in the younger age group. This was relevant for single grandmothers in the last two periods when they are aged above 65.

Wages We calibrate hourly wages according to the wage profiles of workers in the CPS. We first divide gross earnings by hours of work in order to get hourly wages. Wages are then adjusted to 2010 dollars using the BLS CPI calculator. 2% of the sample of workers had earnings or hours information missing and 1.8% had wages of more than $100 per hour, which we drop from the sample. We allow wages to vary according to gender, marital status, age group, household adult composition and education, and take the average across each category. We allow for two education levels: high school or less, and more than high school education. The wage profiles of household members by education level and for each period t is reported in Table 4. Since 65 is the usual Social Security age in the U.S., we assume that those aged 65 and above are retired and therefore do not consider their wages. As can be seen from the Table, those with high school or less have lower wages compared to those with college education. In parent and grandparent households, married individuals tend to earn higher wages compared to single individuals of the same gender, age group and education level. In intergenerational households, single grandfathers earn more than single mothers with the same education level while single grandmothers earn more than single fathers with the same education level only in the first two periods.

4.1.2 U.S. Tax and Benefit System

We base our calibration on the 2010 U.S. tax and benefits system.

Social Security and Federal taxes Social Security taxes are calculated as 6.2% of the first $106,800 in earnings (Social Security Administration, 2010). Federal income tax brackets depend on a tax payer's filing status. We assume that households with a single parent or grandparent file taxes under the single status when there are no children present in the household and file under the head of household status when there are children below 18 present in the household. Married households, on the other hand, file jointly for taxes irrespective of presence of children. For intergenerational households with children aged below 18, we assume that the grandparent files as head of household while the parent files under the single status. If the grandparent is disabled or retired, then the parent files as the head of household.[18]

Taxable income is computed as gross earnings minus exemptions and deductions. Deductions are $\$5,700$ for singles, $\$8,400$ for household heads, and $\$11,400$ for married couples. Each individual and dependent also gets personal exemptions of $\$3,650$. Federal income tax brackets are given in Table 5.

EITC The EITC is a refundable tax credit designed for lower income working families. The phase-in rate, maximum credit, phase-out rate and income limits depend on the number of children aged below 18 in the household. The income limits also depend on a tax payer's filing status, that is, whether filing as single or head or household, or as a married couple. The EITC schedule is given Table 5.

CDCTC The CDCTC is a non-refundable tax credit program available to working families with children under 13. The CDCTC has a tax credit rate of 20% to 35% of child care expenses up to a cap of $\$3k$ for families with one child and $\$6k$ for families with two or more children (Tax Policy Center, 2010). The 35% credit rate applies to families with annual gross income of less than $\$15k$, and declines by 1% for each $\$2k$ of additional income until it reaches a constant tax credit rate of 20% for families with annual gross income above $\$43k$.

CCDF The CCDF is a block grant fund managed by states within certain federal guidelines. CCDF subsidies are available as vouchers or as part of direct purchase programs to working families with children under 13 and with income below 85% of the state median income. We set the CCDF rate to 90% which is the recommended subsidy rate under Federal guidelines although there are a lot of variations across states. We take into account the fact that only a certain proportion of eligible households received the CCDF subsidy: 39% of potentially eligible children living in households below the poverty threshold, 24% of potentially eligible children living in

[18]To qualify as head of household, one must be unmarried, provide for more than half of housing expenses, and have a qualifying dependent who may be a descendant aged below 18 or a disabled relative of any age. *Source:* Inland Revenue Service.

households with income between 101 to 150% of the poverty threshold, and 5% of potentially eligible children living in household with income above 150% of the poverty threshold but below the CCDF eligibility threshold of 85% of state median income (Department of Health and Human Services, 2012). U.S. median household income was \$51,144 in 2010 and the poverty thresholds for different family sizes are given in Table 5.

SSDI To be eligible for disability benefits, one must generally have worked for at least 5 out of the 10 most recent years with the benefits being permanent thereafter. SSDI benefits are based on the age at which one becomes disabled and Average Indexed Monthly Earnings (AIME). SSDI benefits are automatically converted to retirement benefits when the recipient is past the retirement age of 65. We assume that if a person is disabled, that person is disabled at the start of the period and the relevant AIME is a summary of earnings from the previous periods. We compute SSDI benefits on a period basis as follows:

$$SSDI^i_{kt} = \begin{cases} 0.9AIME^i_{kt-1} & \text{if} \quad AIME^i_{kt-1} \in [0, b_1] \\ 0.9b_1 + 0.32\left(AIME^i_{kt-1} - b_1\right) & \text{if} \quad AIME^i_{kt-1} \in [b_1, b_2] \\ 0.9b_1 + 0.32\left(b_2 - b_1\right) + 0.15\left(AIME^i_{kt-1} - b_2\right) & \text{if} \quad AIME^i_{kt-1} > b_2 \end{cases}$$

where b_1 and b_2 are bend points. In 2010, b_1 was equal to \$761 and b_2 was equal to \$4,586 (Social Security Administration, 2014).

We use the following formula to approximate AIME on a period basis:

$$AIME^i_{kt} = \frac{\left\{(AIME^i_{kt-1} + \min\{ssbase_t, y^i_{kt}\}\right\}}{2}$$

where $ssbase_t$ is the Social Security base wage which was \$106,800 in 2010. We assume that all parents are not eligible to claim SSDI in period $t = 0$ while all grandparents are eligible to claim disability benefits when disabled in the first period.[19] The relevant AIME if a grandparent

[19]Less than 3% of SSDI recipients were aged $25 - 29$. Meanwhile, 75% of the working age population are eligible for SSDI benefits with 70% of recipients being aged above 50. When SSI is taken into account, 90% of the working age population are insured against disability (Social Security Administration, 2011a).

is disabled in the first period, is approximated from average earnings of individuals aged $45 - 49$ with the same gender, marital status, and education.

SSI SSI is a means-tested program that provides benefits to low income individuals aged above 65 and to the disabled. The definition of disability is the same as under SSDI although there are no contribution requirements under SSI. It is possible to receive both SSI and SSDI if income is sufficiently low.[20] To be eligible for SSI, countable resources need to be less than $\$2k$ for an individual and $\$3k$ for a couple (Morton, 2014). We use household assets as the measure of resources in our model.

SSI benefits are reduced one-for-one for income from earnings or SSDI. We approximate SSI benefits as follows:

$$SSI^i_{kt} = \max\left\{0, \overline{SSI}^i_{kt} - SSDI^i_{kt}\right\}$$

where $\overline{SSI}^i_{kt}$ is the maximum SSI benefits and benefits are reduced one-for-one by income. In 2010, the maximum monthly benefit available to a single individual was \$674 and the maximum monthly benefit available to a couple was $\$1,011$.

4.1.3 Preference Parameters

We set the 5 year period discount factor $\beta = \frac{1}{(1+r)^5} = 0.95$ corresponding to an annual interest rate of $r = 1\%$. The Frisch elasticity of labor supply is set to 0.5 which corresponds to $\gamma = 2$ (Chetty *et al.* , 2011; Domeij & Klein, 2013; Pistaferri, 2003). The effort cost parameter α^i_k varies by gender, marital status, age group $(25 - 49$ and $50 - 74)$, and household adult structure. α^i_k is internally calibrated so as to match average weekly labor hours of working adults without children aged below 18 in the household in the CPS.

The calibration of α^i_k is done as follows. First, we define a grid of possible values over α. Then, for each household structure, we find the labor supply predicted by our model assuming that

[20] 85% of SSI recipients received the benefits based on disability in 2010 and 34% of SSI recipients also received Social Security benefits (Social Security Administration, 2011b).

household members are healthy and honest, $l_k^i(\boldsymbol{\alpha})$, where $\boldsymbol{\alpha}$ is a vector of grid points associated with the effort cost parameters of all household members. We solve each household's utility maximization problem by taking into account the U.S. Social Security and Federal Taxes and EITC. We then minimize the sum of squares of the distance between expected labor supply predicted by the model and the average weekly labor hours from the CPS, $\widehat{l}_k^i$. For a single adult household,

$$\alpha_k^i = \arg\min\{E[l_k^i(\alpha)] - \widehat{l}_k^i\}^2,$$

and for a two adult household,

$$\{\alpha_k^i, \alpha_k^j\} = \arg\min \sum_{m=i,j} \{E[l_k^m(\boldsymbol{\alpha})] - \widehat{l}_k^m\}^2.$$

The average labor hours and calibrated effort cost parameters are reported in Table 6. As can be seen from the T able, the weekly labor hours predicted by the model are matched very closely to average labor hours in the CPS. We report the life cycle profiles of labor supply assuming honesty for all working adults and including those with children in their household in Figures 1 to 3. The solid lines indicate average hours in CPS data and the dashed lines indicate hours predicted from our model and averaged over all child compositions. As can be seen from Figure 1, the model replicates the life cycle profile of labor supply for working parents very closely. The life cycle profiles of adult members in households with a grandparent are also closely replicated although slightly overestimated especially for married grandmothers in Figures 2 and single grandfathers in Figure 3.

4.1.4 Initial Promised Utility

We calibrate the initial promised utility V_k for each household type according to their adult composition and child composition profiles. V_k is set equal to the expected utility of the household under the U.S. tax and benefit system, when household members are honest about their health status. We take into account the U.S. Social Security and Federal Taxes, EITC, DCTC, CCDF, SSDI and SSI in our calibration.

4.1.5 Computation

We numerically solve the government problem G by backward induction for each of the 208 household types that we have. We first define a grid over promised utility V. Starting from the final period, for each grid point, we find the allocations that minimize expected costs while satisfying promise keeping and incentive compatibility constraints, and find the threatened utilities $\widetilde{V}$ that can be delivered through the threat keeping constraints. In the penultimate period, we repeat the same procedure taking into account the fact that the continuation utilities for truthful and untruthful agents will become respectively the promised and threatened utilities in the final period. We repeat this procedure until the first period. Given the calibrated initial promised utility V_k, we can then find the optimal allocations for each possible disability history.

4.2 Numerical Results

We present our numerical results in this Section. We first characterize the constrained optimal allocations. We then compare those allocations to the case where universal day care is not available, thereby allowing agents to engage in privately observed household child care activities. Finally, we compute the cost savings from implementing the optimal scheme.

4.2.1 Constrained Optimal Allocations

Figures 4 to 10 report the constrained optimal allocations for our 7 household adult structures, averaged over all child arrival profiles and education groups. The optimal allocations as implemented by the scheme described in Proposition 2, are illustrated in Panels (a) and (b) of the Figures.

The solid lines in Panels (a) illustrate the consumption allocated to each household when all adult members of the household are healthy and working. A common feature to note across Figures 4 to 10, is that such consumption is non-decreasing over time. In particular, consumption is strictly increasing in households with a parent present. Similarly, consumption is strictly increasing in grandparent only households, until the retirement period 3, after which consumption is constant.

The increasing consumption profiles are in line with the government providing dynamic work incentives to healthy households. When agents work, they are rewarded with higher consumption and future utilities and therefore higher consumption for the future.

The dashed lines of Panels (a) illustrate consumption allocated to a household when one household member becomes disabled in period $0, 1, 2, 3$ or 4. In two member households, the disabled household member is the mother in Figure 6, the grandmother in Figure 8, and the grandparent in Figures 9 and 10.[21] Consumption of the disabled is influenced by (i) life-cycle wage profiles and (ii) dynamic incentives. On one hand, it may be efficient to have lower consumption for the disabled when agents are more productive. This is so as to discourage healthy agents from mimicking the disabled. On the other hand, preservation of dynamic work incentives imply allocating higher future utilities to working agents, and therefore higher consumption when an agent becomes disabled later in life.

As can be seen from Table 4, parents have increasing wage profiles from age 25 to 49 , except college educated married mothers, whose wage decline in the last two periods. Conversely, grandparents have decreasing wage profiles from age 50 to 64. The wage profile effect seems to dominate for those with rising wage profiles while the wage profile effect reinforces the dynamic incentives effect for those with declining wage profiles. This can be seen from the decreasing consumption profiles of disabled single parents in Figures 4 and 5, and the increasing consumption profiles when grandmothers are disabled in Figures 7 and 8. For two parent households, we have a slightly u-shaped profile of consumption when mothers are disabled in Figure 6. In Figure 9, the wage profile of the grandmother and dynamic work incentives seem important as can be seen from the rising profile of consumption when the grandmother becomes disabled. Meanwhile, the wage profile of grandfathers seem to dominate at first whereas incentivizing mothers subsequently become important once the grandfather is retired, as can be seen from Figure 10.

[21] Note that consumption is constant once all members are disabled or retired since all uncertainty has been resolved. This can be seen in Figures 7 and 8 where consumption is constant after retirement. For two member households, we illustrate the more frequent situation where only the mother or the grandparent becomes disabled.

There are also two forces influencing optimal labor market effort dynamics when all household members are healthy: (i) life-cycle wage profiles and (ii) dynamic incentives. While it may be efficient for agents to work more as they get more productive, the increasing consumption profiles when healthy and the consumption-labor margin in Proposition 1(i) imply that it may also be efficient for agents to work less in future periods in line with the dynamic incentives. Once again, the first effect seems to dominate the second effect for those with rising wage profiles, while the first effect reinforces the second effect for those with declining wage profiles. We therefore observe generally rising labor market effort profiles for parents and declining labor market effort profiles for grandparents as can be seen from Panels (b) of Figures 4 to 10.

4.2.2 Optimal Allocations without Universal Day Care

We now compute the optimal allocations in the case where the government may not use universal day care, but may use non-linear income taxation and asset limits. In other words, we allow agents to engage in privately observed household child care activities. The government problem is similar to problem (G) and delivers the calibrated initial promised utility V_k to households of type k. We also now need to explicitly take into account private child care incentives:

$$\widetilde{\mathbf{h}}(\widetilde{\mathbf{s}}, \mathbf{s}) = \arg\max_{\widetilde{\mathbf{h}}} u\left(c^*(\widetilde{s}) + p\sum_i \widetilde{h}^i(\widetilde{s}, s)\right) - \sum_i v^i\left(\frac{y^{i*}(\widetilde{s})}{w^i} + \widetilde{h}^i(\widetilde{s}, s)\right),$$

where an agent in state s declares to be in state $\widetilde{s} \in S$ and $\widetilde{\mathbf{h}}(\widetilde{\mathbf{s}}, \mathbf{s})$ is a vector of household child care for all household members. We follow the first-order approach (Rogerson, 1985) and impose the private first order conditions of agents with respect to household child care as additional constraints to the government's problem.

The optimal allocations with ($s = 1$) and without ($s = 0$) universal day care are illustrated respectively by the solid and dashed lines in Panels (c) and (d) of Figures 4 to 10. We report the averaged profiles across households with children. Optimal labor market effort profiles (not illustrated) in the case without universal day care are similar and sometimes slightly lower compared to the case with universal day care. The main difference stems from the fact that consumption allocated to households where all members are healthy is higher compared to the case with universal day care, as can be seen from Panels (c). Conversely, consumption allocated to households when one member becomes disabled is lower in earlier periods compared to the case with universal day care, as can be seen from Panels (d).

The intuition behind this result, is that child care needs are relevant in the first three periods $t = 0, 1, 2$. The incentives to mimic the disabled in order to engage in private household child care activities are therefore exacerbated in earlier periods. Since the government cannot use universal day care to counterbalance the child care margin, the only way to do so is by rewarding the healthy through higher consumption and penalizing the disabled through lower consumption. In later periods, when child care needs are less relevant, consumption of the disabled are also higher, in line with the dynamic incentives associated with providing higher future utility to those who were working in earlier periods. The gap in consumptions with and without universal day care were wider for households with more children arriving when parents are aged $30-39$ and grandparents aged $55-64$. For one member households, consumption of the disabled without universal day care was up to \$61 per week lower compared to consumption of the disabled with universal day care. The corresponding figure for two member households was \$117 per week.

4.2.3 Cost Savings from Universal Day Care

We compute the cost savings associated with the case where the government may use universal day care compared to the case where the government may not use universal day care. The costs savings are computed such that both cases deliver the same initial promised utility, V_k. We take the difference between expected costs from both cases and compute the percentage cost savings relative to expected costs in the case where the government may not use universal day care. The average cost savings ranged from 0.05% to 3.31%, with higher cost savings for single parent households and intergenerational households with both a parent and a grandparent present.[22]

[22] The cost savings may be interpreted as the information costs associated with privately observed household child care. We also computed the information costs associated with privately observed disability shocks by taking the difference between expected costs for the full information case and expected costs

We report the average cost savings by education group and by total number of children in Table 7. As can be seen from the Table, single mothers with high school education or less had relatively higher cost savings compared to single mothers with college education. Cost savings for the former range between 1.65% and 2.2% and for the latter between 0.23% and 1.56%. A similar pattern is observed for single fathers, whose cost savings range between 0.1% and 1.23%, and with higher savings for families with more children. Cost savings for two parent households ranged between 0.05% and 0.8% and for grandparent households between 0.20% and 0.36%. Intergenerational households on the other hand, had cost savings ranging between 0.24% and 3.31%, with higher cost savings for intergenerational families with college educated grandmothers and fathers, and for families with a grandfather, a mother, and two children in the household.

In sensitivity analysis, we recalibrate initial promised utilities and recompute the associated cost savings under the assumption of higher cost of formal child care of $p = \$5$, which corresponds to annual formal child care cost of $\$10,000$ for a child in full time day care. As can be seen from Table 8, we have higher relative costs savings for most household types. Cost savings ranged from 0.06% to 2.93% for parent households, 0.27% to 0.68% for grandparent households, and 0.4% to 13.7% for intergenerational households. The more expensive formal child care is, the higher the incentives to mimic the disabled in order to save on formal child care. The role of universal day care in counterbalancing those incentives therefore becomes even more important, thereby leading to higher costs savings.

5 Conclusion

In this paper, we propose an implementation of optimal social insurance when disability shocks are private information. The possibility of engaging in privately observed household child care activities exacerbates the incentives of household members to mimic the disabled, since in addition

under the constrained optimum with universal day care. The information costs ranged between 0.44% to 59.35% with higher cost savings associated with intergenerational households with both a grandparent and a parent present.

to receiving disability benefits, the household may save on formal child care costs. Universal day care helps counteract such incentives for all members of the family, including parents and grandparents. At the same time, non-linear income taxation and asset limits prevent households from oversaving in earlier periods and therefore decrease the private incentives to mimic the disabled in later periods. Calibrating our model to match key features of the U.S. labor and child care markets, we find that the use of universal day care may lead to sizeable cost savings, with higher cost savings for single mothers and for intergenerational households with both a parent and a grandparent present.

While, we have defined child care needs as occurring due to the arrival of a child aged below 5 into the household, we note that child are needs may in fact be broader in definition. For instance, school age children may also have after school care needs. In addition, it is possible that some multi-generational family members, such as grandparents, provide child care to their non-resident grandchildren. Such child care needs would also contribute to the exacerbation of incentives of healthy family members to mimic the disabled. Our computed cost savings therefore provide a lower bound on the potential cost savings from universal day care.

Appendix

Proof of Proposition 1 We drop the constrained optimal * subscript to keep the notation simple. Since only the healthy may mimic the disabled and not vice versa, we also loosely use the notation $\widetilde{s} > s$ to indicate that agents in state $\widetilde{s}$ may mimic agents in state s. The set of first order conditions for each period $0 < t < T$, from the constrained optimization problem (G) are:

$$\begin{aligned}
c(s) &: & \pi(s, s_{-1}) - \psi(\pi, \lambda, \widetilde{\lambda}, \eta) u'\left(c(s)\right) &= 0, \\
y^i(s) &: & \pi(s, s_{-1}) w^i - \psi(\pi, \lambda, \widetilde{\lambda}, \eta) v_i'\left(h^i(s) + \tfrac{y^i(s)}{w^i}\right) &\leq 0, \\
h^i(s) &: & \pi(s, s_{-1}) p - \psi(\pi, \lambda, \widetilde{\lambda}, \eta) v_i'\left(h^i(s) + \tfrac{y^i(s)}{w^i}\right) &\leq 0, \\
V(s) &: & -\pi(s, s_{-1}) G'_{V(s)}\left(V(s), \widetilde{\mathbf{V}}(\mathbf{s}, \widetilde{\mathbf{s}})\right) + \zeta(\pi, \lambda, \widetilde{\lambda}, \eta) &= 0, \\
\widetilde{V}(s, \widetilde{s}) &: & -\pi(s, s_{-1}) G'_{\widetilde{V}(s, \widetilde{s})}\left(V(s), \widetilde{\mathbf{V}}(\mathbf{s}, \widetilde{\mathbf{s}})\right) + \widetilde{\zeta}(\pi, \widetilde{\lambda}, \eta) &= 0,
\end{aligned}$$

where

$$\psi(\pi,\lambda,\widetilde{\lambda},\eta)=\lambda(s_{-1})\pi(s,s_{-1})+\sum_{\widetilde{s}_{-1}>s_{-1}}\widetilde{\lambda}(s_{-1},\widetilde{s}_{-1})\sum_{\widetilde{s}\geq s}\pi(\widetilde{s},\widetilde{s}_{-1})I\{\widetilde{s}\geq s\}+\sum_{\widetilde{s}<s}\eta(\widetilde{s},s)-\sum_{\widetilde{s}>s}\eta(s,\widetilde{s}),$$

$$\zeta(\pi,\lambda,\widetilde{\lambda},\eta)=\lambda(s_{-1})\pi(s,s_{-1})+\sum_{\widetilde{s}_{-1}>s_{-1}}\widetilde{\lambda}(s_{-1},\widetilde{s}_{-1})\pi(\widetilde{s},\widetilde{s}_{-1})I\{s=\widetilde{s}\}+\sum_{\widetilde{s}<s}\eta(\widetilde{s},s),$$

$$\widetilde{\zeta}(\pi,\widetilde{\lambda},\eta)=\sum_{\widetilde{s}_{-1}>s_{-1}}\widetilde{\lambda}(s_{-1},\widetilde{s}_{-1})\pi(\widetilde{s},\widetilde{s}_{-1})I\{\widetilde{s}>s\}-\eta(s,\widetilde{s}).$$

$\lambda(s_{-1})$ denotes the Lagrange multiplier associated with the promise keeping constraint (PK) and $\widetilde{\lambda}(s_{-1},\widetilde{s}_{-1})$ denote the Lagrange multipliers associated with the threat keeping constraints (TK) for agents who declared to be in state s_{-1} in the previous period when they were actually in state $\widetilde{s}_{-1}\in S_{-1}$, and $\eta(s,\widetilde{s})$ denote the Lagrange multipliers associated with the incentive compatibility constraints (IC) of agents who declare to be in state s when they are actually in state $\widetilde{s}\in S$.

$I\{\widetilde{s}\geq s\}$ is an indicator function taking a value of 1 if it is privately optimal for those in state $\widetilde{s}\geq s$ to declare to be in state s, and a value of 0 otherwise. $I\{\widetilde{s}=s\}$ is an indicator function taking a value of 1 if an agent who was previously untruthful happen to be in state s in the current period, and a value of 0 otherwise.[23] $I\{\widetilde{s}>s\}$ is an indicator function taking a value of 1 if an agent who was previously untruthful is in state $\widetilde{s}>s$ in the current period but declares s, and a value of 0 otherwise.

(i) From examining the first order conditions with respect to $y^i(s)$ and $h^i(s)$, we can rule out cases with both $y^i(s)=0$ and $h^i(s)=0$, when agents are healthy and $\pi(s,s_{-1})>0$, since $v_i'(0)=0$ and $w^i>p>0$. In addition, $y^i(s)>0$ and $h^i(s)=0$. Suppose to the contrary that $y^i(s)=0$ and $h^i(s)>0$. Then, it would be possible to decrease $h^i(s)$ by $\epsilon>0$ and increase $y^i(s)$ by $w^i\epsilon$, such that total effort of member i is the same. The promise keeping, threat keeping and incentive compatibility constraints are still satisfied, while the government's expected costs decrease by $\pi(s,s_{-1})\left(w^i-p\right)\epsilon>0$. Thus, $y^i(s)=0$ and $h^i(s)>0$ cannot be optimal. The same argument applies for cases where $y^i(s)>0$ and $h^i(s)>0$. It must therefore be that $y^i(s)>0$ and $h^i(s)=0$ for healthy agents. The consumption-labor margin is then derived directly from the first

[23]By incentive compatibility, such agents will find it optimal to be truthful and declare state s. They therefore get continuation utility $\widetilde{V}(s,s)=V(s)$.

order conditions for $c(s)$ and $y^i(s)$.

(ii) Adding the first order conditions with respect to $V(s)$ and $V(s,\widetilde{s})$ $\forall \widetilde{s} > s$, and taking into account the fact that $I\{s=\widetilde{s}\} + I\{\widetilde{s} > s\} = I\{\widetilde{s} \geq s\}$, we have:

$$\pi(s,s_{-1})\left[G'_{V(s)}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right) + \sum_{\widetilde{s}>s} G'_{\widetilde{V}(s,\widetilde{s})}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right)\right] = \psi(\pi,\lambda,\widetilde{\lambda},\eta).$$

Using the first order condition with respect to $c(s)$, we therefore have:

$$\frac{1}{u'(c(s))} = G'_{V(s)}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right) + \sum_{\widetilde{s}>s} G'_{\widetilde{V}(s,\widetilde{s})}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right). \tag{A1}$$

Adding the first order conditions with respect to $c(s)$ $\forall s$, and taking into account the fact that $\sum_{s\in S}\sum_{\widetilde{s}\geq s}\pi(\widetilde{s},\widetilde{s}_{-1})I\{\widetilde{s}\geq s\} = 1$, we also have:

$$\sum_{s\in S}\frac{\pi(s,s_{-1})}{u'(c(s))} = \lambda(s_{-1}) + \sum_{\widetilde{s}_{-1}>s_{-1}}\widetilde{\lambda}(s_{-1},\widetilde{s}_{-1}).$$

We have an analogous expression for the following period:

$$\sum_{s_{+1}\in S_{+1}}\frac{\pi(s_{+1},s)}{u'(c(s_{+1}))} = \lambda(s) + \sum_{\widetilde{s}>s}\widetilde{\lambda}(s,\widetilde{s}).$$

From the interpretation of the Lagrange multipliers, we have

$$\sum_{s_{+1}\in S_{+1}}\frac{\pi(s_{+1},s)}{u'(c(s_{+1}))} = G'_{V(s)}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right) + \sum_{\widetilde{s}>s} G'_{\widetilde{V}(s,\widetilde{s})}\left(V(s),\widetilde{\mathbf{V}}(\mathbf{s},\widetilde{\mathbf{s}})\right). \tag{A2}$$

The inverse Euler equation follows from (A1) and (A2):

$$\frac{1}{u'(c(s))} = \sum_{s_{+1}\in S_{+1}}\frac{\pi(s_{+1},s)}{u'(c(s_{+1}))}.$$

Applying Jensen's inequality to the inverse Euler equation, we then get the inter-temporal wedge between current and future marginal utilities of consumption:

$$u'(c(s)) < \sum_{s_{+1}\in S_{+1}}\pi(s_{+1},s)u'(c(s_{+1})).$$

Q.E.D.

Proof or Proposition 2 Consider the private problem of a household. The household chooses household child care, assets and declared state $\widetilde{s}$ so as to maximize expected utility:

$$\widetilde{U}(\widetilde{s}_{-1}, s_{-1}) = \underset{\tilde{h},\tilde{A},\widetilde{s}}{Max} \sum_{s\in S} \pi(s, s_{-1}) \left[u\left(\tilde{c}(\widetilde{s}, s)\right) - \sum_i v_i \left(\frac{y^{i*}(\widetilde{s})}{w^i} + \tilde{h}^i(\widetilde{s}, s) \right) + \beta \widetilde{U}(\widetilde{s}, s) \right],$$

s.t.

$$\tilde{c}(\widetilde{s}, s) + (1-s)p\left(n - \sum_i \tilde{h}^i(\widetilde{s}, s) \right) + \tilde{A}(\widetilde{s}, s) = \sum_i y^{i*}(\widetilde{s}) - T(\widetilde{s}) + (1+r)\tilde{A}(\widetilde{s}_{-1}, s_{-1}).$$

Step 1. Universal day care discourages household child care activities.

Suppose that we have $\tilde{h}^i(\widetilde{s}, s) > 0$. Since $s = 1$, households face an effective child care price of $(1-s)p = 0$. Then, by decreasing $\tilde{h}^i(\widetilde{s}, s)$ by $\epsilon > 0$, the household can increase its expected utility by $\pi(s, s_{-1}) \left[v_i \left(\frac{y^{i*}(\widetilde{s})}{w^i} + \tilde{h}^i(\widetilde{s}, s) \right) - v_i \left(\frac{y^{i*}(\widetilde{s})}{w^i} + \tilde{h}^i(\widetilde{s}, s) - \epsilon \right) \right] > 0$ when mimicking $\widetilde{s}$. It must therefore be that $\tilde{h}^i(\widetilde{s}, s) = 0$. The same logic applies for all household members and states.

Step 2. Households accumulate zero assets.

If $\tilde{A}(\widetilde{s}, s) = 0$ for all periods and states, then from the household budget constraints, $\tilde{c}(\widetilde{s}, s) = c^*(\widetilde{s})$ $\forall t$ and $\forall s, \widetilde{s}$. We show that we cannot have $\tilde{A}(\widetilde{s}, s) < 0$ in any period or state. Suppose that assets carried over to the last period are negative, $\tilde{A}(\widetilde{s}_{T-1}, s_{T-1}) < 0$. From the last period's budget constraint:

$$\tilde{c}(\widetilde{s}_T, s_T) = c^*(\widetilde{s}_T) + (1+r)\tilde{A}(\widetilde{s}_{T-1}, s_{T-1}).$$

It must therefore be that $\tilde{c}(\widetilde{s}_T, s_T) < c^*(\widetilde{s}_T)$ $\forall s_T, \widetilde{s}_T$ so that $u'\left(\tilde{c}(\widetilde{s}_T, s_T)\right) > u'\left(c^*(\widetilde{s}_T)\right)$ by concavity of u. For such household choice to be optimal, it must be that the private Euler equation holds:

$$u'\left(\tilde{c}(\widetilde{s}_{T-1}, s_{T-1})\right)) = \sum_{s_T \in S_T} \pi(s_T, s_{T-1}) u'\left(\tilde{c}(\widetilde{s}_T, s_T)\right)).$$

Now, we know from Proposition 1(ii) that we have an inter-temporal wedge at the optimal consumption levels:

$$u'\left(c^*(\widetilde{s}_{T-1})\right)) < \sum_{s_T \in S_T} \pi(s_T, s_{T-1}) u'\left(c^*(\widetilde{s}_T)\right)).$$

From the private Euler equation, the intertemporal-wedge and the fact that $\tilde{c}(\widetilde{s}_T, s_T) < c^*(\widetilde{s}_T)$, it must therefore be that $\tilde{c}(\widetilde{s}_{T-1}, s_{T-1}) < c^*(\widetilde{s}_{T-1})$. From the penultimate period's budget constraint:

$$\tilde{c}(\widetilde{s}_{T-1}, s_{T-1}) + \tilde{A}(\widetilde{s}_{T-1}, s_{T-1}) = c^*(\widetilde{s}_{T-1}) + (1+r)\tilde{A}(\widetilde{s}_{T-2}, s_{T-2}),$$

it must therefore be that $\tilde{A}(\widetilde{s}_{T-2}, s_{T-2}) < 0$. By sequential reasoning, this implies that we must also have $\tilde{A}(\widetilde{s}_0, s_0) < 0$ and $\tilde{c}(\widetilde{s}_0, s_0) < c^*(\widetilde{s}_0)$ in the first period. However, from the first period's budget constraint:

$$\tilde{c}(\widetilde{s}_0, s_0) + \tilde{A}(\widetilde{s}_0, s_0) = c^*(\widetilde{s}_0),$$

so that if $\tilde{A}(\widetilde{s}_0, s_0) < 0$, then $\tilde{c}(\widetilde{s}_0, s_0) > c^*(\widetilde{s}_0)$, a contradiction.

We now show that we cannot have $\tilde{A}(\widetilde{s}, s) > 0$ in any period or state. Suppose that assets carried over from a previous period are positive $\tilde{A}(\widetilde{s}_{-1}, s_{-1}) > 0$. Then, in the current period, the household pays $T(\widetilde{s}) = \sum_i y^{i*}(\widetilde{s})$ as taxes and forfeits $c^*(\widetilde{s})$. The net present value of resources allocated to households would therefore be lower relative to the case where $\tilde{A}(\widetilde{s}_{-1}, s_{-1}) = 0$. Households would therefore choose to accumulate no assets and consume $\tilde{c}(\widetilde{s}, s) = c^*(\widetilde{s})$ $\forall t$ and $\forall s, \widetilde{s}$.

Step 3. Household choices are the same as in the constrained optimum.

Given steps 1 and 2, it follows that when declaring $\widetilde{s}$, households would choose the constrained optimal allocations associated with $\widetilde{s}$. By incentive compatibility constraints (IC), households would therefore reveal their true state and receive the same expected utility as in the constrained optimum. By the design, the government's expected discounted costs would also be the same as in the constrained optimum since $\forall t$ and $\forall s$, $spn - T(s) = c^*(s) + pn - \sum_i y^{i*}(s)$.

Q.E.D.

Model extension The assumption that $w^i > p$ $\forall i$ implies that it is efficient for all healthy household members to devote all of their effort to the labor market. Our model is extendable to cases where some household members have $w^i < p$. For example, if those with market productivity below the minimum wage have no job opportunities, it would be similar to having $w^i = 0$ for those household members. In such cases, it would be efficient for them to devote all of their effort to household child care. Our case for universal day care would still hold. In particular, households where all members are either employed or disabled benefit from free day care. On the other hand, household where there are unemployed non-disabled members benefit from free day care up to the constrained optimum level of formal child care. Formal child care use of households will need to be verifiable in such cases.

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References

Albanesi, S., & Sleet, C. 2006. Dynamic optimal taxation with private information. *Review of Economic Studies*, **73**, 1–30.

Armstrong, M., & Rochet, J-C. 1999. Multi-dimensional screening : A user ' s guide. *European Economic Review*, **43**, 959–979.

Atkeson, A., & Lucas, R. E. 1992. On Efficient Distribution With Private Information. *Review of Economic Studies*, **59**(3), 427–453.

Autor, D., Duggan, M., & Gruber, J. 2014. Moral hazard and claims deterrence in private disability insurance. *American Economic Journal: Applied Economics*, **6**(4), 110–141.

Autor, D. H., & Duggan, M. G. 2006. The growth in the Social Security disability rolls : A fiscal crisis unfolding. *Journal of Economic Perspectives*, **20**(3), 71–96.

Barnett, W. S. 1993. New wine in old bottles: Increasing the coherence of early childhood care and education policy. *Early Childhood Research Quarterly*, **8**(4), 519–558.

Blau, D. 2003. Child care subsidy programs. *In:* Moffitt, R. (ed), *Means-Tested Transfer Programs in the United States*. University of Chicago Press.

Cardia, E., & Ng, S. 2003. Intergenerational time transfers and childcare. *Review of Economic Dynamics*, **6**(2), 431–454.

Center on Budget and Policy Priorities. 2014. *The Earned Income Tax Credit*. Tech. rept.

Chetty, R., Guren, A., Manoli, D., & Weber, A. 2011. Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review P&P*, **101**(3), 471–475.

Child Care Aware of America. 2014. *Child care in america: 2014 state fact sheets*. Tech. rept. Child Care Aware of America.

Compton, J. 2013. Family proximity and the labor force status of women in Canada. *Review of Economics of the Household*, **2007**(Feb.).

Compton, J., & Pollak, R. A. 2013. Family proximity, childcare, and women's labor force attachment. *Journal of Urban Economics*.

Currie, J. 2001. Early childhood education programs. *The Journal of Economic Perspectives*, **15**(2), 213–238.

Department of Health and Human Services. 2012. *Estimates of child care eligibility and receipt for fiscal year 2009*. Tech. rept.

Department of Health and Human Services. 2014. *Child Care and Development Fund*. Tech. rept.

Diamond, P., & Mirrlees, J. A. 1978. A model of social insurance with variable retirement. *Journal of Public Economics*, **10**, 295–336.

Domeij, D., & Klein, P. 2013. Should day care be subsidized? *The Review of Economic Studies*, **80**(2), 568–595.

European Council. 2002. Presidency conclusions. 0–72.

Fernandes, A., & Phelan, C. 2000. A recursive formulation for repeated agency with moral hazard. *Journal of Economic Theory*, **91**, 223–247.

Golosov, M., & Tsyvinski, A. 2006. Designing optimal disability insurance : A case for asset testing. *Journal of Political Economy*, **114**(2), 257–279.

Gruber, J. 2000. Disability insurance benefits and labor supply. *Journal of Political Economy*, **108**(6), 1162–1183.

Guner, N., Kaygusuz, R., & Ventura, G. 2014. Childcare subsidies and household labor supply. *IZA DP No. 8303*.

Haveman, R., Jong, P. D., & Wolfe, B. 1991. Disability transfers and the work decision of older men. *Quarterly Journal of Economics*, **106**, 939–949.

Havnes, T., & Mogstad, M. 2011. Money for nothing? Universal child care and maternal employment. *Journal of Public Economics*, **95**(11-12), 1455–1465.

Havnes, T., & Mogstad, M. 2014. Is universal child care leveling the playing field? *Journal of Public Economics*, May.

Heckman, J., & Cunha, F. 2010. Investing in our young people. *IZA DP No. 5050.*

Ho, C. 2013. Grandchild care, intergenerational transfers and grandparents' labor supply. *Review of Economics of the Household*, **DOI**, 10.1007/s11150–013–9221–x.

Ho, C. 2015. Welfare-to-work reform and intergenerational support: Grandmothers' response to the 1996 PRWORA. *Journal of Marriage and Family*, **77**(April), 407–423.

Ho, C., & Pavoni, N. 2014. Efficient child care subsidies. *Working Paper.*

Hu, J., Lahiri, K., Vaughan, D. R., & Wixon, B. 2001. A structural model of Social Security's disability determination process. *The Review of Economics and Statistics*, **83**(2), 348–361.

Laughlin, L. 2013. *Who's minding the kids ? Child care arrangements : Spring 2011.* Tech. rept. April. U.S. Census Bureau.

Maag, E. 2013. *Perspectives on low-income working families child-related benefits in the federal income tax.* Tech. rept. December. Urban Institute.

Maestas, N., Mullen, K., & Strand, A. 2013. Does disability insurance receipt discourage work? Using examiner assignment to estimate causal effects of SSDI Receipt. *American Economic Review*, **103**(5), 1797–1829.

Marcotte, D., & Wang, Y. 2007. Golden years? The labor market effects of caring for grandchildren. *Journal of Marriage and Family*, **69**(December), 1283–1296.

Maurer-Fazio, M., Connelly, R., Chen, L., & Tang, L. 2011. Childcare, eldercare and labor force participation of married women in urban China, 1982-2000. *Journal of Human Resources*, **46**, 261–294.

Morton, W. R. 2014. *Primer on disability benefits: Social Security Disability Insurance (SSDI) and Supplemental Security Income (SSI)*. Tech. rept. Congressional Research Service.

Pistaferri, L. 2003. Anticipated and unanticipated wage changes, wage risk, and intertemporal labor supply. *Journal of Labor Economics*, **21**(3), 729–754.

Rogerson, W. 1985. The first-order approach to principal-agent problems. *Econometrica*, **53**(6), 1357–1367.

Social Security Administration. 2010. *OASDI and SSI program rates & limits*. Tech. rept. Social Security Administration.

Social Security Administration. 2011a. *Annual statistical report on the Social Security Disability Insurance program, 2010*. Tech. rept. 13.

Social Security Administration. 2011b. *SSI annual statistical report , 2010*. Tech. rept. 13.

Social Security Administration. 2012. *Annual statistical report on the Social Security Disability Insurance program, 2012*. Tech. rept. 13.

Social Security Administration. 2013. *SSI annual statistical report , 2013*. Tech. rept.

Social Security Administration. 2014. *Benefit formula bend points*. Tech. rept.

Tax Policy Center. 2010. *Child and Dependent Care Tax Credit (CDCTC)*. Tech. rept.

Tekin, E. 2007. Childcare subsidies, wages, and employment of single mothers. *Journal of Human Resources*, **42**(2), 453–487.

Weinzierl, M. 2011. The surprising power of age-dependent taxes. *The Review of Economic Studies*, **78**(4), 1490–1518.

Table 1: Household Structure

Household	Adults	Mother	Father	Grandma	Grandpa	Prop.
Parent	Single mother	✔				0.13
	Single father		✔			0.05
	Two parents	✔	✔			0.43
Grandparent	Grandmother			✔		0.04
	Grandparents			✔	✔	0.24
Intergenerational	Grandmother & Father		✔	✔		0.04
	Grandfather & Mother	✔			✔	0.07

Note: Proportions computed from CPS data. Parents are aged 25 to 49 and grandparents from 50 to 74.

Table 2: Proportion with Children Aged below 5

Household	Adults	# Kids	*t*			# Profiles
		< 5	0	1	2	
Parent	Single mother	1 kid	0.45	0.35	0.25	17
		2 kids	0.18	0.09	0.05	
	Single father	1 kid	0.37	0.37	0.25	17
		2 kids	0.17	0.11	0.06	
	Two parents	1 kid	0.40	0.39	0.31	17
		2 kids	0.21	0.21	0.13	
Grandparent	Grandmother	1 kid	0.07	0.07	0.05	4
	Grandparents	1 kid	0.02	0.01	0.01	4
Intergenerational	Grandmother & Father	1 kid	0.07	0.07	0.05	4
	Grandfather & Mother	1 kid	0.24	0.25	0.19	9
		2 kids	0.08	0.08	0.06	

Note: Proportions computed from CPS data.

Table 3: Proportion Disabled

Household	Adults	*t*				
		0	1	2	3	4
Parent	Single mother	0.05	0.07	0.09	0.10	0.12
	Single father	0.02	0.04	0.06	0.09	0.12
	Married mother	0.02	0.02	0.03	0.04	0.05
	Married father	0.03	0.03	0.03	0.03	0.04
Grandparent	Single grandmother	0.15	0.19	0.23	0.23	0.23
	Married grandmother	0.08	0.10	0.13	0.13	0.14
	Married grandfather	0.07	0.11	0.15	0.15	0.16
Intergenerational	Single grandmother	0.10	0.16	0.23	0.23	0.23
	Single father	0.09	0.12	0.15	0.15	0.15
	Single grandfather	0.07	0.11	0.18	0.18	0.19
	Single mother	0.06	0.06	0.06	0.06	0.06

Note: Proportions computed from CPS data.

Table 4: Hourly Wages

Household	Adults	Education	*t*				
			0	1	2	3	4
Parent	Single mother	High School	12.04	12.86	13.85	14.11	14.65
		College	15.80	18.13	20.59	22.14	23.74
	Single father	High School	15.47	16.37	18.30	19.03	19.39
		College	20.13	23.28	25.49	27.48	28.78
	Married mother	High School	13.04	14.00	14.95	15.16	15.23
		College	19.94	23.71	25.42	25.01	24.71
	Married father	High School	16.78	18.78	20.30	21.15	21.76
		College	23.67	28.81	32.38	33.94	34.54
Grandparent	Single grandmother	High School	14.84	14.47	14.03	-	-
		College	24.26	24.22	23.33	-	-
	Married grandmother	High School	15.89	16.22	15.62	-	-
		College	25.22	25.10	24.30	-	-
	Married grandfather	High School	22.39	21.87	21.16	-	-
		College	34.40	33.24	32.23	-	-
Intergenerational	Single grandmother	High School	15.22	15.04	13.77	-	-
		College	24.19	22.95	22.91	-	-
	Single father	High School	13.47	14.78	16.03	18.04	20.99
		College	17.85	19.78	24.49	26.71	32.26
	Single grandfather	High School	21.73	21.05	20.94	-	-
		College	33.77	33.11	33.07	-	-
	Single mother	High School	11.99	12.54	13.98	14.93	15.61
		College	16.85	18.72	23.55	23.99	25.06

Note: Hourly wage in 2010 dollars computed by dividing gross earnings by hours of work from CPS data. We take the mean across adults for each household structure and education level. Grandparents are retired in periods 3 and 4.

Table 5: 2010 US Tax and Benefit System

Federal Income Tax Rates [a]

Tax rate	Taxable income		
	Single	Head	Married
10%	Less than $8,375	Less than $11,950	Less than $16,750
15%	$8,375 - $34,000	$11,950 - $45,550	$16,750 - $68,000
25%	$34,000 - $82,400	$45,550 - $117,650	$68,000 - $137,300
28%	$82,400 - $171,850	$117,650 - $190,550	$137,300 - $209,250
33%	$171,850 - $373,650	$190,550 - $373,650	$209,250 - $373,650
35%	$373,650 and above	$373,650 and above	$373,650 and above

EITC [b]

# Children below 18	All Filing Status			Single and Head		Married	
	Phase-in rate	Maximum Credit	Phase-out rate	Phase-out Income	Income Limit	Phase-out Income	Income Limit
0	7.65%	$457	7.65%	$7,480	$13,460	$12,480	$18,470
1	34%	$3,050	15.98%	$16,450	$35,535	$21,460	$40,545
2	40%	$5,036	21.06%	$16,450	$40,363	$21,460	$45,373
3 or more	45%	$5,666	21.06%	$16,450	$43,352	$21,460	$48,362

Poverty Thresholds [c]

Size of family unit	No. of Children below 18		
	1	2	3
Two people	$15,030		
Three people	$17,552	$17,568	
Four people	$22,859	$22,113	$22,190
Five people	$27,518	$26,675	$26,023

Sources:

[a] http://www.moneychimp.com.

[b] Historical Earned Income Tax Credit Parameters, Tax Policy Center. Phase-out income for married filing jointly status computed by author based on phase-out rate and income

[c] U.S. Census Bureau.

Table 6: Effort Cost Parameter

Household	Adults	Hours		α
		Data	Model	
Parent	Single mother	39.86	39.98	0.58
	Single father	41.54	41.56	0.53
	Married mother	39.63	38.30	0.39
	Married father	43.67	44.20	0.32
Grandparent	Single grandmother	37.27	37.22	0.72
	Married grandmother	37.04	39.15	0.32
	Married grandfather	42.37	43.96	0.34
Intergenerational	Single grandmother	38.73	38.72	0.40
	Single father	41.48	41.34	0.30
	Single grandfather	43.69	43.74	0.33
	Single mother	39.24	39.23	0.35

Note: α calibrated to match average weekly hours of work of working adults in the CPS averaged over time periods.

Table 7: Cost Savings

Household	Adults	High School			College		
		1 kid	2 kids	3 kids	1 kid	2 kids	3 kids
Parent	Single mother	2.20%	1.88%	1.65%	0.23%	0.54%	1.56%
	Single father	0.18%	0.57%	1.23%	0.10%	0.14%	0.41%
	Two parents	0.11%	0.29%	0.80%	0.05%	0.07%	0.15%
Grandparent	Grandmother	0.27%	-	-	0.34%	-	-
	Grandparents	0.20%	-	-	0.36%	-	-
Intergenerational	Grandmother & Father	0.59%	-	-	2.59%	-	-
	Grandfather & Mother	0.27%	2.03%	-	0.24%	3.31%	-

Note: Cost savings are averaged over child arrival profiles. When there are two household members, we report cost savings for cases where both members have the same level of education.

Table 8: Sensitivity Analysis on Cost Savings

Household	Adults	High School			College		
		1 kid	2 kids	3 kids	1 kid	2 kids	3 kids
Parent	Single mother	1.77%	2.23%	2.93%	0.34%	1.01%	2.22%
	Single father	0.26%	0.67%	1.68%	0.15%	0.32%	0.53%
	Two parents	0.11%	0.25%	0.66%	0.06%	0.33%	0.45%
Grandparent	Grandmother	0.27%	-	-	0.45%	-	-
	Grandparents	0.68%	-	-	0.47%	-	-
Intergenerational	Grandmother & Father	0.62%	-	-	13.7%	-	-
	Grandfather & Mother	0.63%	1.18%	-	0.40%	3.80%	-

Note: Cost savings are computed as in Table 7 but with p = $5 per hour, corresponding to a cost of $10,000 per year for a child in full time day care.

Figure 1: Parent Households Labor Supply

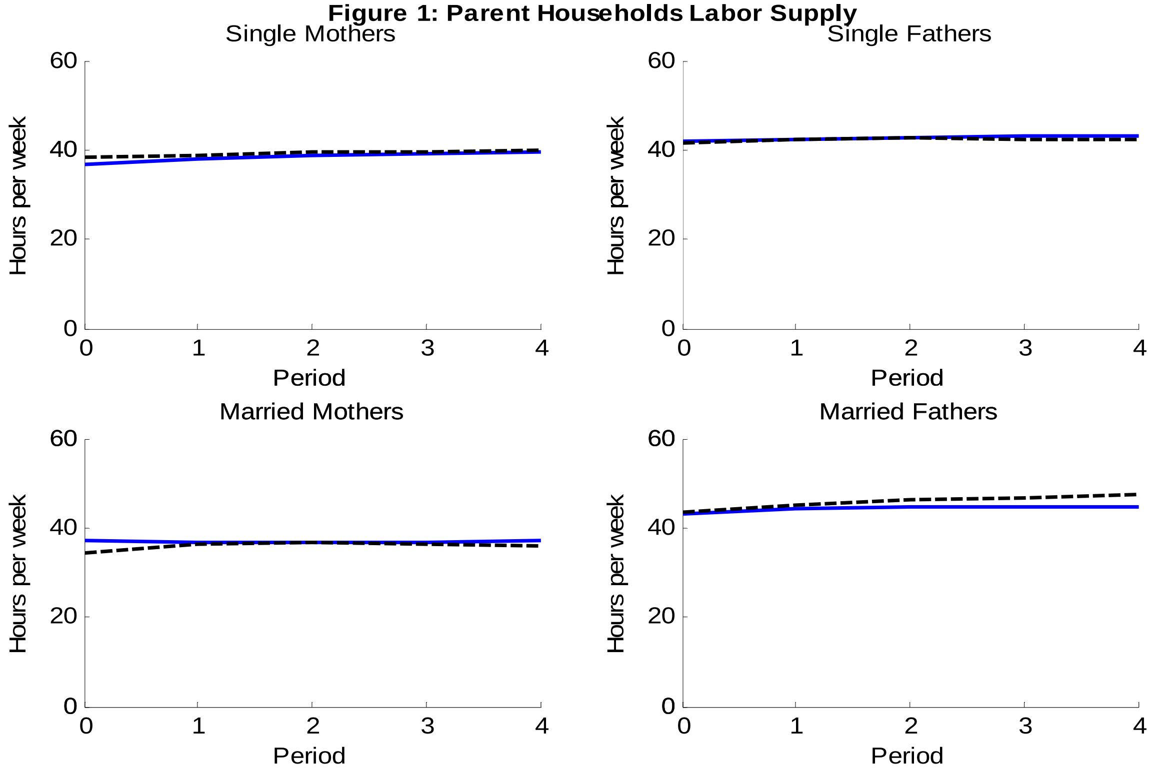
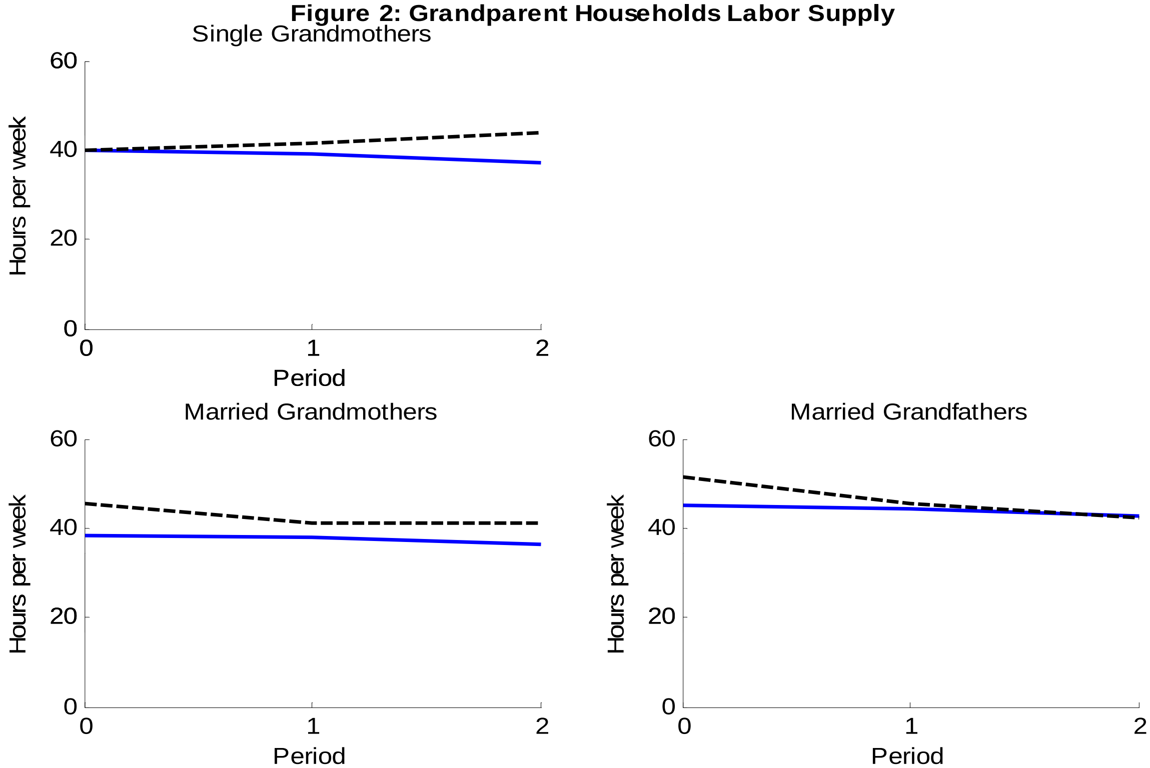


Figure 2: Grandparent Households Labor Supply

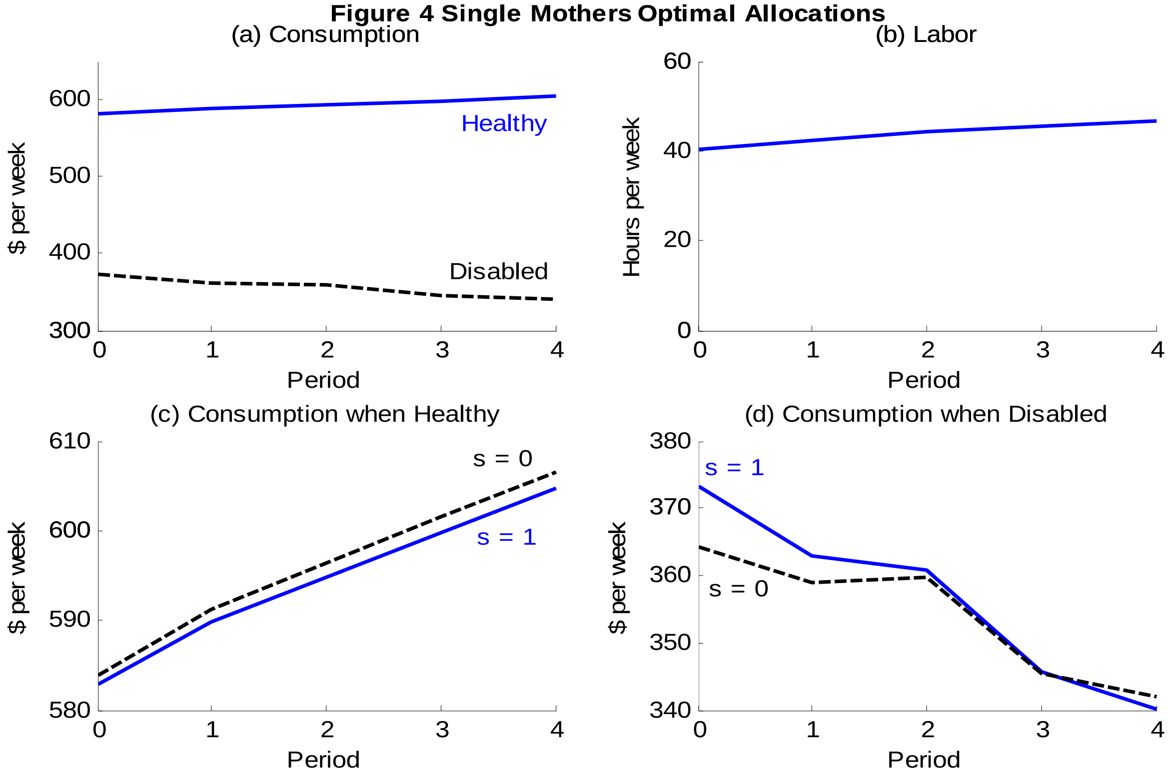


Note: Solid lines represent average hours from CPS data and dashed lines represent average predicted hours from the model.

Figure 3: Intergenerational Households Labor Supply

Note: Solid lines represent average hours from CPS data and dashed lines represent average predicted hours from the model.

Figure 4 Single Mothers Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with (s = 1) and without (s = 0) universal day care, averaged among households with children.

Figure 5 Single Fathers Optimal Allocations

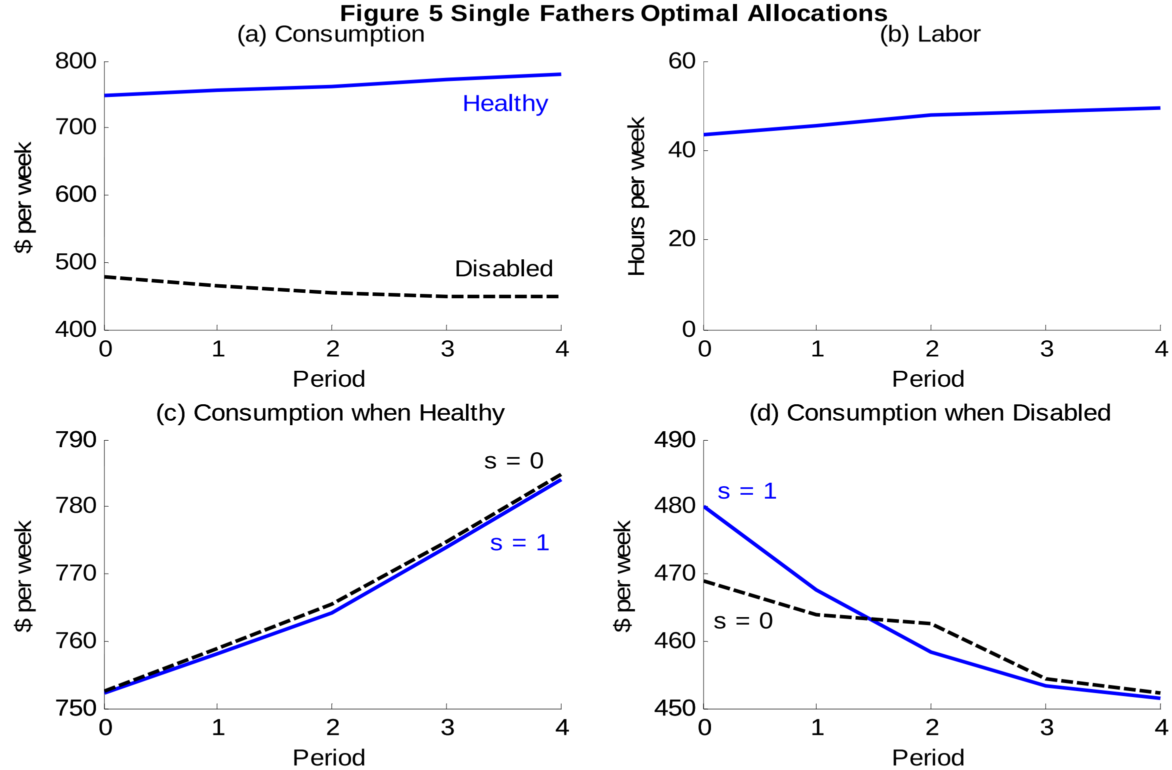
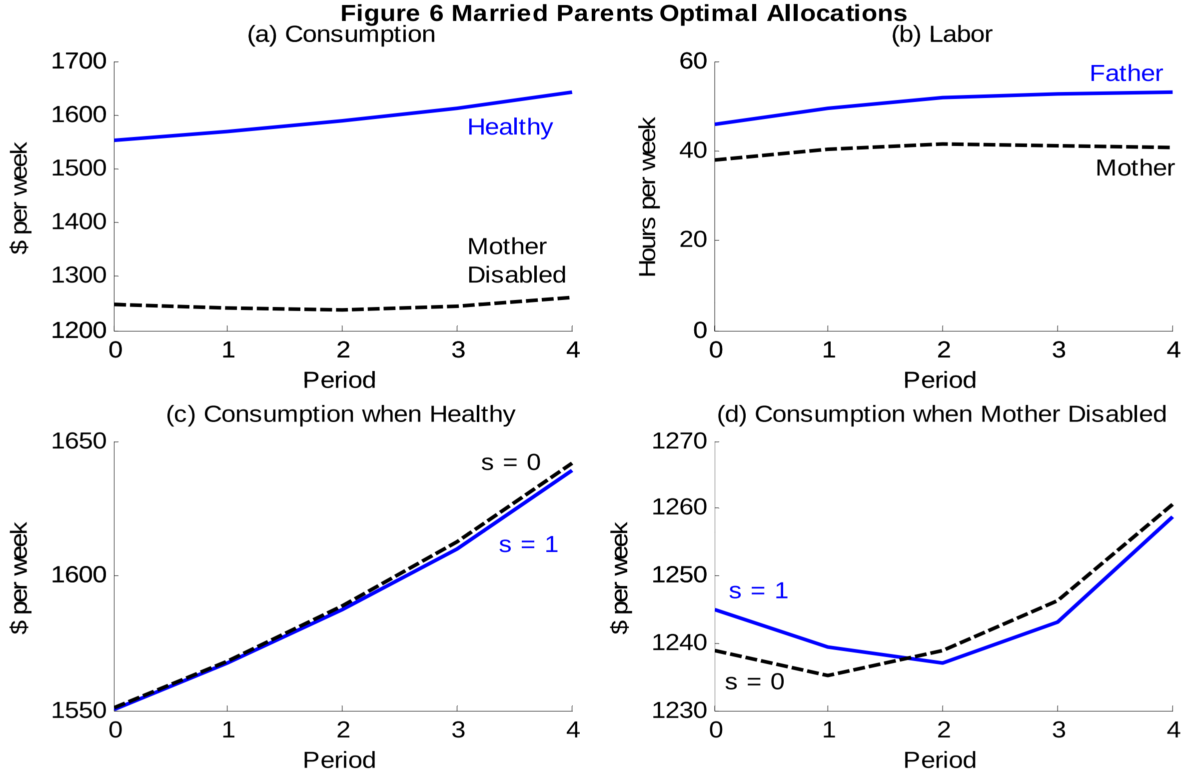


Figure 6 Married Parents Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with (s = 1) and without (s = 0) universal day care, averaged among households with children.

Figure 7 Single Grandmothers Optimal Allocations

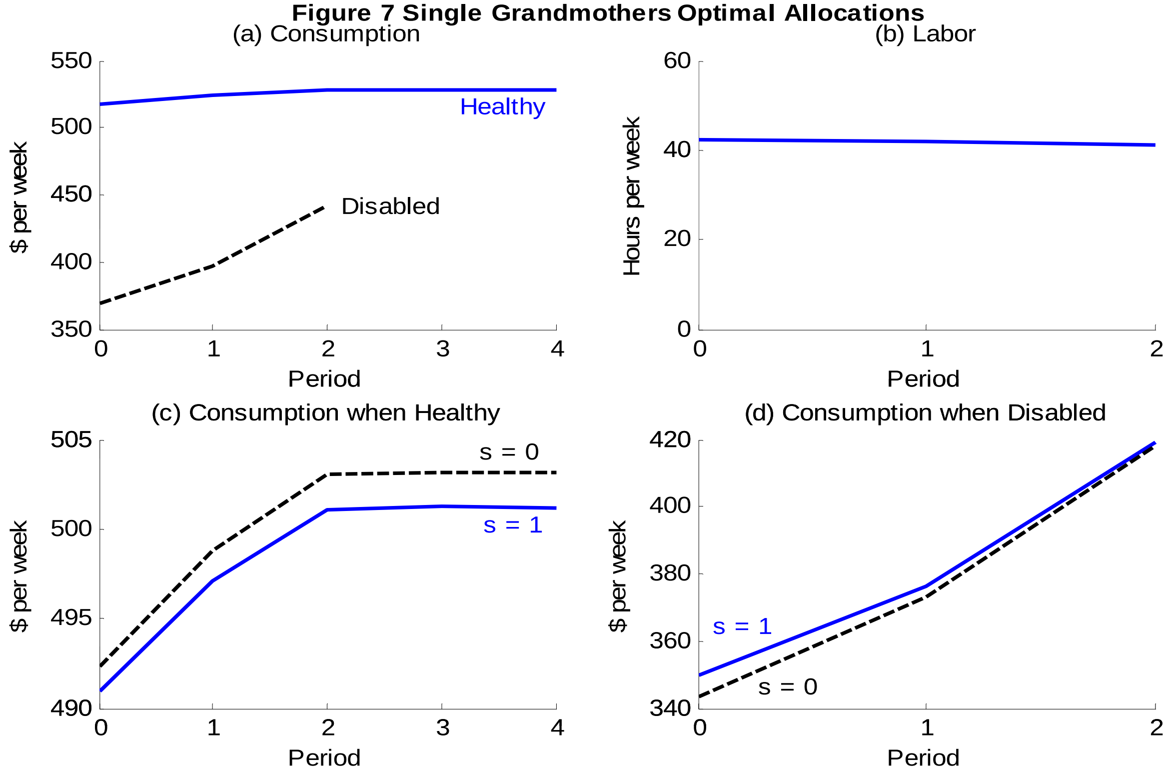
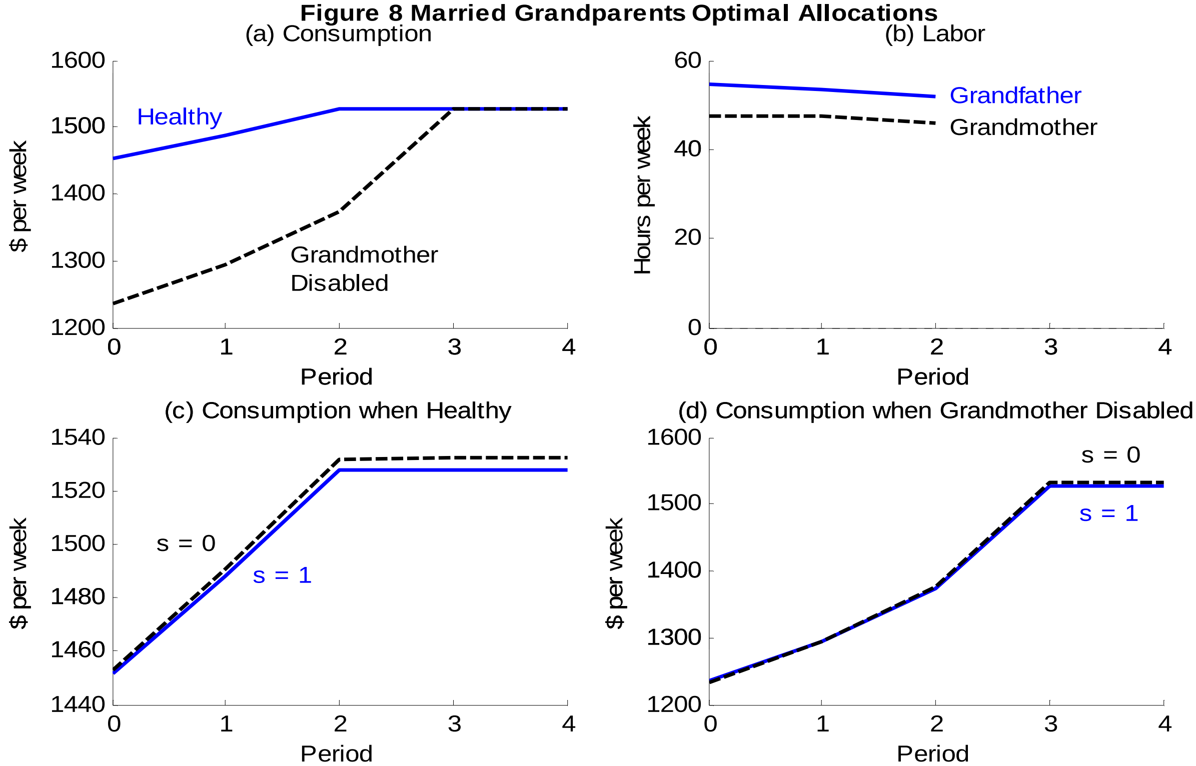


Figure 8 Married Grandparents Optimal Allocations



Note: Top panels report optimal allocations averaged over all households. Bottom panels report optimal allocations with (s = 1) and without (s = 0) universal day care, averaged among households with children. Grandparents are retired in periods 3 and 4. Single grandmothers have zero probability of becoming disabled in period 3 and 4 if they were previously healthy.

Figure 9 Grandmother and Father Optimal Allocations

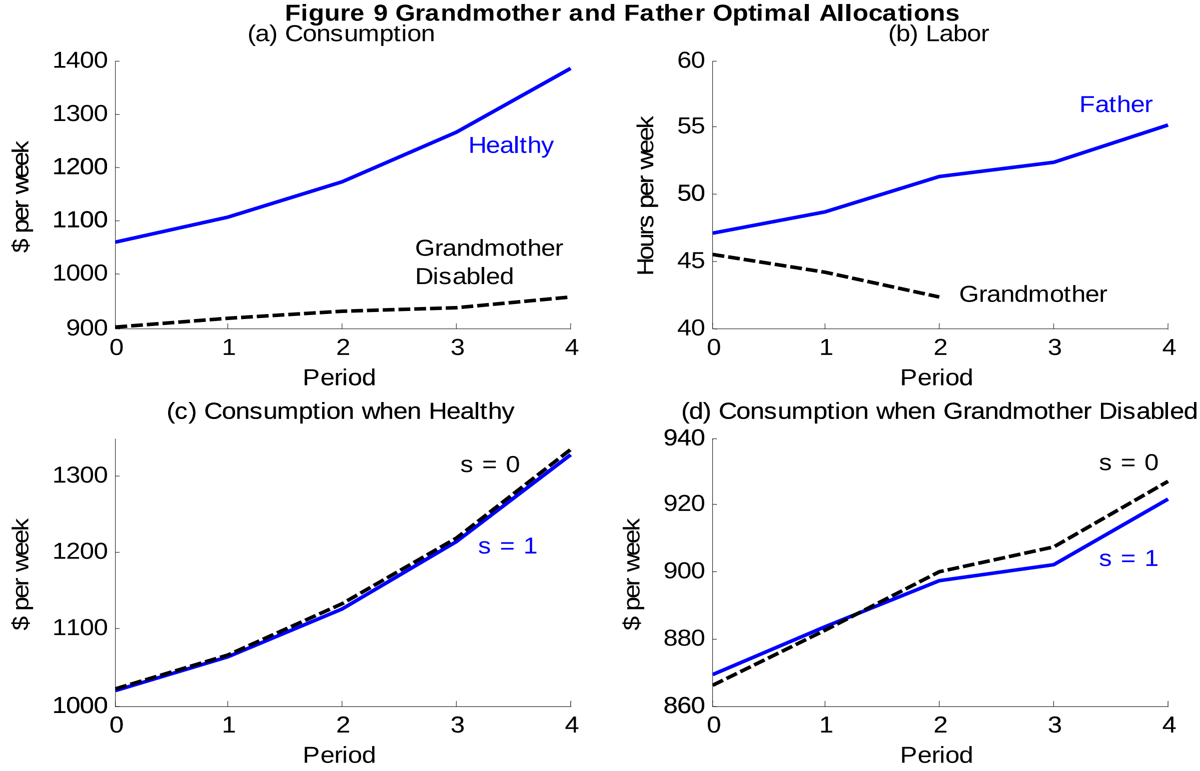
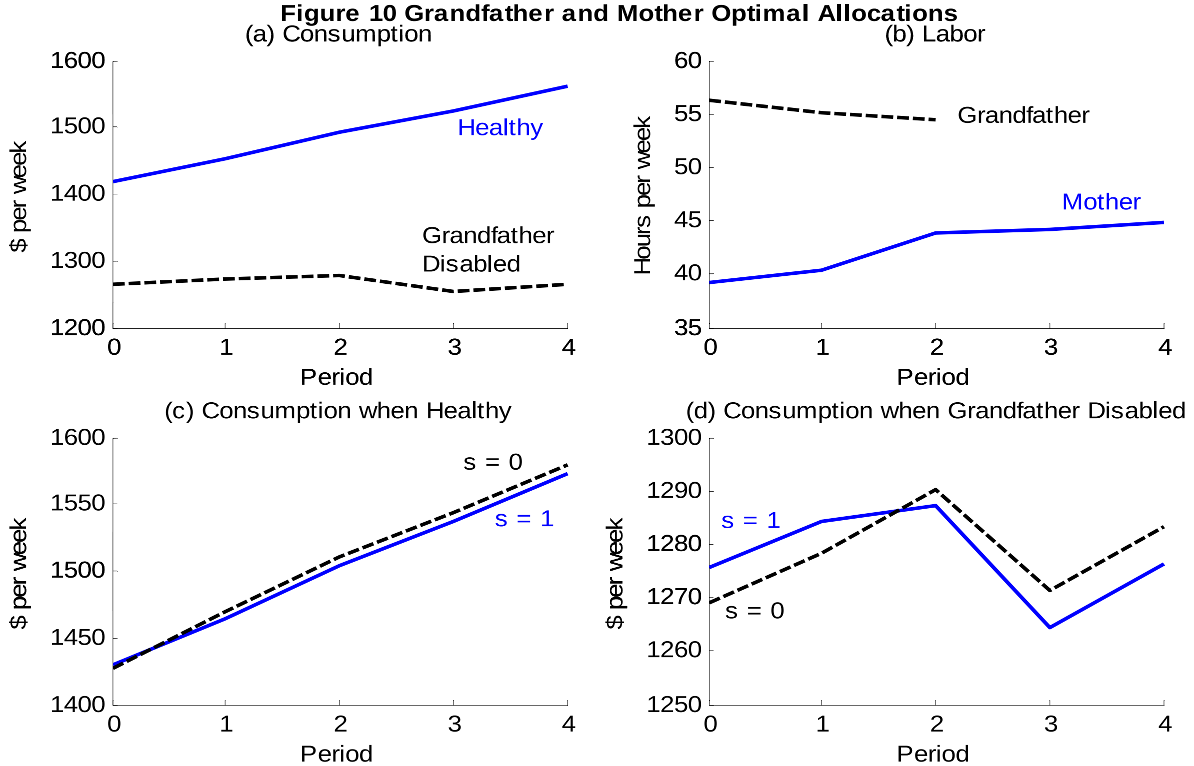


Figure 10 Grandfather and Mother Optimal Allocations



Note: Top panels report optimal allocations averaged over all households and bottom panels report optimal allocations averaged among households with children.